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TATE'S
COMMERCIAL ARITHMETIC.

A COMPLETE SYSTEM
OF
COMMERCIAL ARITHMETIC,
ITS APPLICATION TO THE PRINCIPAL BRANCHES OF COMMERCE
ACCORDING TO
THE EXISTING PRACTICES OF TRADE,
AND NUMEROUS RULES FOR FACILITATING AND PERFORMING
MENTAL CALCULATIONS:
With an Appendix,
EXHIBITING THE METHODS EMPLOYED BY MERCHANTS, BANKERS, AND BROKERS,
FOR THE VALUATIONS OF MERCHANDISE,
MENTAL PER CENTAGES,
INTEREST ACCOUNTS IN ACCOUNTS-CURRENT,
THE PUBLIC FUNDS AND MARINE INSURANCES,
THE STANDARDING OF GOLD AND SILVER,
AND THE
ARBITRATIONS OF EXCHANGE IN BILLS, BULLION, AND MERCHANDISE,
With a Variety of Actual and Pro-formâ Statements of
BRITISH AND FOREIGN INVOICES AND ACCOUNT-SALES.

BY W. TATE, JUN.

Third Edition.

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PREFACE.

The object of the work, now submitted to the public, is, partly to exhibit the theory of Arithmetic freed from several errors which have greatly obscured its principles, and, partly to show its application to the most useful branches of the calculations of Commerce, divested of imaginary statements of the practices of business.

After this exposition of my views, very few remarks seem necessary to be made. It will be seen that the different subjects of this treatise, have been taken in that succession which has been followed by the best writers, without making any attempt to introduce a more scientific arrangement, or one

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that would preclude any reference to subsequent principles. Such a plan could, indeed, not be adopted, without destroying that classification of the Rules, which is so highly useful for the sake of reference, and it has therefore been left to the judgment of the intelligent teacher, to make those selections which he may find best adapted to the capacities, the previous knowledge, and the objects of his pupils.

It has been in deference to these precedents, that the doctrine of fractions and decimals, which should properly follow the theory of whole numbers, has here been placed after the course of those calculations which are required for the common purposes of Commerce; but this does not preclude their being used at any period of the pupil's practice, the earlier the better, because many of the previous exercises can be rendered more easy and concise by the employment of the decimal instead of the usual separations of the integer, and to clearly understand the principles of many operations that may almost be considered but as elementary, requires some knowledge of the nature and modes of using ordinary fractions.

In the arrangement of those branches of calculations which are dependent on mercantile usages, the principle here adopted, is, to place those first which are either the most useful or the most easy: consequently, those which particularly belong to the higher departments of the Counting-house, as well as those which are difficult to be surmounted, have been selected, and formed into an Appendix, that may be employed either in continuation of this work, or as a supplement to any other system of Arithmetic.

As it is more by the exercise of his own judgment, than by a rigid adherence to general rules, that any one can acquire great facility in calculations, advantage has been taken of every opportunity to show the learner the best means by which it can be directed; and as a powerful auxiliary, mental arithmetic has received great attention, as will be found from a variety of new rules and observations, which have been interspersed throughout this treatise.

In conclusion, it is incumbent on me to state, in support of my claims to public confidence, that in all subjects relating to Commerce, very valuable assistance has been afforded me by several gentlemen of great experience in business, and no pains have been spared to procure whatever information might contribute to insure the work a favourable reception.

W. TATE, Junr.

Commercial Academy,
Well Court, Queen Street,
London.

P. S. In a Key, which will shortly be published, further explanations of the best methods of working the more difficult calculations will be given, as well as the routine of business which directs their application.

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ALGEBRAIC CHARACTERS USED IN ARITHMETICAL OPERATIONS.

char.	names.	
$+$	plus	The sign of Addition, and also of excess: thus, $5 + 2$ denotes that 2 is added to 5; also, $5 +$ means that there is excess in the result beyond 5, or that the result is more than 5.
$-$	minus	The sign of Subtraction, and also of deficiency: thus, $5 - 2$ denotes that 2 is taken from 5; also, $5 -$ means, that the result is less than 5.
$\times$	into	The sign of Multiplication: as, 5×4 means that 5 is multiplied by 4.
$\div$	by	The sign of Division: as, $5 \div 2$ means that 5 is divided by 2.
$=$	equal to	The sign of Equality: as, $4 + 3 = 11 - 4 = 7$ means that the sum of 4 and 3 is equal to the difference of 11 and 4, or that each is equal to 7.

ARITHMETIC.

Arithmetic is the science of Numbers.

Numbers are applied to express either the repetitions or the parts of an object or a quantity, and according to these applications they are called either whole or fractional.

A single object is sometimes called unity with reference to several individuals of the same sort; as one man with reference to several men.

A single dimension of value, weight, measure, &c. is called an integer, or an integral quantity; as one pound, one yard.

A simple quantity is a dimension of value, &c. expressed in only one denomination; as seven pounds, or five yards.

A compound quantity is a dimension expressed in more denominations than one; as seven pounds, five shillings, and sixpence, when used to express one sum of money.

NUMERATION.

Numeration is the valuation of numbers expressed by figures.

There are 9 figures, and to fill up a vacant place, a cipher, also called a nought, is used.

Numbers are divided into classes and periods; there are 6 figures in a class, and 3 in a period.

The figures in the first period are called units, tens, and hundreds; those in the second period are called thousands, ten thousands, and hundred thousands.

The first class is called the class of units, the second that of millions, and the third that of billions; therefore if a number expressed at length has not more than 6 figures, it will not contain millions; and if it has not more than 12 figures, it will not contain billions.

Table of the first three periods.

Hundreds	Tens	Millions	Hundreds	Tens	Thousands	Hundreds	Tens	Units
9	8	7,	6	5	4.	3	2	1

In this number 9 8 7 6 5 4 3 2 1, the 9 is hundred-millions, the 8 is ten-millions, the 7 is millions, the 6 is hundred-thousands, the 5 is ten-thousands, the 4 is thousands, the 3 is hundreds, the 2 is tens, and the 1 is a unit.—The whole is read,

Nine hundred and eighty seven millions, six hundred and fifty four thousand, three hundred and twenty one.

Write down in words at length,

17
25
59
172
616

707
1642
2930
8046
9007

27 482
30 303
67 890
70 007
64 000

7 367 424
8 968 700
27 410 869
314 828 416
909 262 900

On the reverse,

Write down in figures,

Seventeen
Twenty four
One hundred and fifty five
Two thousand, six hundred and twenty seven
Twenty five thousand, eight hundred and ninety two
One hundred and seventeen millions, two hundred and sixty five thousand, one hundred and five.

ROMAN NUMERALS.

The following table exhibits this mode of expressing all the units, tens, and hundreds.

1	I	10	X	100	C
2	II	20	XX	200	CC
3	III	30	XXX	300	CCC
4	IV	40	XL	400	CD
5	V	50	L	500	D
6	VI	60	LX	600	DC
7	VII	70	LXX	700	DCC
8	VIII	80	LXXX	800	DCCC
9	IX	90	XC	900	CM
		1000	M		

SIMPLE ADDITION.

Simple Addition is finding the amount of two or more quantities of the same denomination, expressed by whole numbers.

Rule. Place the numbers so that the figures of the same class may be one under the other.

Then, beginning with the units, find the amount of each column of figures, put down the unit figure of each total, and carry the tens to the next column, except for the last, of which set down the whole of the amount.

Proof. Separate the numbers into two or more parts, find the amount of each, and add these amounts together.

Or, reverse the addition; that is, if the work is begun with the bottom figure, let the proof begin with the top figure.

Example.

To add together the following numbers.

Work.	£	Proof.
9 and 2 are 11	6748	Cut off the top line;
and 4 are 15	———	add the other lines
and 4 are 19	8464	together; amount
and 8 are 27;	5074	23039;
set down 7	2372	add the top line
and carry 2	7129	6748;
to the next row;	———	amount as at first
2 and 2 are 4	29787	29787.
and 7 are 11,	———	
and so on.	23039	
	———	
	29787	

N. B. It was formerly the custom to place a dot against every ten found in the addition of each column; and then rejecting the ten thus formed, to add in only the units; as 9 and 2 are 11, dot and go 1; 1 and 4 are 5 and 4 are 9 and 8 are 17, dot and go 7; then putting down 7, carry 2 to the next row because there are 2 dots.

ADDITION TABLE.

1 and	2 and	3 and	4 and	5 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6
2 .. 3	2 .. 4	2 .. 5	2 .. 6	2 .. 7
3 .. 4	3 .. 5	3 .. 6	3 .. 7	3 .. 8
4 .. 5	4 .. 6	4 .. 7	4 .. 8	4 .. 9
5 .. 6	5 .. 7	5 .. 8	5 .. 9	5 .. 10
6 .. 7	6 .. 8	6 .. 9	6 .. 10	6 .. 11
7 .. 8	7 .. 9	7 .. 10	7 .. 11	7 .. 12
8 .. 9	8 .. 10	8 .. 11	8 .. 12	8 .. 13
9 .. 10	9 .. 11	9 .. 12	9 .. 13	9 .. 14

6 and	7 and	8 and	9 and
1 are 7	1 are 8	1 are 9	1 are 10
2 . . . 8	2 . . . 9	2 . . . 10	2 . . . 11
3 . . . 9	3 . . . 10	3 . . . 11	3 . . . 12
4 . . . 10	4 . . . 11	4 . . . 12	4 . . . 13
5 . . . 11	5 . . . 12	5 . . . 13	5 . . . 14
6 . . . 12	6 . . . 13	6 . . . 14	6 . . . 15
7 . . . 13	7 . . . 14	7 . . . 15	7 . . . 16
8 . . . 14	8 . . . 15	8 . . . 16	8 . . . 17
9 . . . 15	9 . . . 16	9 . . . 17	9 . . . 18

EXERCISES.

£	£	£	£	£	Days.
6212	7183	9268	4484	4809	3164
7344	6404	4487	5573	4295	7276
8162	7328	5165	6731	5582	3152

Pence.	Years.	Miles.	Tons.	Pounds.
4816	5555	418764	228267	776584
4816	4444	83490	36392	776584
4816	6666	725124	4717	776584
4816	7777	44953	626	776584
4816	8888	371546	717	776584
4816	9999	53484	9464	776584

SIMPLE SUBTRACTION.

Simple Subtraction is finding the difference between two simple quantities of the same denomination, expressed by whole numbers.

RULE.

Place the less number under the greater, with the figures of the same class under each other, as in Addition.

Then, begin with the lower unit figure, (and so proceed with the rest,) and find the difference between that and the figure above it, when the lower figure is not the greater; but when the lower figure is greater, borrow 1 from the next column, call it 10, add it to the upper figure, subtract the lower figure from the amount, set down the remainder, and carry one to the next lower figure.

Proof. Add the remainder to the lower line.

EXAMPLE.

To subtract 6725 from 8457.*

5 from 7 leaves 2	8457
2 from 5 leaves 3	6725
7 from 4 cannot be taken	——
borrow 1, 7 from 14	1732
leaves 7 and carry 1 to 6,	——
6 and 1 are 7 from 8 leaves 1.	8457 Proof.
Proof. Add 6725 to 1732.	——

EXERCISES.

6748	72873	34254	670203
2416	10621	17182	81724
——	——	——	——
——	——	——	——

---

* Note. In the Examples and Exercises to this and the following Rules on simple quantities, any denomination that is preferred may be given.

SIMPLE MULTIPLICATION.

Simple Multiplication is a contraction of simple Addition, when the numbers to be added are repetitions of the same number.

The multiplicand is the quantity given to be multiplied.

The multiplier denotes how many times the multiplicand is to be taken, or how many times it would be expressed if the calculation were performed by Addition.

The product is the quantity produced by the multiplication, and is always of the same denomination as the multiplicand.

N. B. The product is sometimes called a multiple of the given quantity; as 20 pence are called a *multiple* of 5 pence, being 4 times 5 pence; so on the reverse, the quantity multiplied is called a *part* of the product; as 5 pence are called a part of 20 pence.

Multiplication is called *short*, when it is performed in one line; *double*, when the product of a product is taken; and *long*, when the whole product is formed by the addition of two or more products.

Rule. Beginning with the unit figure, take the product of each figure of the multiplicand, and set down the unit figure and carry the tens as in addition.

In long multiplications be careful to put the unit figure of each separate product under the multiplying figure to which it belongs.

Proof. The usual proof of a long multiplication is to reverse it, making the multiplier the multiplicand, and the multiplicand the multiplier, otherwise the best proof of multiplication is division.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

In reference to the product, the numbers of the multiplicand and multiplier are sometimes called *factors;* as, 5 and 7 are called factors of 35; and 2 and 18, 3 and 12, 4 and 9, 6 and 6, are called factors of 36.

EXAMPLE 1.

To multiply 437 by 7

437
7
3059

7 times 7 are 49, set down 9 and carry 4; 7 times 3 are 21 and 4 are 25, set down 5 and carry 2; 7 times 4 are 28 and 2 are 30, set down 30.

EXAMPLE 2.

To multiply 8437 by 10, by 100, and by 20

84370 product by 10
843700 - - - 100

The multiplication by 10 is performed by placing 1 cipher on the right of the number to be multiplied; and by 100, by placing 2 ciphers.

8437
20
168740 product by 20

This is the most convenient form of multiplying by a number which has one or more ciphers on the right hand, that is by a number of tens or hundreds, &c. but the following form is in general better understood by beginners.

84370 10 times 8437.
2
168740 20 times.

EXAMPLE 3.

To multiply 8437 by 21 and 204

	8437	8437	
	3	204	
3 times	25311	33748	4 times
	7	16874	200 times
21 times	177177	1721148	204 times

Exercises.

674626	183467	463076	544879
2	2	3	4
317104	448756	220704	662482
5	6	7	8
425726	127246	202162	314203
9	10	11	12

Ex. 13.	44821	by	10	Ex. 17.	41672	by	20
14.	21462	-	100	18.	20342	-	30
15.	11072	-	100	19.	36120	-	500
16.	23450	-	1000	20.	44643	-	700

Double Multiplications.

Ex. 21.	44867	by	22	Ex. 25.	35726	by	42
22.	51543	-	24	26.	16871	-	63
23.	23762	-	25	27.	29579	-	84
24.	17124	-	36	28.	91627	-	144

Long Multiplications.

Ex. 29.	54267	by	37	Ex. 34.	80400	by	85
30.	38729	-	82	35.	73100	-	412
31.	46581	-	117	36.	82000	-	670
32.	37975	-	251	37.	76500	-	620
33.	62682	-	370	38.	41400	-	710

Products.

1.	1349252	2.	366934	3.	1389228	4.	2179516
5.	1585520	6.	2692536	7.	1544928	8.	5299856
9.	3831534	10.	1272460	11.	2223782	12.	3770436
13.	448210	14.	2146200	15.	1107200	16.	23450000
17.	833440	18.	610260	19.	18060000	20.	31250100
21.	987074	22.	1237032	23.	594050	24.	616464
25.	1500492	26.	1062873	27.	2484636	28.	13194288
29.	2007879	30.	3175778	31.	5449977	32.	9531725
33.	23192340	34.	6834000	35.	30117200		
36.	54940000	37.	47430000	38.	29394000		

EXAMPLES OF THE CONTRACTION OF LONG MULTIPLICATION.*

Ex. 1. To multiply 6375 by 17 and by 23.

6375	6375
17	23
108375	146625

In the first calculation, for ten times, we take in at once the back figure to each product, except the first; and in the second for twenty times, we take in twice the back figure.

Thus: 7 times 5 are 35, 5 and carry 3; 7 times 7 are 49 and 3 are 52 and 5 (the back figure) are 57, 7 and carry 5; 7 times 3 are 21 and 5 are 26 and 7 (the back figure) are 33, 3 and carry 3; 7 times 6 are 42 and 3 are 45 and 3 (the back figure) are 48, 8 and carry 4; 4 and 6 (the back figure) are 10.

In the second we say, 3 times 5 are 15, 5 and carry 1; 3 times 7 are 21 and 1 are 22 and 10 (for twice 5) are 32, &c.

In a similar manner we may take in 3 times the back figure for 30, &c. but unless there are only a few figures this method is very liable to occasion error.

When there are only two figures concerned in each factor, the multiplication may always be done in one line; thus,

Ex. 2. To multiply 62 by 46 and 307 by 85.

62	307
46	85
2852	26095

Here we say 6 times 2 are 12, put down 2 and carry 1; 4 times 2 are 8 and 1 are 9, carry 9; 6 times 6 are 36 and 9 are 45, put down 5 and carry 4; 4 times 6 are 24 and 4 are 28.

Also 5 times 7 are 35, put down 5 and carry 3; 8 times 7 are 56 and 3 are 59, put down 9 and carry 5; 5 times 3 are 15 and 5 are 20, put down 0 and carry 2; 8 times 3 are 24 and 2 are 26.

---

* To be omitted by the learner until he has made some proficiency in Arithmetic.

When the multiplier is nearly 100 or 1000, we may multiply by 100 or 1000, and add or subtract the difference.

Ex. 3. To multiply 6247 by 99 and by 997

	624700	100 times	6247000	1000 times
Subtract	6247	once	18741	3 times
	618453	99 times	6228259	997 times.

If the multiplier had been 101 or 1003, it is evident that we should have used addition instead of subtraction.

Sometimes two of the left hand figures of a long multiplier are the product of a lower figure, in this case we can take the product by the lower figure: as in the following example.

Ex. 4. To multiply 8365 by 246

	8365	
	246	
6 times	50190	
240 times	200760	(6 times multiplied by 4)
	2057790	

Exercises.

Multiply,

Ex. 39.	4827	by	13		Ex. 42.	5381	by	16
40.	5583	by	14		43.	2352	by	17
41.	9017	by	15		44.	4516	by	18
Ex. 45.	81	by	37		Ex. 47.	303	by	61
46.	67	by	53		48.	705	by	73
Ex. 49.	5374	by	99		Ex. 51.	5262	by	98
50.	4156	by	999		52.	3351	by	996
Ex. 53.	6291	by	328		Ex. 55.	3136	by	549
54.	3672	by	427		56.	2917	by	728

Products.

62751 78162 135255 86096 39984 81288
2997 3551 18483 51465
532026 4151844 515676 3337596
2063448 1567944 1721664 2123576

Sometimes when the unit figure is 1, the following form of multiplication may be used with advantage.

Example 5. To multiply 7462 by 21.

7462 by 21
14924
156702

In which, putting the multiplier by the side, we let the multiplicand stand for once, and take the product by 2 for 20 times.

When the figure of tens is 1, as in the first example of these contractions, the multiplication may be made by the units, and placed one figure to the right.

Example 6. To multiply 6375 by 17.

6375
44625 for 7 times
108375

In this form, the first line is considered as a number of tens, and represents 10 times.

In some cases of multiplication by numbers between 100 and 112, or between 1000 and 1012, the same principle may be acted upon, by extending the first products 2 or 3 places to the right, according as the first line is intended to stand for 100 or 1000 times.

Example 7. To multiply 673 by 108, 112, 1004, and 1012.

673.. by 108
5384 8 times
72684

673.. by 112
8076 12 times
75376

673... by 1008
5384 8 times
678384

673... by 1012
8076 12 times
681076

Exercises.

Ex. 57. Multiply 4272 by 21, 41, and 61.
58. Multiply 3067 by 13, 17, and 19.
59. Multiply 2142 by 104 and 112.
60. Multiply 3343 by 1005 and 1012.

Products.

Ex. 57. 89712-175152-260692. Ex. 59. 222768- 239904.
58. ~~3971~~- 52139- 58273. 60. 3359715-3383116.
39871

SIMPLE DIVISION.

Simple Division is finding the value or amount of a part of a simple quantity. It is the reverse of Multiplication, for its object is to find a quantity, the product of which multiplied by a given number, will either be equal to a given simple quantity, or be the greatest product that can be obtained out of it.

The Dividend is the quantity to be divided.

The Divisor is the number by which the division is made, and shows the part which the quotient is to be of the dividend.

The Quotient is the product sought by the division, and is always of the same denomination as the dividend.

The Remainder is the excess of the dividend above the product of the multiplication of the quotient by the divisor,[*] when the dividend is greater than the product of any simple quantity multiplied by the given divisor.

Rule. Find how often the divisor is contained in the same number (or in one more if it is not contained in the same number) of the highest figures of the dividend; multiply the divisor by this number, subtract the product, and to the remainder bring down the next higher figure of the dividend; then find again how many times the divisor is contained in this increased remainder, and so continue the division until all the figures of the dividend are used; observing, that when the bringing down of one figure from the dividend is not sufficient to make the remainder equal to the divisor, one, two, or more extra figures are to be brought down, but then as many ciphers as extra figures must be placed in the quotient.

Observe, also, that when the quotient figure is right, its product will not be greater than the number from which it is to be taken, and the remainder will not be greater than the divisor.—When there are ciphers in the divisor, cut off both these ciphers and the same number of the right hand figures of the dividend.

The best Proof of division is multiplication, multiplying the quotient by the divisor, and adding in the remainder, if any.

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[*] The Remainder may be considered, either as a deficiency of the product of the quotient and the divisor in comparison with the dividend, or it may be valued as a fraction and annexed to the quotient, by making the remainder a numerator and the divisor a denominator. Thus in Ex. 1. the remainder 3 divided by 4 gives 3-4ths expressed by $\frac{3}{4}$; or if the quantity divided is not in the lowest denomination of its kind, the remainder may be reduced and then divided; thus if the 6375 are pounds of money, then the 3 over are 3 pounds or 60 shillings, which divided by 4 give 15 s.; and thus it makes up the complete quotient £1593 15 s. Otherwise if the 3 are shillings or 36 pence they will give 9 d. or if the 3 are pence, then they are equal to 12 farthings, and will give 3 farthings.

When the divisor is a number of which the products are given in the Multiplication Table, it is called *short* divisor, because the division can then be performed in one line; if the divisor is a composite divisor, that is, one which is *exactly* the product of two short divisors, it is called a *double* divisor; and if the divisor is not a short divisor nor exactly a double divisor, it is called a *long* divisor.

Ex. 1. To divide 6375 by 4
4 in 6 gives 1 and 2 over
4 in 23 gives 5 and 3 over
4 in 37 gives 9 and 1 over
4 in 15 gives 3 and 3 over

4) 6375
1593-3 over

Proof. Multiply 1593 by 4 and add in 3.

Ex. 2. To divide 16734 by 32 and by 37.

32-4) 16734
8) 4183-2
522-30

37) 16734 (452
148
193
185
84
74
10 remainder.

Observe. In double division if there is a remainder to the second division, we must multiply that remainder by the first divisor, and take in the first remainder; thus, in dividing by 4 we have 2 over, and in dividing by 8 we have 7 over, therefore we multiply 7 by 4=28 and add in 2=30.

1st Proof $522\times32+30=16734$
2nd Proof $452\times37+10=16734$

Ex. 3. To divide 56740 by 5300

53,00) 567,40 (10
53
3740 remainder.

There being 2 ciphers in the divisor, we cut off the 40 or the two right hand figures in the dividend, 53 in 56 goes once and 3 over; bring down 7; 53 will not go in 37, set down 0 in the quotient, and to the 37 over bring down the 40 cut off.

Ex. 4. Divide 5367 by 100

Quotient 53—67 over.

To divide by 10, by 100, or by 1000, cut off 1, 2, or 3 of the lower figures of the dividend; the other figure or figures will be the quotient, and the figure or figures cut off will be the remainder.

EXERCISES.

2) 33682
3) 41679
4) 61062

5) 23456
6) 81204
7) 21272

8) 12345
9) 74217
12) 54321

Divide,

Ex. 10. 61472 by 18
11. 34703 - 27
12. 50414 - 35
Ex. 13. 49832 by 44
14. 10627 - 56
15. 37940 - 72

16. 631371 by 17
17. 404342 - 34
18. 673011 - 67
19. 673706 by 107
20. 511342 - 346
21. 141705 - 633

22. 745736 by 2100
23. 212905 - 3600
24. 844147 - 4200
25. 414040 by 6170
26. 606926 - 3900
27. 414070 - 5100

QUOTIENTS.

Ex. 1. 16841
Ex. 2. 13893
Ex. 3. 15265-2
4. 4691-1
5. 13534
6. 3038-6
7. 1543-1
8. 8246-3
9. 4526-9
10. 3415-2
11. 1285-8
12. 1440-14
13. 1132-24
14. 189-43
15. 526-68
16. 37139-8
17. 11895-12
18. 10044-63
19. 6296-34
20. 1477-300
21. 223-546
22. 355-236
23. 59-505
24. 200-4147
25. 67-650
26. 155-2426
27. 81-970

CONTRACTED METHOD OF LONG DIVISION.

The operation of Long Division may be contracted by the omission of the partial products, in the following manner.

Rule. When the product of the figure of the quotient multiplied by each figure of the divisor, is not greater than the corresponding figure of the dividend, subtract it and put down the remainder; but when the product is greater, add to the dividend figure as many tens as are necessary to admit of the subtraction, and then putting down only the remainder, carry the number of the tens employed to the next product.

EXAMPLE.

To divide 58420 by 37

```
37 ) 58420 ( 1578
     214
      292
       330
        34
```

Beginning with 58 we say 7 from 8 leaves 1—3 from 5 leaves 2, bring down 4—7 times 5 are 35, from 44 leaves 9 and carry 4—3 times 5 are 15 and 4 are 19, from 21 leaves 2, bring down 2—7 times 7 are 49, from 52 leaves 3 and carry 5—3 times 7 are 21 and 5 are 26, from 29 leaves 3, bring down 0—7 times 8 are 56, from 60 leaves 4 and carry 6—3 times 8 are 24 and 6 are 30, from 33 leaves 3, &c.

For other contractions in the operations of Long Division, see the end of Compound Division, and the end of Decimal Division.

EXERCISES TO BE PROVED.

Ex. 1. Divide 61268 by 29
2. Divide 44842 by 47
3. Divide 18067 by 53
4. Divide 24809 by 79

Ex. 5. Divide 41735 by 101
6. Divide 35387 by 343
7. Divide 91075 by 373
8. Divide 63733 by 675

TABLES OF MONEY, WEIGHTS, AND MEASURES.

MONEY.

4	Farthings	make	1 Penny.*	*d.*
12	Pence	—	1 Shilling.	*s.*
20	Shillings	—	1 Pound.	£

¼ d. = 1 Farthing. ½ d. = 2 Farthings. ¾ d. = 3 Farthings.

Farthings 4 = 1 Penny.
48 = 12 = 1 Shilling.
960 = 240 = 20 = 1 Pound.

Pence Tables for reducing small Quantities of Money.

Pence		s.	d.	Pence		s.	d.
20	are	1	8	24	are	2	0
30	—	2	6	36	—	3	0
40	—	3	4	48	—	4	0
50	—	4	2	60	—	5	0
60	—	5	0	72	—	6	0
70	—	5	10	84	—	7	0
80	—	6	8	96	—	8	0
90	—	7	6	108	—	9	0
100	—	8	4	120	—	10	0
110	—	9	2	132	—	11	0
120	—	10	0	144	—	12	0

TROY WEIGHT.

24	Grains, *grs.*	make	1 Pennyweight.	*dwt.*
20	Pennyweights	—	1 Ounce.	*oz.*
12	Ounces	—	1 Pound.	*lb.*

Grains 24 = 1 Pennyweight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

By this weight are weighed Gold, Silver, and Jewels.

100 lb. = 1 Hundred Weight, and 2000 lb. = 1 Ton.

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* The penny is also nominally divided into 8 ths, or half farthings, 16 ths, or quarter farthings, and 32 nds, or half quarter farthings.

AVOIRDUPOIS WEIGHT.*

16 Drams	make	1 Ounce.	*oz.*
16 Ounces	—	1 Pound.	*lb.*
28 Pounds	—	1 Quarter.	*qr.*
4 Qrs. or 112 lb.	—	1 Hundred Weight.	*cwt.*
20 Cwt.	—	1 Ton.	

Drams 16 =	1 Ounce.				
256 =	16 =	1 Pound.			
7168 =	448 =	28 =	1 Quarter.		
28672 =	1792 =	112 =	4 =	1 Cwt.	
573440 =	35840 =	2240 =	80 =	20 =	1 Ton.

Note. 1 pound Avoirdupois is 7000 grains Troy = 14 oz. 11 dwts. 16 grs. Troy; and 175 lb. Troy are equal to 144 lb. Avoirdupois. The oz. Avoirdupois is equal to 437½ grains Troy, and 175 oz. Troy = 192 oz. Avoirdupois.

WOOL WEIGHT.

1 Clove = 7 lb.
1 Stone = 14 lb.
1 Tod = 28 lb.
1 Wey = 13 Stone = 182 lb.
1 Sack = 2 Weys = 364 lb.
1 Last = 12 Sacks = 4368 lb.

MISCELLANEOUS WEIGHTS.

A Stone of Meat = 8 lb.
A Gallon of Oil = 9 lb.
A Truss of Straw = 36 lb.
A Truss of New Hay = 60 lb.
— Old Hay = 56 lb.
36 Trusses make 1 Load.

APOTHECARIES' WEIGHT.

20 Grains	make	1 Scruple	℈
3 Scruples	—	1 Dram	ʒ
8 Drams	—	1 Ounce	℥
12 Ounces	—	1 Pound	lb.

Note. This weight is the same in grains, ounces, and pounds, as Troy weight; it is used in prescriptions, and therefore in the compounding and mixing of medicines,† but not in the buying or selling of Drugs or Chemicals.

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* Avoirdupois weight is the only weight that can be legally used in this kingdom, except for Jewels, and the precious Metals, and for the compounding, but not the selling, of Medicines.

† In medicinal directions, 16 fluid ounces are reckoned equal to 1 pint Wine measure, and by weight the fluid ounce is estimated at 455½ Troy grains.

ASSAY WEIGHTS.

Gold.			Silver.		
4 Grains	make	1 Carat.	20 Dwts.	make	1 Oz.
24 Carats	—	1 Pound.	12 Oz.	—	1 lb.

The Assay weights are only used to show the purity of Gold and Silver.—Standard Gold is 22 carats of fine Gold with 2 carats of alloy.*—Standard Silver is 222 dwts. or 11 oz. 2 dwts. of fine Silver, with 18 dwts. of alloy.

LONG MEASURE.

12	Inches	make	1 Foot.
3	Feet	—	1 Yard.
5½	Yards	—	1 Rod, Pole, or Perch.
40	Poles	—	1 Furlong.
8	Furlongs	—	1 Mile. (Statute Mile.)
3	Miles	—	1 League.

Inches 12 = 1 Foot.
36 = 3 = 1 Yard.
198 = 16½ = 5½ = 1 Pole.
7920 = 660 = 220 = 40 = 1 Furlong.
63360 = 5280 = 1760 = 320 = 8 = 1 Mile.

An inch was formerly divided into 3 barleycorns, but is now usually divided into either halves, quarters, and eighths, or into tenths or twelfths.

A degree of the Meridian in the latitude of London is nearly 69 miles, 1 furlong, 37 yards.

N. B. For Land measure, the furlong is divided into 10 chains, each of 100 links; the chain measuring 66 feet or 792 inches.

Also, in the measuring of depths, the fathom of 2 yards, or 6 feet, is sometimes used.

CLOTH MEASURE.

4	Nails	make	1 Quarter of a Yard.
4	Quarters	—	1 Yard.
5	Quarters	—	1 Ell.

The yard is 36 inches; the quarter, 9 inches; and the nail, 2¼ inches.

Flemish and French ells of 3 and 6 qrs. of a yard are not now used. English ells are nominally used in measures of width, but, with very few exceptions, not in those of length.

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* That is, inferior metal; as silver or copper is an alloy to Gold; copper is an alloy to Silver.

IMPERIAL LIQUID MEASURE.

FOR WINE AND SPIRITS.

4	Gills	make	1 Pint.
2	Pints	—	1 Quart.
4	Quarts	—	1 Gallon.
42	Gallons	—	1 Tierce.
63	Gallons	—	1 Hogshead, Hhd.
84	Gallons	—	1 Puncheon.
126	Gallons	—	1 Pipe.
252	Gallons	—	1 Tun.

N. B. These measures of the Tun, Hhd. Puncheon, and Pipe, are not used in trade.

THE MERCHANTS' MEASURES FOR WINES ARE AS FOLLOWS:

Port . . .	Pipe	115	Galls.
Sherry . .	—	108	—
Madeira & Cape	—	92	—
Sicilian . .	—	93	—
Teneriffe & Vidonia		100	—
Malaga . .	—	105	—
Lisbon & Bucellas		117	—
Hock . .	Hhd.	30	Galls.
Claret & Hermitage		46	—
Tent . . .	—	52	—
Spanish Red	Tun	210	—

The last, which is called the merchants' tun, is equal to the late tun of 236 wine gallons.

The tun of Fish oil is 252 gallons, but the tun of Seed oil is 236 gallons.

FOR BEER AND ALE.

9	Gallons	make	1 Firkin.
18	Gallons	—	1 Kilderkin.
36	Gallons	—	1 Barrel.
54	Gallons	—	1 Hhd.
108	Gallons	—	1 Butt.

N. B. The Imperial gallon, with its divisions, is the only legal liquid and dry measure; but the gallon of the late Wine measure is still used, particularly with some of the Seed oils.* An Imperial gallon of water weighs 10 lb. Avoird. and the cubic contents of a gallon are reckoned equal to $277\frac{274}{1000}$ cubic inches. The late Wine gallon was 231 cubic inches, and the late Beer gallon, 282 cubic inches.

6 Wine Gallons	nearly make	5 Imperial Gallons.
100 Beer Gallons	— —	101 Imperial Gallons.

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* The Imperial gallon of oil is reckoned by weight at 9 lb. Avoirdupois. The Wine gallon of oil is reckoned at 7¼ lb. Avoirdupois.

IMPERIAL DRY MEASURE.

2	Pints	make	1 Quart.
2	Quarts	—	1 Pottle.
4	Quarts	—	1 Gallon.
2	Gallons	—	1 Peck.
8	Gallons	—	1 Bushel.
8	Bushels	—	1 Quarter.
10	Quarters	—	1 Last.

Gallons 2 = 1 Peck.
8 = 4 = 1 Bushel.
64 = 32 = 8 = 1 Quarter.
640 = 320 = 80 = 10 = 1 Last.

The Imperial Bushel contains 80 lb. by weight of distilled water, and is computed at $2218\frac{192}{1000}$ cubic inches.

For the late Coal measure, 3 bushels made 1 sack—3 sacks made 1 vat; and 12 sacks or 36 bushels made 1 chaldron, a score of coals being 21 chaldrons : but by the present regulations coals are sold by weight and not by measure. A sack of coals is commonly reckoned to weigh 2 cwt.

For heaped measure, the Imperial bushel must be 19½ inches in diameter, and the cone or heaped part above the measure must be, at least, 6 inches in height, making the whole contents nearly 2818½ cubic inches.

The late bushel used for Dry measure, was called the Winchester bushel; its contents were nearly 2150½ cubic inches, and therefore

32 Winchester Bushels nearly make 31 Imperial Bushels.

SQUARE MEASURE.

144	Square Inches	make	1 Square Foot.
9	Square Feet	—	1 Square Yard.
100	Square Feet	—	1 Square of Flooring, &c.
272¼	Square Feet	—	1 Rod of Brickwork.

Square measure is also Superficial measure, from its being used in the measurement of surfaces, without any regard to depth or thickness, although in a great many cases it has reference to the depth or thickness of 1 inch; as in the measurement of planks, and what are termed areas in Gauging; in Brickwork it refers to the thickness of a brick and a half.

LAND MEASURE.

30¼	Square Yards	make	1	Square Perch, Pole, or Rod.
16	Square Poles	—	1	Square Chain.
40	Square Rods	—	1	Rood.
4	Roods	—	1	Acre.
640	Acres	—	1	Square Mile.

The square rod is 5½ yards in length, and the same in breadth. The terms roods and acres are used only in Square measure, and it is therefore unnecessary to say a square rood, or a square acre.

In the measurement of large tracts of land, the square league of 9 square miles is sometimes used.

Sq. In. 144 = 1 Sq. Foot.
1296 = 9 = 1 Sq. Yard.
39204 = 272¼ = 30¼ = 1 Sq. Pole.
627264 = 4356 = 484 = 16 = 1 Sq. Chain.
1568160 = 10890 = 1210 = 40 = 2½ = 1 Rood.
6272640 = 43560 = 4840 = 160 = 10 = 4 = 1 Acre.
100 Acres make 1 Hide of Land.

CUBIC OR SOLID MEASURE.

1728	Cubic Inches	make	1	Cubic Foot.
27	Cubic Feet	—	1	Cubic Yard.
40	Cubic Feet	—	1	Ton of Shipping.
50	Cubic Feet	—	1	Load of Timber.

The cubic yard is sometimes called a load (meaning a cart-load) of earth, &c.

The ton of shipping is a weight of various quantities as well as a measure.

TIME.

60	Seconds	make	1	Minute.
60	Minutes	—	1	Hour.
24	Hours	—	1	Day.
7	Days	—	1	Week.
365	Days 6 hours	—	1	Julian Year.
365	Days, 5 h. 48 m. 48 s.	—	1	Solar Year.

365 days are reckoned 1 common year, and to allow for the quarter of a day omitted in each year, every 4th year, called Leap year, is reckoned 366 days.—12 calendar months make 1 year, viz,

January	31 days.	May	31 days.	September	30 days.
February	28	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

In Leap Years the additional day is given to February, which then contains 29 days.

N. B. To find Leap year, divide the number of the years by 4, if none remains it is Leap year; if 1, 2, or 3 remains, the year is so many years after Leap year.

Old Style, still used in Russia, is the day according to the old style of reckoning; it now differs 12 days from the new or present style; thus the 6th of January, new style, is the 25th of December, or Christmas day, old style.

The difference between the old and new style has been occasioned by the observance of every 4th year as a Leap year, giving to each year 11 minutes 12 seconds more than the proper Solar year, which from the Council of Nice, in the year 325, when the festivals of the church were regulated, to the year 1752, when the alteration was made in England, made an excess of upwards of 11 days; and a further correction being made of 1 day in the year 1800, which was not reckoned a Leap year, the difference has been increased to 12 days.

The Leap year retains its Roman appellation Bissextiles, or twice the 6th day, from the Roman method of intercalating the extra day, by calling two of days in March, as, each, the 6th of the Calends of March.

REDUCTION.

Reduction is the changing of a quantity from one form or denomination into another, without altering its value.

Reduction is called descending, when the quantity is to be reduced into a lower denomination, and ascending, when it is to be brought into a higher denomination.

Rule. All great denominations are brought into small by multiplication, and all small into great by division, multiplying or dividing by as many of the less denomination as will make one of the greater.

N. B. This Rule would be more correctly expressed thus— To reduce any quantity, consider it as changed into the denomination of the quantity required; and when the change has been made into a lower denomination, multiply it, or when it has been made into a higher denomination, divide it, by the number of the less quantity that is equal to an integer of the greater quantity.

MONEY.

To reduce £ 14 17 10½ d. into shillings, pence, and farthings.

£. s. d.
14 17 10½
20

297 shillings
12

3574 pence
4

14298 farthings

The 14 pounds are brought into shillings, by considering them as 14 shillings, and multiplying them by 20,* taking in the given shillings; then the 297 shillings are brought into pence, by considering them as 297 pence, and multiplying them by 12, taking in the given pence; and, lastly, the 3574 pence are brought into farthings, by considering them as 3574 farthings, and multiplying them by 4, taking in the ½ d. as 2 farthings.

Ex. 1. Reduce £ 1, £ 7, £ 11, £ 27, and £ 321 into farthings.
2. Reduce £ 17 14 s. £ 81 11 s. 4 d. £ 17 17 s. 4½ d. into farthings.
3. In 617 s. 6 d. how many farthings?
4. In 8147¾ d. how many eighths of pence?
5. Reduce 12345 pence into shillings.
6. Reduce 27674 farthings into shillings.
7. In 14379 half farthings how many shillings?
8. In 42708 farthings how many pounds?

PRODUCTS.

Ex. 1. f. 960 f. 6720 f. 10560 f. 25920 f. 308160
2. f. 16992 78304 17154 Ex. 3. f. 29640
4. ½f. 65182 Ex. 5. s. 1028 9 d. Ex. 6. s. 576 6½ d.
Ex. 7. s. 149 9⅜ d. Ex. 8. £ 44 9 9 d.

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* It is evident that we take 20 times 14 shillings, in the place of taking 14 times 20 shillings, and so on with the rest of the operation; and although we commonly say that we multiply pounds by 20 to bring them into shillings, there is no question but that 20 times 14 pounds are 280 pounds and not 280 shillings.—To say we multiply by 20 shillings, that is, take 20 shillings worth of times, is highly incorrect.

TROY WEIGHT.

Example.

To reduce 7965 grains into oz.

$$\begin{array}{rl} & \text{grs.} \\ 24\left\{\begin{array}{r} 4\,) \\ 6\,) \end{array}\right. & \begin{array}{l} \underline{7965} \\ \underline{1991\ \ 1} \end{array} \\ 20\,) & \underline{\ 331\ \ 21} \\ \text{Oz.} & \underline{\ \ 16\ \ 11\ \ 21} \end{array}$$

In describing such operations as the one above, which are called Reduction ascending, it is usual to say, as with this calculation, that we divide grains by 24 to bring them into dwts. and dwts. by 20 to bring them into oz.—the remainder at each step being of the same denomination as the quantity divided—but, if we properly consider the operation according to the invariable principle of division, viz. that the quotient is of the same denomination as the quantity divided, we must consider these 7965 grains as so many dwts. and take the 24 th part, or divide by 24; and the 331 dwts. as so many oz. and divide by 20, or take the 20 th part; and the reason that the 21 over represents so many grains, is, that as the 24 th part of 1 dwt. is one grain, so the 24 th part of 21 dwts. is 21 grs.; and thus we must reason with every calculation of this nature. It would be incorrect to say that we divide by 24 grains, because the divisor only shows the part to be taken, and we cannot with propriety say, we take the 24 grainth part.

Exercises.

Ex. 9. Reduce 17 lb. into oz. dwts. and grs.
10. Reduce 11 lb. 10 oz. 14 dwts. 19 grs. into grains.
11. In 24 lb. 11 oz. 17 dwts. 17 grs. how many grains?
12. In 41475 grains how many oz.?
13. In 57934 grains how many lb.?
14. In 43592673 grains how many tons?
15. In 153 lb. 11 oz. 2 dwts. 12 grs. of gold how many grs.?
16. In 362 lb. 9 oz. 5 dwts. of silver how many grs.?

Products.

Ex. 9. grs. 97920 — 10. grs. 68515
11. grs. 143945 — 12. oz. 86 8 3
13. lb. 10 0 13 22 — 14. tons 2 15 2 11 3673 grs.
15. grs. 886620 — 16. grs. 2089560

AVOIRDUPOIS WEIGHT.

EXAMPLE.

To reduce cwt. 29 3 11 lb. into lb.

cwt. qrs. lb.	cwt. qrs. lb.	cwt. qrs. lb.
29 3 11	(or) 29 3 11	(or) 29 3 11
4	999	348
119 qrs.	222	95 for 3 qr. 11 lb.
28	84 = 3 qrs.	3343 lb.
963 with 11	11	
238	3343 lb.	
3343 lb.		

In the second and third methods we multiply the 29 cwt., considered in the operation as so many lb., by 112, in order to bring them at once into lb.:—in the second method we first take 111 times 9, and then 111 times 2 or 20, and add in these products with the 29, in order to get 112 times 29.

In the third method, we take 12 times 29, and by placing the figures two spaces to the right, we let the 29 be added in as so many hundreds.

EXERCISES.

Ex. 17. Reduce 27 cwt. into drams.
18. Reduce 12 cwt. 3 qrs. 15 lb. into pounds.
19. Reduce 17 tons 11 cwt. 2 qrs. 22 lb. into oz.
20. Reduce 482741 drams into cwt.
21. Reduce 62097 oz. into cwt.
22. Reduce 74247 lb. into tons.

To Method 2.

*23. In 12 cwt. 1 qr. 16 lb. how many lb.?
*24. In 29 cwt. 2 qr. 22 lb. how many lb.?

To Method 3.

*25. In 43 cwt. 3 qr. 13 lb. how many lb.?
*26. In 150 cwt. 3 qr. 27 lb. how many lb.?

PRODUCTS.

Ex. 17. drs. 774144	Ex. 18. lb. 1443
19. oz. 630240	20. cwt. 16 3 9 11 5
21. cwt. 34 2 17 1	22. tons 33 2 3 19
23. lb. 1388	24. lb. 3326
25. lb. 4913	26. lb. 16911

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* To be worked by the 1st Method, if found too difficult as directed.

CLOTH MEASURE.

Ex. 27. In 311 yards how many quarters and nails ?[*]
28. In 47 yards 3 qrs. 2 nails how many nails ?
29. In 61742 nails how many quarters and yards ?
30. In 50543 nails how many inches, quarters, and yards ?

LONG MEASURE.

31. In 18 miles how many furlongs and poles ?
32. In 11 miles 3 furlongs how many chains, yards, and inches ?
33. In 61743 inches how many furlongs ?
34. In 4243892 inches how many leagues ?

LIQUID MEASURE.

35. In 161 gallons how many gills ?
36. In 67 tuns 162 gallons how many gallons ?
37. In 44967 pints how many Excise tuns of 252 gallons ?
38. In 18264 gallons of olive oil how many tuns, and what is the weight in cwt. qrs. lb. at 9 lb. per gallon ?
39. In 3827 gallons how many regular pipes of Port wine ?
40. In 67346 gallons how many regular pipes of either Teneriffe wine or Madeira wine ?
41. In 1645 gallons how many barrels of beer ?
42. In 16560 gallons how many butts of beer ?

DRY MEASURE.

43. In 616 quarters of wheat how many bushels, pecks, and gallons ?
44. In 113 lasts 8 qrs. 5 bushels how many bushels ?
45. In 36409 gallons how many lasts ?
46. In 4373 bushels how many quarters, and what is the weight at the rate of 62 lb. per bushel ?
47. In 992 Winchester bushels how many Imperial bushels ?
48. In 992 Imperial bushels how many Winchester bushels ?

LAND MEASURE.

49. In 27 acres how many roods and perches ?
50. In 4320 perches how many acres ?

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[*] For the Products of these Exercises see the following page.

SQUARE MEASURE.

Ex. 51. In 64 square yards how many square inches?
52. In 436431 square inches how many square yards?

CUBIC MEASURE.

Ex. 53. In 64 cubic yards how many cubic inches?
54. In 436431 cubic inches how many cubic yards?

TIME.

Ex. 55. In 1830 Julian years, how many seconds?
56. In 41211 days how many Julian years?
57. In 890082895 seconds how many Solar years?
58. How many days from March 21 the Vernal equinox, to September 23 the Autumnal equinox?
59. How many days from June 21 the Summer solstice, to December 21 the Winter solstice?
60. How many days from Christmas day December 25, to Midsummer day June 24?
61. How many days from Lady day March 25, to Michaelmas day September 29?

Products.

Ex.
27. nails 4976
28. nails 766
29. yards 3858 3 2
30. inches 113721. 3 ns.
yards 3158 3 3
31. poles 5760
32. inches 720720
33. furl. 7 175 0 3
34. leagues 22 0 7 185 2 8
35. gills 5152
36. gallons 17046
37. tuns 22 76 7
38. tuns 77 92 cwt. 1467 2 16
39. pipes 33 32
40. pipes 673 46 p. 732 2
41. bars 45 25
42. butts 153 36
43. gallons 394 2 4

Ex.
44. bush. 9109
45. lasts 56 8 7-0 1
46. qrs. 546 5 cwt. 2420 3 2
47. Imp. bush. 961
48. Winc. bush. 1024
49. poles 4320
50. acres 27
51. inches 82944
52. yards 336 6 111
53. inches 2985984
54. yards 9 9 975
55. sec. 57750408000
56. years 112 303
57. years 28 75 2 28 31
58. days 186
59. days 183
60. days 181
61. days 188.

COMPOUND ADDITION.

Rule. Find the amount of each denomination, beginning with the lowest, and, except for the highest amount in which the whole amount is required, reduce each amount into the next higher denomination; then set down the remainder and carry the integers produced to the next superior denomination.

EXAMPLE.

£	s.	d.
67	11	8½
48	19	11¾
72	17	10½
84	4	5¾
273	14	0½

In adding up the shillings, we may put down the unit figure of the units of shillings, and carry the tens to the tens, then take half the amount of these tens for pounds, and if 1 is over set it down as 10 shillings.

MONEY.

(1) £	s.	d.	(2) £	s.	d.	(3) £	s.	d.	(4) £	s.	d.
27	7	2	35	17	3	75	3	7	21	14	7¼
34	14	7½	59	14	7½	54	17	1½	75	16	0
57	19	2¼	97	13	5¼	91	15	4¼	79	2	4¼
91	16	1	37	16	8¼	35	16	5¾	57	16	5½
75	18	7¾	97	15	7	29	19	7¼	26	13	8¾
97	13	5	59	16	5½	91	17	3¼	54	2	7

(5) £	s.	d.	(6) £	s.	d.	(7) £	s.	d.	(8) £	s.	d.
257	1	5½	525	2	4¼	127	4	7½	261	17	1¼
734	3	7¾	179	3	5	525	3	5	379	13	5
595	5	3	250	4	7¼	271	0	5	257	16	7¾
152	14	7½	975	3	5¼	524	9	1	184	13	5
207	5	4	254	5	7	379	4	3¼	725	2	3¼
798	16	7¾	379	4	5	215	5	8¾	359	6	3

TROY.						AVOIRDUPOIS.					
(9) oz.	dwt.	gr.	(10) lb.	oz.	dwt.	(11) lb.	oz.	dr.	(12) cwt.	qr.	lb.
5	11	4	7	1	2	15	15	15	25	1	17
7	19	21	3	2	17	27	14	10	72	3	26
3	15	14	5	1	15	30	15	11	54	1	16
7	19	22	7	10	11	25	10	4	24	1	16
9	18	15	2	7	13	17	6	2	17	0	19
8	13	12	3	11	16	63	13	3	55	2	16

CLOTH.			LONG.			LAND.			WINE.		
(13) yd.	qr.	n.	(14) m.	fur.	po.	(15) a.	r.	p.	(16) hhd.	gal.	qrt.
135	3	3	2	1	19	726	1	31	31	57	1
70	2	2	1	7	22	219	2	17	97	18	2
95	3	0	2	5	31	1455	3	14	76	13	1
176	1	3	0	6	12	879	1	21	55	46	2
26	0	1	1	6	17	1195	2	14	87	38	3
279	2	1	0	5	21	365	1	12	55	17	1

BEER.			DRY.			TIME.					
(17) b.	fir.	gal.	(18) ch.	bu.	pk.	(19) w.	d.	ho.	(20) h.	m.	sec.
25	2	7	75	2	1	71	3	11	15	42	41
17	3	5	41	24	1	51	2	9	21	27	51
96	2	6	92	16	1	76	0	21	15	37	28
75	1	4	70	13	2	95	3	21	21	42	27
96	3	7	54	17	3	79	1	15	18	47	38
75	0	5	79	25	1	62	2	12	20	50	21

Amounts.

Ex. 1.	£	385	9	1½	Ex. 11.	lb.	181	11	13
2.		388	14	0½	12.	cwt.	249	3	26
3.		379	9	5	13.	yds.	784	1	2
4.		315	5	8¾	14.	ms.	10	1	2
5.		2745	6	11½	15.	acres	4842	0	29
6.		2563	3	9¾	16.	hhds.	404	2	2
7.		2042	7	6½	17.	b.	387	2	7
8.		2168	9	1¼	18.	ch.	413	27	1
9.	oz.	43	18	16	19.	wks.	436	0	17
10.	lb.	29	11	14	20.	hrs.	114	8	26

APPLICATION OF ADDITION.

Ex. 1. A, B, C, D, went partners in the purchase of a quantity of goods; A laid out £ 7, half a guinea,* and a crown,* B 49 s., C 54 s. 6 d., and D 87 d.—What was laid out in all?

Ex. 2. A man lent his friend at different times these several sums, viz. £ 63, £ 25 15 s., £ 32 7s., £ 15 14 10, and fourscore and 19 pounds, half a guinea, and a shilling—How much did he lend in all?

Ex. 3. Suppose I am indebted to A £ 8 17 3½, to B £ 33 10 8¼, to C £ 6 17 11½, to D £ 5 15 5¼, to E 20 guineas, to F £ 2 18 7½, and to G 100 crowns—How much is the whole of my debts?

Ex. 4. Suppose there is due to me from A L £ 3 16 4½, £ 7 11 10¼, £ 18 3 10¼, £ 1 12 6, £ 37 9 4¾; from T B £ 22 12 6¼, £ 6 5 0½, £ 6 16 5¾; from R S £ 8 11 5¼, £ 7 17 6, £ 8 18 6, £ 9 3 9; from W G £ 117 5 2, £ 0 7 8¼, £ 31 11 3, £ 20, £ 100, £ 6 11 5, £ 3 14 4¼; and from A C £ 80, £ 47, £ 33, £ 11 4 7, £ 41, and 100 guineas—What is due to me from each of these persons, and what is the amount of all their debts?

Ex. 5. A person owes to A £ 67 11 5¼, to B £ 3 11 1 more than to A; to C £ 16 17 5 more than to B, to D £ 4 17 11 more than to A, and to E £ 17 2 7 more than to B—How much is he indebted to each of these five persons, and what is the amount of all their claims?

Ex. 6. How many days are there from the Vernal equinox, March 21, to the Autumnal equinox, September 23; how many days are there from the Autumnal equinox to the Vernal equinox; and how many days are there in both these portions of the year?

Ex. 7. How many days are there from the Summer solstice June 21, to the Winter solstice December 21?

Ex. 8. How many days have elapsed from the 17th May, 1801, to May 31, 1830; and how many minutes and seconds are there in these days?

N. B. Attention must be paid here to the leap years.

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* The guinea is 21 s. or £1 1 s. The crown is 5 s.

Ex. 9. If the successive distances of the principal primary planets are as follows: viz. Mercury, 36 millions 814,721 miles from the Sun; Venus, 31 millions 977,031 miles from Mercury; the Earth 26 millions 208,248 miles from Venus; Mars, 49 millions 907,630 miles from the Earth; Jupiter, 349 millions 591,478 miles from Mars; Saturn, 412 millions 589,924 miles from Jupiter; and Georgian or Herschel 912 millions 910,968 miles from Saturn—What is the distance of each of these celestial bodies from the centre of the system?

Ex. 10. The first of five fields contains 6 acres 1 rood 27 poles; the second 17 acres 3 roods 31 poles; the third 17 acres; the fourth 12 acres 1 rood 30 poles; and the fifth 23 acres 3 roods 23 poles—What are their whole contents?

Ex. 11. The estimated value of a tradesman's Stock in trade is £ 626 13 7; he has in Cash £ 34 18 2; the amount of the Debts owing to him is £ 1133 19 9; and his Household Furniture, Fixtures, &c. are considered to be worth £ 223 10—What is the whole amount of his estate?

Ex. 12. I have bought a certain quantity of Mahogany for £ 150, and in disposing of it I wish to gain £ 25—What must I sell it for?

Ex. 13. If the prime cost of certain goods which I have purchased for my correspondent at Manchester, be £ 83 11 3, and the amount of my charges be £ 3 15 4; In what amount will he be indebted to me?

Ex. 14. If the cost of 4 bales of Osnaburghs, shipped by me for my correspondent at Jamaica, be £ 125 4 4, and the shipping Charges be £ 13 8 2, the Commission on the Goods and Charges be £ 3 9 4, the amount of the Insurance be £ 14 0 5, and the Commission on the Insurance be 16 s. 2 d.—What will be the whole amount of the Invoice? And if I have to charge his account with £ 1 10 6 for Postages, &c.—How much will he be indebted to me for the whole?

PRODUCTS.

Ex. 1.	£ 13 6 3	Ex. 8.	sec. 916,380,000
2.	236 8 4	9.	Georgian 1,820,000,000
3.	104 0 0	10.	acr. 77 2 31
4.	735 13 8	11.	£ 2019 1 6
5.	387 8 4¼	12.	175 0 0
6.	days 365	13.	87 6 7
7.	days 183	14.	Invoice £ 156 18 5 Whole amount £ 158 8 11

COMPOUND SUBTRACTION.

Rule. Begin with the lowest part of the quantity to be subtracted, and take each part from the similar part of the other quantity; but when the latter is not large enough to allow of the subtraction, take (or borrow) an integer of the next higher denomination, reduce it into the same denomination, add it to the less quantity, and subtract the other from the amount; then carry 1 to the next higher denomination of the quantity to be subtracted.

The proof is made by adding the remainder to the quantity subtracted.

EXAMPLE.

To subtract £ 416 13 4¼ from £ 715 2 7¾.

In subtracting 13 s., as they cannot be taken from 2 s., £ 1 is borrowed and (as 20 s.) added to the 2 s.; then the 13 s. are taken from 22 s. and 1 is carried to the £ 6.

	£	s.	d.
	715	2	7¾
	416	13	4¼
	298	9	3½
Proof £	715	2	7¾

Instead of making the subtraction in this manner, the quantity to be subtracted may be taken from the integer borrowed, and then the given part of the greater quantity is to be added to the remainder: thus, instead of subtracting 13 from 22, we may take 13 from the 20 borrowed, and to the remainder 7 we must then add the 2.

Sometimes, in this manner, it is better to take the amount of all the lower denominations from a borrowed integer of the highest, as thus,

To subtract £ 17 19 8 from £ 23 15 7.

	£	s.	d.	
	23	15	7	
	17	19	8	
£	5	15	11	remainder.

In which, taking 19 s. 8 d. from £ 1 or 20 s. we add the remainder 4 d. to the 15 s. 7 d. and carry the 1 £ to the 17 £.

So in the first Example, subtracting 13 s. 4 d. from £ 1, we may add the remainder 6 s. 8 d. to 2 s. 7 d.

EXERCISES.

Ex. 1.

£	s.	d.
147	14	10
82	11	6

Ex. 2.

£	s.	d.
354	11	6½
246	12	4¼

Ex. 3.

£	s.	d.
404	10	6½
219	17	8

Ex. 4.

£	s.	d.
538	17	1¼
207	19	2¼

Ex. 5.

£	s.	d.
715	17	7
318	18	10½

Ex. 6.

£	s.	d.
507	0	0
279	16	7⅜

Subtract

Ex. 7. £ 71 16 1⅛ from £ 84 15 10¼.
8. £ 42 11 10¾ from £ 61 17 8⅝.
9. 64 lb. 7 oz. 11 dwts. from 82 lb. 11 oz. 13 dwts.
10. 23 lb. 11 oz. 17 dwts. from 27 lb. 4 oz. 9 dwts.
11. 10 oz. 17 dwts. 3 grs. from 17 oz. 15 dwts. 11 grs.
12. 27 cwt. 1 qr. 14 lb. from 35 cwt. 2 qrs. 22 lb.
13. 14 cwt. 0 qr. 24 lb. from 18 cwt. 1 qr. 26 lb.
14. 17 cwt. 3 qrs. 21 lb. from 19 cwt. 2 qrs. 17 lb.
15. 3 tons 11 cwt. 3 qrs. from 5 tons 0 cwt. 1 qr.
16. 17 miles 1 fur. 17 po. from 25 miles 7 fur. 11 po.
17. 16 leagues 2 m. 7 fur. from 25 leagues 1 m. 3 fur.
18. 11 yards 1 qr. 3 n. from 14 yards 2 qrs. 2 n.
19. 4 sacks 1 bush. 2 pecks from 7 sacks 2 bush. 1 peck.
20. 5 sacks 0 bush. 1 peck from 1 chaldron.
21. 21 days 17 h. 25 m. from 34 days 1 h. 35 m.
22. 4 days 11 h. 28 m. 37 s. from 1 week.
23. £ 4 11 4½ from £ 1 10 6, added to £ 1 18 4½, £ 3 17 5⅛, and £ 4 6 7¼.
24. £ 17 16 5⅜ from £ 2 3 6, £ 11 1 3, £ 10 14 6, and £ 3 11 5.

When the amount of several quantities is to be subtracted from another given quantity, the subtraction may be made in the manner of the following Example.

EXAMPLE.

To subtract the amount of £ 17 3 11, £ 28 13 5, £ 42 16 4, £ 22 11 10, and £ 62 15 7, from £ 193 7 2.

£	s.	d.
193	7	2
17	13	11
28	13	5
42	16	4
22	11	10
62	15	7
£ 18	16	1

In working this without putting down the amount of the given sums of money to be subtracted, we add up the pence, and as they amount to 3 s. 1 d. we borrow 3 s. in addition to the given 2 d., and subtracting the 3 s. 1 d. we say 1 d. remains; the 3 borrowed is then added to the given shillings to be deducted, which then amount to 71; we therefore borrow £ 4 or 80 s. and subtract 71 from 87; the 4 is then carried to the units of pounds, which make 25, and we say, 25 from 33 (borrowing 30) there remains 8 and carry 3; and lastly adding up the tens, we say, 18 from 19 there remains 1.

Ex. 1.

£	s.	d.
637	11	5
123	10	2
67	17	7
226	11	5
47	4	8

Ex. 2.

£	s.	d.
480	0	7
62	6	3
143	0	7
88	19	8
161	2	6

Ex. 3.

cwt.	qrs.	lb.
347	3	22
62	2	20
121	1	25
18	3	11
22	1	22

REMAINDERS.

Ex. 1. £ 172 7 7 Ex. 2. £ 24 11 7 Ex. 3. cwt. 122 2 0

APPLICATION OF SUBTRACTION.

Ex. I. If the amount of the debts owing by me is £ 177 2 11, and the amount of the debts owing to me is £ 300, what is the balance of these debts in my favour?

Ex. 2. If the amount of the debts owing to me is £ 1801 17 6, and of the Cash and other property in my possession is £ 917 8 4, and at the same time the debts owing by me is £ 1600 14 4, what is the net amount of my present estate?

Ex. 3. If the net of my capital originally employed in trade is £ 800, and I find the net of my present worth to be £ 617 3 7, what has been the loss attending my trade?

Ex. 4. If £ 6 10 has been paid for goods which sell for £ 7 3 10, what is the amount of the gain?

Ex. 5. If goods which cost £ 172 14 1 were sold for £ 158 15 6, what was the amount of the loss?

Ex. 6. What was the cost of a hhd. of Sugar, when the selling price was £ 62 10, and the gain £ 7 15 3?

Ex. 7. What was the selling price of 10 bags of Pimento, which cost £ 48 15, and upon which a loss of £ 3 17 11 was sustained?

Ex. 8. What is the balance or difference between the Dr. and Cr. sides of this account?

Dr. Side.				Cr. Side.		
£	s.	d.		£	s.	d.
71583	11	7		81735	15	3
3761	0	11		6626	16	6
82651	18	2		9854	19	9
5070	16	7		56487	2	10
9856	14	11		3373	13	3
74825	12	9		828	16	8
8376	16	6		9272	14	7
675	18	2		897	15	5
£				£		

Ex. 9. How many days difference is there between what are usually called the Summer and Winter portions of the year, or between the days from the Vernal to the Autumnal equinox, and the days from the Autumnal to the Vernal equinox? *(See Ex. 6. Compound Addition.)*

Ex. 10. If I have to pay for certain goods purchased at twelve months' credit, £ 171 3 6—What shall I have to pay for them at present, if a discount of £ 17 3 6 be allowed me for ready money?

Ex. 11. What must be paid for goods which amount to £ 3 7 7, upon which a discount is allowed of 3 s. 6 d.?

Ex. 12. If the produce of certain goods sold by my agent on my account be £ 571 14 4, and the amount of his charges be £ 47 3 11—What would be the net proceeds, or the amount that would remain after the charges were deducted?

Ex. 13. I borrowed of my friend £ 200, and I have repaid him £ 18 10, £ 25, £ 17 6 4, £ 100, and £ 13 18 6—How much am I still indebted to him?

Ex. 14. An agent in London had consigned to him by a planter in Jamaica 60 barrels of Coffee, which produced £ 1074 13 0, upon which the charges were £ 477 16 4; 10 tons of Logwood, which produced £ 142 9 2, upon which the charges were £ 46 2 2; 16 bags of Pimento, which produced £ 41 13 11, upon which the charges were £ 15 9 6; and 20 hhds. of Sugar, which produced £ 1155 13 6, upon which the charges were £ 621 14 0—What were the net proceeds of each of these sales, and if the planter had drawn £ 1000 on the credit of these consignments, what balance would be due to him?

ANSWERS.

Ex. 1.	£ 122	17	1	Ex. 8.	Dr. Side	£ 256802	9	7
2.	1118	11	6		Cr. Side	169077	14	3
3.	182	16	5		Balance	87724	15	4
4.	0	13	10	9.	days 8			
5.	13	18	7	10.		£ 154	0	0
6.	54	14	9	11.		3	4	1
7.	44	17	1	12.		524	10	5
				13.		25	5	2

14. Net Proceeds—Coffee £ 596 16 8—Logwood £ 96 7 0
Pimento £ 26 4 5—Sugar £ 533 19 6
Balance £ 253 7 7

COMPOUND MULTIPLICATION.

Rule. Multiply each part of the given quantity, beginning with the lowest, by the given number, and put down the product as in Compound Addition.

Observe. When the multiplier is above 12, if it is a double multiplier, multiply successively by each factor; if it is not a double multiplier, either multiply by the nearest double multiplier, and add or subtract the product of the top line by the difference, or multiply by the units, tens, &c. and add the products together.

If the multiplier is very large, take 10 times, 100 times, 1000 times, &c. as far as the highest denomination of the multiplier; then multiply each product and the top line by the proper figures in the multiplier, and add the products together.—Or, reduce the given quantity into its lowest denomination, multiply it as in Simple Long Multiplication, and reduce the product into as high a denomination as is required.

Example 1.

Multiply £ 62 11 7 by 7, 17, and 27.

£	s.	d.	£	s.	d.	£	s.	d.
62	11	7	62	11	7	62	11	7
		7			17			3
			438	1	1	187	14	9
£ 438	1	1	625	15	10			9
			£ 1063	16	11	£ 1689	12	9

To multiply by 17, we take 7 times and 10 times, and add the products together; but we may take 4 times 4 for 16 times, and add in once the top line.

Observe. In multiplication of money, farthings multiplied by 4 produce the same number of pence; pence multiplied by 12 produce the same number of shillings; and shillings by 20, the same number of pounds.* So, generally, a quantity in a lower denomination multiplied by the number making 1 of a higher denomination, produces the same number of that higher denomination; as 7 inches multiplied by 12 produce 7 feet.

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* See the Examples and Exercises in page 40.

EXERCISES.

£	s.	d.	£	s.	d.	£	s.	d.
14	7	10	15	16	1½	16	12	7
		2			3			5

£	s.	d.	£	s.	d.	£	s.	d.
121	13	6	211	17	11½	129	13	3½
		3			6			4
		7			5			12

When the nearest double multiplier is used, the number of times to be added or subtracted may be put against the 1st multiplier, with the sign + (plus) or — (minus).

Example 2.—To multiply by 29, 58, and 59.

	£	s.	d.
	42	14	7⅜
			4+1
	170	18	5½
			7
£	1239	3	9⅞

	£	s.	d.
	81	9	8½
			8+2
	651	17	8
			7
	4563	3	8
	162	19	5
£	4726	3	1

	£	s.	d.
	22	16	6
			6—1
	136	19	0
			10
	1369	10	0
£	1346	13	6

When we have to add in only once, we take it in while we work the second product; or when we have to subtract only once, we subtract the top line after we have taken the second product; but if we have to add or subtract more than once, it is necessary to express the additional product.

EXERCISES.

		£	s.	d.					£	s.	d.		
Ex.	7.	47	4	7½	by	32	Ex.	13.	10	16	2½	by	23
	8.	61	18	10	-	36		14.	48	4	6	-	31
	9.	12	15	3¾	-	42		15.	27	12	8⅛	-	43
	10.	77	0	7	-	84		16.	33	14	10	-	57
	11.	69	13	11¼	-	96		17.	27	15	5⅜	-	76
	12.	99	19	11¾	-	99		18.	64	18	8	-	79

Ex. 3. To multiply £ 87 16 4 by 1237.

	£	s.	d.			£	s.	d.	
	87	16	4	(or)		87	16	4×7	
	1756	s.						10	
	21076	d.				878	3	4×3	
	1237							10	
	147532					8781	13	4	
	63228							12	
	252912					105380	0	0	- 1200 times
d.	26071012					2634	10	0	- 30 times
s.	2172584	4				614	14	4	- 7 times
£	108629	4	4		£	108629	4	4	

In the multiplication of money by 10, it is to be observed, that we may at once put down the shillings and pence that make the product of the given *pence;* and that half the number of the *shillings* is the unit figure of the number of the pounds, putting down 1 ten shillings if there is 1 over; then the figures of the given pounds may be put down as so many tens, without the trouble of multiplying them.

EXERCISES.

Ex.	£	s.	d.			Ex.	£	s.	d.		
19.	41	13	6	by	236	22.	53	13	9	by	1632
20.	38	19	1	-	754	23.	72	16	11	-	2751
21.	67	11	10	-	833	24.	94	3	10	-	6438

PRODUCTS.

Ex.	£	s.	d.	Ex.	£	s.	d.	Ex.	£	s.	d.
7.	1511	8	0	13.	248	12	9½	19.	9835	6	0
8.	2229	18	0	14.	1494	19	6	20.	29371	8	10
9.	536	3	1½	15.	1188	5	1⅜	21.	56303	17	2
10.	6470	9	0	16.	1923	5	6	22.	87618	0	0
11.	6690	18	0	17.	2110	14	0½	23.	200398	17	9
12.	9899	17	11¼	18.	5129	14	8	24.	606405	19	0

N. B. As these Exercises, as well as those to the following Examples, and those on the Weights and Measures in the following page, are rather difficult for a young beginner, they may be omitted the first time of the pupil's going through Compound Multiplication; therefore from Ex. 18. he may go forward to Compound Division.

EXERCISES ON WEIGHTS AND MEASURES.

AVOIRDUPOIS WEIGHT.

lb.	oz.	dr.	cwt.	qr.	lb.	tons.	cwt.	qr.
16	7	7	14	3	11	21	17	3
		7			8			10

TROY WEIGHT.

oz.	dwt.	gr.	oz.	dwt.	gr.	lb.	oz.	dwt.
4	17	21	31	11	18	16	10	17
		5			6			9

CLOTH MEASURE.

yds.	qr.	na.	yds.	qr.	na.	yds.	qr.	na.
16	2	2	28	3	3	121	1	1
		11			12			16

LONG MEASURE.

yds.	ft.	in.	mi.	fur.	po.	fur.	ch.	lin.
13	2	11	14	7	31	21	7	33
		3			8			8
		7			10			9

TIME.

ho.	m.	sec.	ho.	m.	sec.	wks.	da.	ho.
64	38	31	27	57	59	17	5	17
		10			11			12

PRODUCTS.

Ex. 1.	lb.	115	4	1	Ex. 9.	yds.	1941	0	0
2.	cwt.	118	3	4	10.	yds.	293	1	3
3.	tons	218	17	2	11.	m.	1197	6	0
4.	oz.	24	9	9	12.	fur.	1564	7	76
5.	oz.	189	10	12	13.	ho.	646	25	10
6.	lb.	152	1	13	14.	ho.	307	37	49
7.	yds.	182	3	2	15.	wk.	213	5	12
8.	yds.	347	1	0					

When the multiplier is a large number, and the multiplicand is a simple quantity expressed by a small number, the multiplication may be made, as it is called from above; that is, in making the calculation, the two terms may change places—thus:

Ex. 4. To multiply 8¼ d. by 1442.

d.	(or)	£	s.	d.
¼..1442		¼..6	0	2
8				8
11536		48	1	4
360½		1	10	0½
d. 11896½		£ 49	11	4½
s. 991 4½				
£ 49 11 4½				

In the second form, the multiplier, 1442, (reckoned as so many pence, equal to 120 s. 2 d. or £ 6 0 2,) is used in the place of the multiplicand, and the number of the multiplicand is used as the multiplier; which form may conveniently be adopted in some of the following exercises.

EXERCISES.

Ex. 1.	Multiply	£ 8	by	341	*Product*	£ 2728	0	0
2.	—	s. 11	-	203	—	111	13	0
3.	—	d. 4	-	369	—	6	3	0
4.	—	d. 11	-	160	—	7	6	8
5.	—	d. 17	-	247	—	17	9	11
6.	—	d. 21	-	365	—	31	18	9
7.	—	d. 7½	-	127	—	3	19	4½
8.	—	d. 9½	-	135	—	5	6	10½
9.	—	d. 6¼	-	284	—	7	7	11
10.	—	d. 11¼	-	488	—	22	17	6
11.	—	d. 13½	-	504	—	28	7	0
12.	—	d. 16½	-	743	—	51	1	7½

To multiply pence by 12.

Rule. **Reckon the number of the pence as so many shillings, adding 3 d. for 1 farthing, 6 pence for 1 halfpenny, or 9 d. for 3 farthings.**

Example. To multiply $17\frac{3}{4}$ d. by 12.
Product 17 s. 9 d.

Obs. If the fraction of the penny is in eighths take $1\frac{1}{2}$ d. for each 8 th.

To multiply pence by 24.

Rule. Reckon the tens of pence as so many pounds, and twice the number of the units of pence as so many shillings; adding 6 d. for 1 farthing, 1 s. for 1 halfpenny, or 1 s. 6 d. for 3 farthings.

Example. To multiply $25\frac{3}{4}$ d. by 24.
Product £ 2·11 6.

Here, the 2 in the 25 shows the number of the pounds in the product, and for the shillings we say twice 5 s. are 10 s., and 1 s. 6 d. for 3 farthings, are 11 s. 6 d.

Obs. 8 ths of pence produce 3 d. for each 8th.

To multiply shillings by 20.

Rule. Reckon the number of the shillings as so many pounds, adding 10 s. for 6 d., 5 s. for 3 d., or 1 s. 8 d. for each penny.

Example. To multiply 27 s. 8 d. by 20.
Product £ 27 13 4.

Here 27 s. 6 d. give £ 27 10 s. and for the remaining 2 d. we take twice 20 d. or 3 s. 4 d.

EXERCISES.

Multiply

by 12	by 24	by 20
d. 4	d. 11	s. 7
$7\frac{1}{4}$	$17\frac{1}{4}$	6 3
$11\frac{1}{2}$	$21\frac{1}{2}$	8 6
$13\frac{3}{4}$	$34\frac{3}{4}$	11 9
$17\frac{1}{8}$	$32\frac{1}{8}$	13 4
$11\frac{3}{8}$	$47\frac{3}{8}$	16 8
$16\frac{4}{8}$	$54\frac{5}{8}$	17 $2\frac{1}{2}$
$21\frac{7}{8}$	$63\frac{7}{8}$	18 $6\frac{1}{2}$

Sometimes the multiplication of a compound quantity (particularly of a sum of money in which the pence are 3 pence or 6 pence) may be made in the following manner.

Ex. 5. To multiply 12 s. 6 d. by 1271.

	s.	d.	
	12	6	
	1271		
	635	6	for 1271 sixpences.
	15252	0	1271 times 12 s.
s.	15887	6	= £ 794 7 6

When the pence are 6 we take half the multiplier in shillings, when they are 3 we take the fourth. In a similar manner we may also use 4 d. as the third, and 1½ d. as the eighth of a shilling. When the shillings are 10 or 5, by valuing the first product we can conveniently extend this principle.—Thus:

Ex. 6. To multiply £ 3 10 3 by 875.

£	s.	d.	
3	10	3	
		875	
10	18	9	875 times 3 d.
437	10	0	— — 10 s.
2625	0	0	— — 3 £
£ 3073	8	9	

For 875 times 3 d. we take the fourth of 875 s. or 218 s. 9 d.; and then for 875 times 10 s. we take the half of £ 875, or £ 437 10 s.

EXERCISES.

Ex. 1.	Multiply	7 s. 6 d.	by 327	*Product*	£ 122	12	6
2.	—	13 s. 6 d.	- 245	—	165	7	6
3.	—	27 s. 3 d.	- 413	—	562	14	3
4.	—	£ 7 10 6	- 209	—	1572	14	6
5.	—	£ 11 10 3	- 338	—	3891	4	6
6.	—	£ 17 5 6	- 184	—	3178	12	0

COMPOUND DIVISION.

Rule. Divide each part of the given quantity, beginning with the highest, and set down each quotient as in Simple Division.

Obs. 1. The remainder of each part must be reduced into the next lower denomination, adding in that part, if any, of the given quantity which is in the same denomination.

2. If the divisor is not contained in that part of the dividend which is in the highest denomination, that part must be reduced until the divisor is contained.

3. If the divisor is not a *short* divisor, or a *double* divisor,* it can be used only as a *long* divisor; that is, we cannot use an imperfect divisor, and add or subtract for the difference, as we can in Multiplication.

EXAMPLE 1.

To divide £ 17 16 7¼ by 4 and by 32.

	£	s.	d.	
4)	17	16	7¼	
£	4	9	1¾	1 over.

	£	s.	d.	
4)	17	16	7¼	
8)	4	9	1¾	1 over.
£	0	11	1¼	29 over.

With respect to the remainder, we may either consider it as a surplus of the dividend, or we may value it as a fraction: thus £ 4 9 1¾ is the fourth part of £ 17 16 7, and the extra farthing makes the remainder.—Or, by making the remainder the numerator (the upper figure) and the divisor the denominator (the lower figure) of a fraction, we give the exact value;—as for the first quotient £ 4 9 1¾, ¼, that is, 1-4 th of a farthing; and in the second quotient £ 0 11 1½ $\frac{29}{32}$, meaning as we may further explain it, that if instead of stopping the division at the reduction into farthings, we were to reduce the remainder into 32 nd parts of farthings, then there would be a continuation of the quotient in 29 of these parts.

---

* N. B. A double divisor is one of the products in the Multiplication Table. Larger numbers can sometimes be made triple divisors, and used as such; as 4 times, 8 times, 6 times, for 192.

EXERCISES.

£ s. d.	£ s. d.	£ s. d.
2) 67 14 8	5) 49 15 5	7) 72 11 7
£	£	£

£ s. d.	£ s. d.	£ s. d.
3) 17 12 4½	6) 28 0 0	8) 81 14 9¼
£	£	£

£ s. d.	£ s. d.	£ s. d.
3) 44 11 10½	5) 67 14 8½	6) 87 10 6¼
5)	7)	8)
£	£	£

£ s. d.	£ s. d.	£ s. d.
4) 87 18 9½	3) 78 15 3¼	6) 63 12 9½
5)	8)	7)
8)	7)	9)
£	£	£

Ex. 2. To divide £ 181 14 10 by 100.

£ s. d.
£ 1,81 14 10
s. 16,34 multiply 81 by 20 and take in 14
d. 4,18 — 34 - 12 — 10

Quotient £ 1 16 $4\frac{18}{100}$.

In this method of dividing by 100 or by 1000, we cut off the 2 or 3 right-hand figures, and then the figures so cut off are the remainder, which is to be reduced into the next lower denomination.

For another method of dividing by 100 or 1000 see page 50.

EXERCISES.

	£	s.	d.				£	s.	d.		
Ex. 13.	234	15	6	by	100	Ex. 17.	4167	19	7½	by	1000
14.	123	14	5	-	100	18.	8222	11	6¼	-	1000
15.	89	16	7	-	100	19.	763	17	10¼	-	1000
16.	84	0	0	-	100	20.	852	15	9¾	-	1000

Ex. 3. To divide £ 827 13 7 by 37.

```
          £     s.   d.    £    s.  d.
   37 ) 827    13    7 (  22    7   4½ 34/37
         87
         13
       ------
       ) 273 ( 7 s.
          14
       ------
       ) 175 ( 4 d.
          27
       ------
       ) 108 ( 2 f.
          34
         ---
```

Observe that, in general, it is not necessary to value the remainder; and therefore when we are sure of the last quotient figure, we may put it down without working it. In the practice of business it is usual to take the last figure at the nearest number of times, or if the last remainder is above half the divisor, to call the quotient figure one more.

EXERCISES.

	£	s.	d.				£	s.	d.		
Ex. 21.	673	13	6	by	13	Ex. 26.	717	12	4½	by	67
22.	826	14	7	-	23	27.	536	17	2¼	-	78
23.	717	0	0	-	34	28.	456	13	7½	-	89
24.	444	15	6	-	46	29.	629	16	5¼	-	91
25.	827	17	2	-	51	30.	555	15	7¾	-	109

DIVISION OF WEIGHTS AND MEASURES.

TROY WEIGHT.

	lb.	oz.	dwt.		lb.	oz.	dwt.		oz.	dwt.	gr.
3)	616	11	6	5)	343	10	17	7)	555	17	21

AVOIRDUPOIS WEIGHT.

	cwt.	qr.	lb.		cwt.	qr.	lb.		cwt.	qr.	lb.
8)	371	1	17	9)	454	1	23	10)	614	1	11
	cwt.	qr.	lb.		cwt.	qr.	lb.		cwt.	qr.	lb.
21)	444	1	12	73)	836	1	26	100)	843	3	23

CLOTH MEASURE.

	yds.	qr.	na.		yds.	qr.	na.		yds.	qr.	na.
44)	672	1	2	56)	407	3	1	65)	814	1	3

QUOTIENTS.

Ex. 1.	£	33	17	4	Ex. 22.	£	35	18	10½-22
2.		5	17	5½	23.		21	1	9-6
3.		9	19	1	24.		9	13	4½-12
4.		4	13	4	25.		16	4	7¾-11
5.		10	7	4¼-5	26.		10	14	2½-20
6.		10	4	4-5	27.		6	17	7¾-39
7.		2	19	5½	28.		5	2	7½
8.		1	18	8¼-31	29.		6	18	5-25
9.		1	16	5½-25	30.		5	1	11½-115
10.		0	10	11¾-22	31.	lb.	205	7	15-1
11.		0	9	4½-13	32.		68	9	7-2
12.		0	3	4¼-236	33.	oz.	79	8	6-3
13.		2	6	11¼-84	34.	cwt.	46	1	19-5
14.		1	4	8¾-72	35.		50	1	27-4
15.		0	17	11½-36	36.		61	1	20-7
16.		0	16	9½-40	37.		21	0	17-19
17.		4	3	4¼-262	38.		11	1	23-27
18.		8	4	5¼-673	39.		8	1	21-23
19.		0	15	3¼-337	40.	yds.	15	1	0-22
20.		0	17	0½-679	41.		7	1	0-29
21.		51	16	5-1	42.		12	2	0-31

PARTICULAR METHODS OF FURTHER CONTRACTING OPERATIONS IN DIVISION.

Sometimes in the division of a simple quantity by a large divisor, instead of reducing the remainder in the ordinary method, the divisor may be exactly divided by the number that would be employed in the reduction : as,

Ex. 4. To divide £ 7567 by 480.

£ £ s. d.
480) 7567 (15 15 3½
2767
24) 367 (15 s.
127
2) 7 (3½ d.

In this calculation, instead of reducing the first remainder 367 £ into shillings, we divide the divisor by 20, and use the quotient 24 as the divisor upon 367 s.—so also instead of reducing the 7 s. into pence, and dividing by 24, we divide 24 by 12 and divide 7 d. by 2.

In case of having such a divisor as the above, or one which is the number of some lower denomination that is equal to the given integer, the result is obtained at once in that lower denomination: thus, as 480 half pence make 1 £, the 480 th part of 7567 £ is so many half pence: thus,

7567 half pence.
d. 3783½
s. 315 3½
£ 15 15 3½

This principle or mode of operation can frequently be well applied to the remainder, after the regular division has been made upon the given quantity ; thus, instead of reducing and dividing the remainder of the preceding calculation, viz. £ 367, by 480, we may reckon for the result 367 half pence, or 183½ pence, or 15 s. 3½ d.

As a summary of the chief contractions that can be thus employed with particular divisors used in the division of money, it may be observed, that in dividing pounds of money by 4, the remainder is so many times 5 s.; by 8, so many half crowns;

by 16	so many	times	1 s. 3 d.	by 24	so many	times	10 d.
30	—	—	8 d.	40	—	—	6 d.
60	—	—	4 d.	80	—	—	3 d.
120	—	—	2 d.	160	—	—	1½ d.
240	as so many pence			480	as so many half pence, and		
960	as so many farthings.						

In dividing shillings by certain numbers, similar valuations may be made.—Thus, when the divisor is any multiple of 12, we may consider the shillings as so many pence, and divide by the number of the multiple: as for 24, divide by 2; 48, by 4; 60, by 5, &c. If 120 is the divisor, 10 s. produce 1 d.; 5 s., ½ d.; and 2 s. 6 d., ¼ d.

Other contractions will often suggest themselves to the skilful calculator; as in dividing shillings by 30 or 40 we may consider them as pence, multiply them by 4 or 3, and cut off the unit figure.

In other divisions besides those of money we can sometimes employ a similar method.—Thus, to divide cwt. by 28, we may consider them as qrs. (which divides them by 4) and divide them by 7; the quotient is so many qrs. and the remainder is so many times 4 lb.; each qr. of an cwt. in the dividend, in this case, produces 1 lb.

EXERCISES.

To divide

Ex.	£	s.		Ex.	s.	d.		Ex.	£	s.	
1.	24	-	by 20	9.	7	-	by 12	17.	33	-	by 16
2.	36	10	— 20	10.	10	6	— 12	18.	38	-	— 30
3.	42	15	— 20	11.	13	3	— 12	19.	144	-	— 60
4.	347	-	— 240	12.	17	9	— 12	20.	172	-	— 80
5.	834	-	— 240	13.	21	6	— 12	21.	137	10	— 120
6.	561	-	— 240	14.	37	3	— 12	22.	364	15	— 120
7.	827	-	— 720	15.	45	9	— 12	23.	567	10	— 240
8.	999	-	— 960	16.	57	3	— 12	24.	738	15	— 240

25. 37 cwt. by 28
26. 55 cwt. by 28
27. 31 cwt. 2 qrs. by 28
28. 51 cwt. 3 qrs. by 28

RULE FOR DIVIDING A SUM OF MONEY BY 100.

Cut off the units and tens of pounds, and the other figures will give the amount in pounds.

Divide the number of the tens and units of pounds by 5, and consider the quotient as so many shillings.

Reckon the remainder of the pounds with half the number of the shillings, taking the nearest amount,* as so many farthings, abating 1 when they exceed 12, and 2 when they exceed 37.

N. B. This Rule applies also to the division by 1000, cutting off three figures, dividing the hundreds and tens by 5, and reckoning the remaining pounds, taking the nearest amount as farthings, with the same abatement.

It is founded on the valuation of the decimal parts of a pound, and is here inserted on account of its great utility in valuations of per centages.—The result is generally true to the nearest farthing, which is fully as precise as is ever required in the calculations of business.

EXAMPLE.

To divide £ 1767 16 4 by 100 and by 1000.

first product £ 17 13 6¾

second — 1 15 4¼

£1700 give £17; 67 considered s., and divided by 5, give 13 s.; the 2 over with 8 (half the number of the shillings) give 28 parts, called 27 farthings, or 6¾ d.

Again; £ 1000 give £ 1; 76 s. divided by 5 give 15 s.; and 1 over with 8 (the nearest number of pounds) give 18 parts, called 17 farthings, or 4¼ d.

EXERCISES.

To be proved by the common method.

	£	s.	d.				£	s.	d.		
Ex. 1.	234	15	6	by	100	Ex. 5.	4167	19	7	by	1000
2.	123	14	5	-	100	6.	8222	11	6	-	1000
3.	89	16	7	-	100	7.	763	17	10	-	1000
4.	84	0	0	-	100	8.	852	15	9	-	1000

---

* When there is an odd number of shillings, we generally reckon them as 1 more, in taking half their number, particularly if there be any pence, and the number of the remainder is between 12 and 25, or between 37 and 50, as the half of 17 s. 2 d. may be reckoned 9.

When the tens and units of the divisor are 25 or 75, it is sometimes convenient to multiply both the divisor and dividend by 4, and use the products in their places.

Example. To divide £ 1486 17 2 by 175

	£	s.	d.		£	s.	d.
175	1486	17	2				
4			4				
700)	5947	8	8 (		8	9	11
	347						
)	6948 (9 s.						
	648						
)	7784 (11 d.						

Or, using the Rule just given for dividing by 100, we may make the division thus;

	£	s.	d.
7)	5947	8	8
100)	849	12	8
	£ 8	9	11

We may here observe, that this method is frequently of use, when the two lower figures of the divisor are other parts than fourths of 100;—as 12½, multiply by 8; with 31¼, multiply by 16, or 32;—but to know when these can be employed, requires some proficiency in fractions.

Other contractions of divisions may be made by employing decimal valuations, and by dropping the dividing figures, for which the student may refer to the contractions of division at the end of that part of this work which treats on Decimals.

EXERCISES.

Ex.	1.	Divide	£ 3162	0	0	by	125 *	*Quotient*	£ 25	5	11
	2.	—	4070	10	0	-	175	—	23	5	2½
	3.	—	5264	15	0	-	625 *	—	8	8	5½
	4.	—	7142	17	6	-	875	—	8	3	3
	5.	—	5163	18	10	-	1225	—	4	4	3½
	6.	—	3487	10	0	-	8475	—	0	8	2¾

---

* In Ex. 1. we may also multiply by 8, and divide by 1000. In Ex. 3. we may multiply 16, and divide by 10000.

MULTIPLICATION

BY A FRACTIONAL NUMBER.

Case 1. When the multiplier is a simple fraction.

Rule. Multiply the given quantity by the numerator of the fraction, and divide the product by the denominator.

Or, when the fraction can conveniently be separated into parts, find the product for each part, and take the amount.

N. B. The *numerator* is the *upper* number of the fraction, and the *denominator* is the *lower* number.

EXAMPLE.

To multiply £ 84 16 4½ by ¾.

	£	s.	d.
	84	16	4½
			3
4)	254	9	1½
	£ 63	12	3⅜

(or)

	£	s.	d.	
½ . . .	84	16	4½	
½ . . .	42	8	2¼	for 2-4 ths
	21	4	1⅛	- 1-4 th
£	63	12	3⅜	

(or)

	£	s.	d.	
¼ . . .	84	16	4½	
	21	4	1⅛	deduct
	£ 63	12	3⅜	

It must be observed, that when the result is found by the deduction of a part, if there is any remainder, the lowest part or quantity of the amount to be subtracted must be reckoned as 1 more :—thus, in the third method, if the 4½ d. had been 4¾ d., instead of 1⅛, we should have reckoned 1¼ d.

EXERCISES.

Ex. 1.	Multiply	£	8	17	6	by	¾	*Product*	£	6	13	1½
2.	—		6	14	4½	-	⅗	—		4	0	7½
3.	—		7	0	10½	-	⅚	—		5	17	4⅛
4.	—		10	2	4	-	⅝	—		6	6	5½
5.	—		12	14	8	-	7/16	—		5	11	5
6.	—		18	16	10	-	11/16	—		12	19	0⅞

Case 2. When the multiplier is a mixed number

Rule. Multiply separately by the whole number and by the fraction, and add the products together.

EXAMPLE.

To multiply £ 8 11 5 by $5\frac{3}{4}$.

£	s.	d.					£	s.	d.		
8	11	5	×	5		=	42	17	1	for	5
8	11	5	× 3	÷	4	=	6	8	$6\frac{3}{4}$	-	$\frac{3}{4}$
						£	49	5	$7\frac{3}{4}$	-	$5\frac{3}{4}$

(or)

	£	s.	d.			
$\frac{1}{2}$...	8	11	5			
			5			
	42	17	1	for		5
$\frac{1}{2}$...	4	5	$8\frac{1}{2}$	-		$\frac{1}{2}$
	2	2	$10\frac{1}{4}$	-		$\frac{1}{4}$
£	49	5	$7\frac{3}{4}$	-		$5\frac{3}{4}$

(or)

	£	s.	d.			
$\frac{1}{4}$...	8	11	5			
			6			
	51	8	6	for		6
subtr.	2	2	$10\frac{1}{4}$	-		$\frac{1}{4}$
£	49	5	$7\frac{3}{4}$	-		$5\frac{3}{4}$

As the following form of multiplication sometimes occurs, it is here given to show how it may be performed.

To multiply £ $67\frac{1}{2}$ by $69\frac{1}{2}$

£ $67\frac{1}{2}$
$69\frac{1}{2}$
$34\frac{1}{2}$ 69 times $\frac{1}{2}$
603
402
$33\frac{3}{4}$ $\frac{1}{2}$ of $67\frac{1}{2}$
$4691\frac{1}{4}$

		£	s.	d.				£	s.	d.
Ex. 1.	Multiply	£ 8	10	6	by	$4\frac{1}{2}$	*Product*	£ 38	7	3
2.	—	11	13	4	-	$5\frac{3}{4}$	—	67	1	8
3.	—	16	17	6	-	$7\frac{5}{8}$	—	128	13	$5\frac{1}{4}$
4.	—	17	9	8	-	$11\frac{11}{16}$	—	204	6	$8\frac{3}{4}$
5.	—	23	13	1	-	$17\frac{6}{7}$	—	422	7	11
6.	—	16	11	6	-	$23\frac{11}{12}$	—	396	8	$4\frac{1}{2}$

DIVISION

BY A FRACTIONAL NUMBER.

Rule. Multiply both the divisor and the dividend by the denominator of the fraction, and divide the latter product by the former.

N. B. When a whole number with a fraction, that is a mixed number, is multiplied by the denominator of the fraction, it requires only the whole number to be multiplied, and the numerator of the fraction to be added in.

Divide £ 71 5 6 by $3\frac{3}{4}$

		£	s.	d.
$3\frac{3}{4}$		71	5	6
4				4
15 -	3)	285	2	0
	5)	95	0	8
		£ 19	0	$1\frac{1}{2},\frac{2}{5}$

We multiply the dividend and the divisor by 4 the denominator of this fiaction $\frac{3}{4}$, saying, in working the divisor, 4 times 3 are 12 and 3 (the numerator) are 15.

If the divisor is only a fraction (as $\frac{3}{4}$) we have to multiply by the denominator (as 4) and divide the product by the numerator (as 3).

Exercises.

	£	s.	d.				£	s.	d.		
Ex. 1.	84	16	4	by	$2\frac{1}{2}$	Ex. 4.	56	13	5	by	$4\frac{3}{5}$
2.	60	0	0	-	$3\frac{1}{2}$	5.	21	16	4	-	$5\frac{3}{7}$
3.	45	17	7	-	$5\frac{1}{2}$	6.	82	15	7	-	$6\frac{3}{8}$

Products.

Ex. 1.	33	18	$6\frac{1}{4},\frac{3}{5}$	Ex. 4.	12	6	$4\frac{1}{2},\frac{22}{23}$
2.	17	2	$10\frac{1}{4},\frac{1}{7}$	5.	4	0	$4\frac{1}{2},\frac{4}{38}$
3.	8	6	10	6.	12	19	$8\frac{1}{4},\frac{29}{51}$

DIRECT PROPORTION.

Four quantities are said to be in direct proportion, when the third term is the same product of the first, that the fourth is of the second, or, when the fourth term is the same product of the third, that the second is of the first.

If the first three terms of a direct proportion are given, the fourth may be found by the following Rule.

THE RULE OF THREE DIRECT.

State the three given terms in the form of a question, making the second term of the same kind as the answer required; then, of the two remaining terms, make that the first term of which the second term is the result, and make that the third term of which the answer required is to be the result. Bring the first and third terms into the same denomination; then multiply the second term by the number of the third, and divide the product by the number of the first, and the quotient will be the answer in the same denomination as the second term.

N. B. 1. If the three terms are of the same kind, as all in money, either the first and second, or the first and third, may be reduced into the same denomination.

2. When the first and third terms are simple quantities, if they are in the same denomination, they of course require no reduction; if they are not in the same denomination, the higher must be reduced into the denomination of the lower.—When either the first or the third term is a compound quantity, reduction must be made into the lowest denomination that either term contains.

3. When the second term is a compound quantity, it may be reduced into its lowest denomination for convenience of multiplication, when the third term is expressed by a large number; otherwise, the second term may be multiplied, without reducing it, according to any form of Compound Multiplication.

4. When the first and third terms are of the same denomination, if the first term is an integer, only a multiplication is required; if the third term is an integer, only a division is required.

5. If the first and second terms, or the first and third terms, can be exactly divided by any number, they may be divided, and their quotients be used in their places.

EXAMPLE 1.

To find the cost of 54 lb. of Sugar at 4½ d. per lb.

	lb.		d.		lb.
If	1	———	4½	———	54?
			6		
			2 3		
			9		
Answer	£ 1		0 3		

As we want to find the cost of 54 lb., and the cost of 1 lb. is given, we place this given cost as the second term, and the 1 lb. as the first term; and we complete the statement, by placing 54 lb. as the third term.

The question being thus stated, we see that the first and third terms do not want reduction, because they are both simple quantities in the same denomination; and the first term being an integer, 1 lb, we see that we have only to multiply the second term by the number of the third; that is, the third term being 54 times the first, we know that the fourth term must be 54 times the second.*

EXERCISES.

Ex. 1. If 1 lb. of Soap costs 7½ d. what cost 10 lb.?
2. If 1 lb. of Sugar costs 11 d. what cost 17 lb.?
3. What cost 12 lb. of Raisins when 1 lb. costs 8½ d.?
4. What cost 12 lb. of Rice when 1 lb. costs 4¾ d.?
5. What cost 12 yards of Cloth when 1 yard costs 17¼ d.?
6. What will 12 yards of Cloth cost at 19¾ d. per yard?

6. If the multiplication of the first and second or of the first and third terms by any number, will make the operation easier, by cancelling either a fraction, or the lower denominations of a compound quantity, they may be multiplied, and the products used in their places.

7. In these divisions and multiplications, it is only the first, with either the second or the third, that is to be operated upon, and not all three terms.

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* We may say, that four quantities are proportional when the same multiplier which produces the third term from the first, produces also the fourth from the

Ex. 7. What cost 2 dozen pair of Gloves, at 16½ d. per pair?
8. What cost 20 gallons of Rum, at 13 s. per gallon?
9. What cost 20 gallons of Brandy, at 27 s. 6 d. per gal.?
10. What cost 20 pair of Shoes, at 5 s. 9 d. per pair?
11. What cost 27 yards of Cloth, at 4 s. 3 d. per yard?
12. What cost 83 yards of Silk, at 7 s. 9 d. per yard?
13. If 1 lb. of Cotton costs 10 d., what will 7½ lb. cost?
14. If 1 lb. of Tea costs 7 s. 9 d., what will 11¾ lb. cost?
15. What is the value of 24¾ lb. of Indigo, at 4 s. 10 d. per lb.?
16. What is the value of 10⅝ yards of Cambric, at 10 s. 6 d. per yard?
17. What cost 27¾ lb. of Tea, at 6 s. per lb.
18. What cost 50½ lb. of Loaf Sugar, at 13 d. per lb.?
19. What cost 85⅜ yards of Silk, at 7 s. 6 d. per yard?
20. What cost 117¾ gallons of Oil, at 6 s. 8 d. per gallon?
21. When 1 foot of Mahogany costs 2 s. 7 d., what will 7 yards 2 feet cost?
22. When 1 lb. of Sugar costs 7 d., what will 1 cwt. 1 qr. 17 lb. cost?
23. When 1 lb. of Cheese costs 8 d., what will 2 cwt. 2 qrs. 23 lb. cost?
24. If 1 cwt. of Figs costs 57 s. 6 d., what will 2 tons 11¾ cwt. cost?

PRODUCTS.

Ex.					Ex.					Ex.				
1.	s.	6	3		9.	£	27	10	0	17.	£	8	6	6
2.		15	7		10.		5	15	0	18.		2	14	8½
3.		8	6		11.		5	14	9	19.		32	0	3¾
4.		4	9		12.		32	3	3	20.		39	5	0
5.		17	3		13.	s.	6	3		21.		2	19	5
6.		19	9		14.	£	4	11	0¾	22.		4	11	7
7.	£	1	13	0	15.		5	19	7½	23.		10	2	0
8.		13	0	0	16.		5	11	6¾	24.		148	15	7½

---

second, &c. which multiplier, when the first and third terms are in the same denomination, may be obtained in the fractional form, by making the number of the third term the numerator, and the number of the first term the denominator; thus, in the above Example 1.

the multiplier or index of the production is $\frac{54}{1}$, hence
the first term, 1 lb. $\times \frac{54}{1} = 54$ lb., the third term.
the second term, 4½ d. $\times \frac{54}{1} = 20$ s. 3 d., the fourth term.

EXAMPLE 2.

If £ 502 7 s. is the cost of 10 cwt. 2 qrs. 6 lb., what is the worth of 1 lb.?

```
    cwt. qrs. lb.            £  s.                 lb.
If   10   2   6 ————————— 502  7 ——————————————— 1 ?
    ————                  ———     s. d.
lb. 1182            1182 ) 10047 ( 8  6 per lb.—Answer.
    ————                    591
                          ————
                    1182 ) 7092 ( 6 d.
                           ····
```

We make 1 lb. the third term because it is required to find its value, we make 10 cwt. 2 qr. 6 lb. the first term because its value is given, and we make this value the second term.

We then reduce the first term into pounds, to make it in the same denomination as the third, and because the 3 rd term is the 1182 nd part of the first, we take the 1182 nd part of the second term in order to find the fourth term, or answer to the question; beginning the division by 1182 after reducing the pounds into shillings, because the divisor is not contained in the number of the pounds.*

EXERCISES.

Ex. 1. If 4 lb. of Tea cost 32 s., what costs 1 lb.?
2. If 6 lb. of Sugar cost 7 s. 6 d., what costs 1 lb.?
3. If 12 lb. of Cotton cost 10 s., what costs 1 lb.?
4. If 12 pair of Gloves cost 13 s. 6 d., what costs 1 pair?
5. If 12 yards of Muslin cost 17 s. 9 d., what costs 1 yard?
6. If 12 yards of Calico cost 5 s. 3 d., what costs 1 yard?
7. If 20 lb. of Cochineal cost £ 11 10 s., what costs 1 lb.?
8. If 20 lb. of Indigo cost £ 7 15 s., what costs 1 lb.?

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* The formula of this calculation may be thus expressed:
The first term, cwt. 10 2 6 × $\frac{1}{1182}$ = 1 lb. the third term.
The second term, £ 502 7 0 × $\frac{1}{1182}$. = 8 s. 6 d. the fourth term.

Ex. 9. If 25 men receive £ 76 5, what does each man receive?

Ex. 10. If 48 days' pay are £ 39 18, what is one day's pay?

Ex. 11. If 67 cwt. cost £ 79 16 10, what will 1 cwt. cost?

Ex. 12. If 123 cwt. cost £ 279 6 3, what will 1 cwt. cost?

Ex. 13. If 100 pieces of Gingham cost £ 122 10, what is the cost of 1 piece?

Ex. 14. If 100 yards of Muslin cost £ 7 18 4, what is the cost of 1 yard?

Ex. 15. If 100 yards of flowered Silk cost £ 56 13 4, what is the cost of 1 yard?

Ex. 16. If 1000 pieces of Nankeen cost £ 372 18 4, what costs 1 piece?

Ex. 17. If 4½ lb. of Wax cost 10 s. 1½ d., what will 1 lb. cost?

Ex. 18. If £ 4 9 1½ is given for 7¾ yards of Velvet, what will 1 yard cost?

Ex. 19. When the value of 3⅗ oz. of Silver is 27 s., what is the worth of 1 oz.?

Ex. 20. What is the worth of 1 lb. of Nutmegs, when 6⅝ lb. cost 24 s. 3½ d.?

Ex..21. If 3 lb. 11 oz. of Tea cost 29 s. 6 d., what is the value of 1 oz.?

Ex. 22. If 2 cwt. 3 qrs. 17 lb. of Cotton Wool cost £ 13 10 10, what costs 1 lb.?

Ex. 23. If 5 yds. 2 qrs. 2 nls. of Silk cost £ 1 16 6, what is the value of 1 yard?

Ex. 24. If 11 cwt. 1 qr. of Bitter Almonds cost £ 14 10 8, what is the price of 1 lb.?

ANSWERS.

	£	s.	d.		£	s.	d.
Ex. 1.		8	0	Ex. 13.	1	4	6
2.		1	3	14.		1	7
3.		0	10	15.		11	4
4.		1	1½	16.		7	5½
5.		1	5¾	17.		2	3
6.		0	5¼	18.		11	6
7.		11	6	19.		7	6
8.		7	9	20.		3	8
9.	3	1	0	21.		0	6
10.		16	7½	22.		0	10
11.	1	3	10	23.		6	5¾
12.	2	5	5	24.		0	2¾

EXAMPLE 3.

To find the worth of cwt. 3 1 17 of Hops, when cwt. 7 3 22 cost £ 44 12 6.

	cwt. qrs. lb.	£ s. d.	cwt. qr. lb.
If	7 3 22 ———	44 12 6 ———	3 1 17?
	784	892 s.	336
	106 = 3 qrs. 22 lb.		45 = 1 qr. 17 lb.
	890 lb.	10710 d.	381 lb.
		381	
		10710	
		85680	
		32130	

89·0) 408051·0 (4584 d.

520

755 382 s.

431

75 £ 19 2 0 Answer.

...

The first and third terms being compound quantities, require to be reduced into their lowest denomination, viz. pounds, and then in order to more conveniently multiply the second term by 381, it is reduced into pence.

The reason of this calculation is, that if 890 lb. cost 10710 pence, then 1 lb. will cost 890 th part of these pence, and consequently at this rate, 381 lb. will cost 381 times the 890 th part, or what is the same thing, the 890 th part of 381 times.*

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* To express this operation according to the formula of the preceding example, the index or multiplier being $\frac{381}{890}$, we say, that

1 st term — cwt. 7 3 22 × $\frac{381}{890}$ = cwt. 3 1 17 — 3 rd term.
2 nd term — £ 44 12 6 × $\frac{381}{890}$ = £ 19 2 0 — 4 th term.

Respecting the working of such a calculation as the above, it should be well understood, that although in abstract numbers, 381 times 10710, are the same product as 10710 times 381, yet 381 times 10710 pence are not the same amount as 10710 times 381 lb.; and therefore, if in calculations of the Rule of Three, for convenience sake we make the multiplication of the numbers under the third term, or say that we multiply the second and third terms together, yet it is properly only the product of the second term that is to be taken, for to say that we can multiply by any quantity, as by 381 lb. or by 10710 pence, is very incorrect, the multiplier never has any other object than to show how many times the multiplicand is to be taken, that is, how many times it would be repeated by addition, and we cannot very properly say we take 381 lb. weight of times, or 10710 pennyworth of times. It is equally as improper

EXERCISES.

Ex. 1. When 7 pipes of Wine cost £ 350, what is the worth of 4 Pipes?

Ex. 2. What is the worth of 13 pints of Alcohol, when 7 pints cost £ 1 11 6?

Ex. 3. What is the worth of 27 pair of Shoes, at the rate of £ 5 per dozen pair?

Ex. 4. What is the worth of 7 cwt. 3 qrs. of Figs, at £ 2 15 per cwt.?

Ex. 5. Find the value of 4 lb. 11 oz. of Isinglass, at 27 s. 6 d. per lb.?

Ex. 6. If 52 weeks' Wages are £ 5 5, what is the amount of 17 weeks' Wages?

Ex. 7. What is due to a servant for 25 days' Wages, at 10 guineas per annum?

Ex. 8. What is the amount of a clerk's Salary, from the 25 th of December to the 17 th of February, at 80 guineas per annum?

Ex. 9. If the Charges upon Goods be 8 per cent., that is, £ 8 upon every £ 100 of Goods, what is the amount of the Charges upon Goods amounting to £ 467 12 6?

Ex. 10. If Interest at 5 per cent. amounts to £ 67 4 4, what will it amount to at 4 per cent.?

Ex. 11. If 12 s. 6 d. will buy 6 lb. 4 oz. of Coffee, how much will 20 s. purchase?

Ex. 12. How much Sugar can I buy with £ 100 at the rate of 66 s. 6. d. per cwt.?

Ex. 13. The amount of the Debts upon a bankrupt's estate is £ 6842, and the amount of the Assets is £ 3364, at what rate will be the Dividend in the pound upon these amounts?

Ex. 14. If Interest upon a given sum at £ 5 per cent. be £ 4 11 6, what will it be at 4 per cent.?

Ex. 15. If £ 5 of Interest requires £ 100 in money, upon what capital does the Interest amount to £ 13 12 6?

Ex. 16. If Mercury performs 32 revolutions round the Sun in 2813 of our days, how many revolutions does he perform in 87 of our years, considering each year to contain exactly 365 days 6 hours?

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to say we divide by 890 lb., for the divisor only shows what part is to be taken of the dividend, or how many times the quotient would require to be repeated to make it equal to the dividend; and it is ridiculous to say we take the 890 th lb. part.

Ex. 17. If the progress of sound be uniformly 1142 feet per second of time, in what time will the report of a piece of ordnance be heard at the distance of 23 miles ?

Ex. 18. If light move with the velocity of 200,000 miles per second, in what time would an eclipse of one of Jupiter's satellites be seen on the surface of the earth, at the distance of 437 millions of miles ?

Ex. 19. In what time would a vessel, making good a direct distance of 126 miles per day, arrive from Jamaica, supposing the whole distance to be 67½ degrees of 69½ miles each ?

Ex. 20. If a hhd. of Sugar weighing 13 cwt. 1 qr. 17 lb. cost £ 37 7 2, what was it rated at per cwt. ?

Ex. 21. If 37 cwt. 3 qrs. 17 lb. of Goods is carried 157 miles for £ 6 13 2, how much may be carried the same distance for £ 11 4 6?

Ex. 22. If £ 2865 bear a loss of £ 1117, how much at this rate will £ 1735 have to bear ?

Ex. 23. If my Income is £ 350 per annum, and I spend on an average 15 s. 6 d. per day, how much can I lay by in 1 year ?

Ex. 24. If my Income is £ 800 per annum, and I wish to lay by 150 guineas yearly, how much shall I have left to spend per day ?

PRODUCTS.

Ex. 1.	£	200	0	0	Ex. 13.	£	0	9	10
2.		2	18	6	14.		3	13	2¼
3.		11	5	0	15.		272	10	0
4.		21	6	3	16.	revs.	361		
5.		6	8	10¾	17.	min.	1	46	
6.		1	14	3¾	18.	min.	36	25	
7.		0	14	4½	19.	days	37$\frac{29}{126}$		
8.		12	8	6½	20.	£	2	15	9
9.		37	8	2¼	21.	cwt.	63	3	16
10.		53	15	5½	22.	£	676	8	9
11.	lb.	10	0	0	23.		67	2	6
12.	cwt.	30	0	8	24.		1	15	2¼

INVERSE PROPORTION.

Inverse Proportion is when of four quantities, arranged as in Direct Proportion, the fourth term is the same product of the second term, that the first term is of the third, and not the third of the first.*

If, therefore, when the given terms of a proportion are placed in the form of question, and the third, being greater than the first, requires the fourth term to be in the same degree less than the second; or when the third is less than the first, and the fourth term is required to be in the same degree greater than the second term, the calculation of the fourth term belongs to the following Rule.

RULE OF THREE INVERSE.

Rule. Reduce, if required, the first and third terms into the same denomination; then multiply the second term by the number of the first term, and divide the product by the number of the third term.

Observe. In questions of Inverse Proportion we may divide the first and third, or the second and third, terms, by any number that will exactly divide them, but not the first and second, as in Direct Proportion.

EXAMPLE.

If 8 men can do a piece of work in 12 days, how many days will 16 men require to perform the same?

	men		days		men	
If	8	require	12	what will	16	require?
			8			
		16)	96			
		Answer	6	days.		

---

* This supposes that the arrangement of the first three terms is made in the form of a question; otherwise, if the quantities are arranged according to the regular form of a proportion, that is, with the similar terms in pairs, they are then said to be inversely, or rather reciprocally, proportional, when the second term is the same product of the first that the third term is of the fourth.

Exercises.

Ex. 1. If the provisions of a Garrison will last 2500 men for 73 days, how long will they last when the Garrison is reinforced by 750 men?

Ex. 2. If 10 yards of Muslin 4 quarters wide are sufficient for a dress, how much will be required of Muslin which measures 5 qrs. wide?

Ex. 3. If 15 yards of Calico 4 quarters wide are sufficient to form a lining, how much will be required of Calico 3 quarters wide?

Ex. 4. If a journey be performed in 7 days, travelling at the rate of 120 miles per day, how long will it require at 150 miles per day?

Ex. 5. If I lend a friend £ 200 for 12 months, how long ought he in return lend me £ 150?

Ex. 6. If I sell out £ 754 10 0 of 3 per cent. Stock at $92\frac{1}{2}$ per cent. net, how much $3\frac{1}{2}$ per cent. Stock can I procure in exchange, if the full cost of the latter is $101\frac{1}{4}$ per cent.?

Ex. 7. If I sell 127 oz. 17 dwts. of old silver Plate at 5 s. 6 d. per oz. how much new Plate can I purchase with the amount, at the rate of 7 s. 9 d. per oz.?

Ex. 8. How much Silver 11 oz. 2 dwts. fine,* is equal to 37 lb. 11 oz. 10 dwts. of Silver 10 oz. 15 dwts. fine?

Products.

Ex. 1.	Days $56\frac{2}{13}$	Ex. 5. m. 16
2.	Yds. 8	6. £ 689 5 11
3.	Yds. 20	7. oz. 90 14 15
4.	Days $5\frac{3}{5}$	8. lb. 36 9 2 18

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* 11 oz. 2 dwts. fine means, that out of the 12 oz. of metal, here called Silver, 11 oz. 2 dwts. are fine silver, the remaining 18 dwts. being copper or other alloy.

COMPOUND PROPORTION.

Compound Proportion is a combination of two or more simple proportions, occurring in the same calculation, when the product or *fourth* term of the first simple proportion, forms the second term of the second simple proportion; and the product of this proportion also forms the second term of the third simple proportion, and so on, with as many simple proportions as may be given.

Instead of separately working these proportions, they may be contracted by making the multiplications in succession, and dividing the product either by the divisors in succession, or by the product of the divisors, according to the following Rule.

Rule. Make the statements of the simple proportions, making that as the common term, which is similar to the product required; mark them as being direct, or inverse, and reduce the terms, if necessary, as in simple proportion.

Then, if both, or all, the proportions are direct, or both, or all, are inverse, take the product of the numbers of the first terms, and also the product of the numbers of the third terms, and use them upon the common term as in the Rule of Three Direct or Inverse.

But if one proportion is direct and the other inverse, repeat the statement of the direct, and invert the terms of the inverse proportion, and proceed as before.

Observe. Any first term, together with any third term, may be divided by whatever number will exactly divide them; and in direct proportions, any first and second terms may also be divided; then the quotients thus obtained are to be used instead of the original terms.

EXAMPLE 1.

If 14 horses eat 56 busnels of oats in 16 days, how many bushels will be sufficient for 20 horses for 24 days?

Dir. If 14 horses *Dir.* in 16 days	} require	B. 56 480	what will {	20 horses 24 days?
224		4480 224		480

224) 26880 (120 bushels.—Answer.

To be read thus—If 14 horses in 16 days require 56 bushels, what will 20 horses in 24 days require? Both questions are direct, because the more horses the more bushels, and the more days the more bushels?

EXAMPLE 2.

If 8 men in 14 days can mow 112 acres of grass, how many men must there be to mow 2000 acres in 10 days?

Dir. If 112 acres *Inv.* in 14 days	} require	MEN 8	what will {	2000 acres in 10 days?

repeated

112 acres 10 days }		MEN 8 28000	{ 2000 acres 14 days
1120	1120)	224000	28000
	Answer.	200	men.

The first proportion is direct, because a greater number of acres requires a greater number of men to mow them; but the second is inverse, because the less number of days requires a greater number of men.

It may also be worked thus;

Divide 14 and 112 by 14, the quotients are 1 and 8, and dividing 8 and 8 by 8, the quotients are 1 and 1, and using these quotients instead of the original numbers, we have only to divide 2000 by 10.

EXERCISES.

Ex. 1. If £ 100 in 12 months gain £ 6 Interest, how much will £ 75 gain in 9 months?

Statement.

$$\left.\begin{array}{l}\textit{Dir.}\ \text{If £ 100}\\ \textit{Dir.}\ \text{mo. 12}\end{array}\right\}\ \text{gain}\ \overset{\text{£}}{6}\ \text{what will}\ \left\{\begin{array}{l}\text{£ 75}\\ \text{9 mo.?}\end{array}\right.$$

Ex. 2. If a regiment of Soldiers consisting of 600 men, consume 162 quarters of wheat in 108 days, how many quarters of wheat will 11232 men consume in 56 days.

Statement.

$$\left.\begin{array}{l}\textit{Dir.}\ \text{If 600 men}\\ \textit{Dir.}\ \text{108 days}\end{array}\right\}\ \text{require}\ \overset{\text{QUARTERS.}}{162}\ \text{what will}\ \left\{\begin{array}{l}\text{11232 men}\\ \text{56 days?}\end{array}\right.$$

Ex. 3. If 40 acres of Grass be mowed by 8 men in 7 days, how many acres could be mowed by 24 men in 28 days?

Statement.

$$\left.\begin{array}{l}\textit{Dir.}\ \text{If 8 men}\\ \textit{Dir.}\ \text{7 days}\end{array}\right\}\ \text{mow}\ \overset{\text{ACRES}}{40}\ \text{what will}\ \left\{\begin{array}{l}\text{24 men}\\ \text{28 days?}\end{array}\right.$$

Ex. 4. If £ 100 in 12 months gain £ 6 Interest, what Principal will gain 67 s. 6 d. in 9 months?

Statement.

$$\left.\begin{array}{l}\textit{Dir.}\ \text{If £ 6}\\ \textit{Inv.}\ \text{12 months}\end{array}\right\}\ \text{require}\ \overset{\text{£}}{100}\ \text{what will}\ \left\{\begin{array}{l}\text{67 s. 6 d.}\\ \text{9 months?}\end{array}\right.$$

Ex. 5. If £ 100 in 365 days gain £ 5 Interest, what will £ 847 gain in 87 days?

Statement.

$$\left.\begin{array}{l}\textit{Dir.}\ \text{If £ 100}\\ \textit{Dir.}\ \text{days 365}\end{array}\right\}\ \text{gain}\ \overset{\text{£}}{5}\ \text{what will}\ \left\{\begin{array}{l}\text{£ 847}\\ \text{87 days?}\end{array}\right.$$

Ex. 6. If by working 16 hours per day, a piece of work requires 100 men for 15 days, how many men will be required to perform the same in 25 days, working 12 hours per day.

Statement.

$$\left.\begin{array}{l}\textit{Inv.}\ \text{If 15 days}\\ \textit{Inv.}\ \text{16 hours}\end{array}\right\}\ \text{require}\ \overset{\text{MEN}}{100}\ \text{what will}\ \left\{\begin{array}{l}\text{25 days}\\ \text{12 hours?}\end{array}\right.$$

PRODUCTS.

Ex. 1.	£ 3 7 6	Ex. 4.	£ 75 0 0
2.	Q. $1572\frac{12}{25}$	5.	£ 10 1 11
3.	A. 480	6.	Men 80

CONTINUED PROPORTION.

Continued Proportion is a combination of two or more Proportions, when the product or fourth term of each simple proportion forms the third term of the following proportion.

Instead of working these proportions separately, they may be performed together like Compound Proportion, by the following Rule, called the Chain Rule.

CHAIN RULE.

Arrange the given quantities in the following order; make the quantity whose value, weight, or other result is required, the term of demand; and in order to distinguish it, place against it a note of interrogation; then place the first and second terms of the given proportions, so that the first term of the first proportion may be similar to the term of demand, and that the first term of each following proportion may be similar to the last second term.

Reduce, if necessary, the similar terms into their lowest denomination, multiply the last term by the numbers in succession of all the second terms, and also the term of demand, and divide the product by the product of the numbers of the first terms.

Observe 1. The first terms are usually called the antecedents, and the others consequents; and the statement of them is called an equation, because each consequent is the value or equivalent of its antecedent, each pair of terms being called a rate of the equation.

2. Any antecedent and any consequent may be divided by a number, that will exactly divide them, and their quotients may be used in their places; and this may be repeated as often as practicable, taking only two terms, one on each side, at a time.

3. When the equation is complete, the last quantity will be of the same species as the quantity which is required to be equivalent to the term of demand.

4. The rule may be thus expressed: "Having made the necessary reductions, multiply the numbers of the consequents together for a dividend, and the numbers of the antecedents together for a divisor, and the quotient will be the number of the product required."

EXAMPLE 1.

If 22 oz. of fine gold make 24 oz. of standard gold, and 12 oz. make 1 pound, and for the journey of 15 lb. of standard gold the Royal Mint delivered 701 sovereigns, what number of sovereigns were delivered in return for 367 oz. of fine gold?

$$\begin{array}{llrcrl}
 & & & & 367 & \text{oz. fine?} \\
\text{Oz.} & \text{fine} & 22 & = & 24 & \text{oz. standard.} \\
\text{Oz.} & & 12 & = & 1 & \text{lb.} \\
\text{lb.} & & 15 & = & 701 & \text{Sovereigns.}
\end{array}$$

$$\begin{array}{rcrl}
22 & & 701 & \text{Sovereigns.} \\
12 & & 367 & \\
\hline
264 & & 257267 & \\
15 & & 24 & \\
\hline
\end{array}$$

$$\begin{array}{rl}
396{,}0\,) & 617440{,}8\ (\ 1559\ \text{Sovereigns.} \\
 & \ \ 2214 \\
 & \ \ \ \ 2344 \\
 & \ \ \ \ \ \ 3640 \\
 & \ \ \ \ \ \ \ \ 768\ \text{remainder}
\end{array}$$

The term of demand is here 367 oz. of fine gold, and the value of this quantity is required in sovereigns, which therefore forms the last consequent.

Or, dividing 12 and 22 by 12 and by 2 for 24, and dividing 24 by 24, we have on one side 11, 1, and 15, and on the other 367, 1, 1, and 701, and the statement of the calculation is expressed thus,

$$\frac{701}{15} \times \frac{\overset{1}{24}}{\underset{1}{12}} \times \frac{367}{\underset{11}{22}}$$

and is worked thus,

$$\begin{array}{rrrr}
 & & \text{S.} & \\
 & & 701 & \\
 & & 367 & \\
\cline{3-3}
165\left\{\begin{array}{l}11\\15\end{array}\right. & \begin{array}{l})\\)\end{array} & \begin{array}{r}\overline{257267}\\\overline{23387}\end{array} & \begin{array}{r}\\10\end{array} \\
\cline{3-4}
 & \text{S.} & 1559\frac{32}{165} & \\
\cline{3-4}
\end{array}$$

Exercises.

Ex. 1. The Cologne mark weighs 3608 grains Troy. What is then the value of this weight of fine silver, when 37 oz. of fine silver make 40 oz. of British standard silver, and 1 oz. of standard silver is worth 60 d. sterling ?

1 Cologne mark fine ?
1 = 3608 grains Troy.
480 = 1 oz. Troy.
Statement. Oz. fine 37 = 40 oz. standard.
Oz. standard 1 = 60 pence.

Product £ 2 0 8.

Ex. 2. 10 ells of Vienna are equal to 3454 old French lines, 44329½ French lines are equal to 100 metres, 1000 metres are equal to 39371 English inches, and 36 English inches are equal to 1 English yard. How many English yards are equal to 1000 Vienna ells ?

1000 Vienna ells ?
10 = 3454 French lines.
44329½ = 100 Metres.
Statement. 1000 = 39371 English inches.
36 = 1 Yard.

Product 852 English yards.

Ex. 3. The Berlin quart contains 64 Prussian cubic inches, 1728 cubic inches make 1 cubic foot, 10 P. cubic feet equal 15585 old French cubic inches, 50412½ French cubic inches equal 1000 litres, and 10,000 litres equal 2201 Imperial gallons. How many Imperial gallons are therefore equal to 1000 Berlin quarts ?

1000 Berlin quarts ?
1 = 64 Pr. cub. in.
1728 = 1 Cubic foot.
Statement. 10 = 15585 Fr. cub. in.
50412½ = 1000 Litres.
10000 = 2201 Imperial gallons.

Product 252 Imperial gallons.

PRACTICE.

Practice is the method of finding the fourth term of a proportion in parts of the producing term.

Operations in Practice are divided into those which are direct, and those which are indirect. Direct is when the required product is directly obtained from a given amount; Indirect Practice is when the product is obtained from an assumed amount.

DIRECT PRACTICE.

Case 1. When the given quantity is less than the integer of the rate.[*]

Rule. Take the same parts out of the given amount, that the given quantity is of the integer.

EXAMPLE 1.

To find the amount of 11 oz. of Tea, at 8 s. 6 d. per lb.

	s.	d.		
½	8	6	value of	1 lb.
¼	4	3	. .	8 oz.
½	1	0¾	. .	2 oz.
		6⅜	. .	1 oz.
	s. 5	10⅛	. .	11 oz.

In calculations of Practice, the statement of a question may be made in the same manner as in the calculations of the Rule of Three; and for the information of the young student the production of the third term in parts of the first term may be first made, in order to show how the fourth term is to be obtained from the second.

---

[*] A rate is the two terms given in order to find a proportional result from another given quantity, and consequently answers to the first and second terms of the general mode of stating questions to be worked by the Rule of Three. An integral rate is therefore one in which the number of the first term would be expressed by unity, a per centage rate, by 100, &c.

The reduction of the remainders from the pence may be made into either 4 ths or farthings, or 8 ths, that is half farthings, according to the nicety that the calculation may require; and when the parts are numerous or complicated, the remainders of the pence may be reduced into 10 ths or 100 ths; or the same may be done with the remainders of the farthings.—Thus, repeating the foregoing calculation, and showing the production of the third term in parts of the first,

	lb.		s. d.		oz.
If	½ . . . 1	————	½ . . . 8 6	————	11 ?
	¼ . . . 8 oz.		¼ . . . 4 3		
	½ . . . 2 -		½ . . . 1 0.75-100 ths.		
	1 -		6.37		
	11 oz.		s. 5 10.12		

The amount of the 100 ths is 112, of which the 100-100 ths are carried forwards as 1 penny.

Exercises.

Find the amount of

			s.	d.
Ex. 1.	5 oz. at 8 s. per lb.	*Product*	s. 2	6
2.	7 oz. at 4s. 6 d. per lb.	—	- 1	11⅝
3.	11 oz. at 5s. 6 d. per lb.	—	- 3	9⅜
4.	7 dwts. at 5 s. 6 d. per oz. Troy	—	- 1	11
5.	13 dwts. at 6 s. 2½ d. per oz. Troy	—	- 4	0⅜
6.	17 dwts. at 4 s. 11¾ d. per oz. troy	—	- 4	2¾
7.	21 lb. at 14 s. 6 d. per cwt.	—	- 2	8⅝
8.	17 lb. at 18 s. 3 d. per cwt.	—	- 2	9 2/8
9.	23 lb. at 29 s. 6 d. per cwt.	—	- 6	0⅝
10.	5 oz. 7 drs. at 4 s. 6 d. per lb.	—	- 1	6¼
11.	7 oz. 11 drs. at 3 s. 7 d. per lb.	—	- 1	8⅝
12.	13 oz. 13 drs. at 5 s. 10½ d. per lb.	—	- 5	0¾
13.	1 qr. 14 lb. at 14 s. 0 d. per cwt.	—	- 5	3
14.	2 qrs. 21 lb. at 28 s. 0 d. per cwt.	—	- 19	3
15.	1 qr. 23 lb. at 37 s. 6 d. per cwt.	—	- 17	0⅞

Case 2. When the given quantity is greater than the integer of the rate.

Rule. Multiply the given rate by the number of the integers, and take parts for the remaining quantity.

EXAMPLE 2.

To find the amount of 7 lb. 11 oz. at 8 s. 1½ d. per lb.

	£	s.	d.		
½ ...		8	1½	value of 1 lb.	
			7		
	2	16	10½	— 7 lb.	
¼ ...		4	0¾	—	8 oz.
½ ...		1	0-.75	—	2 oz.
			6-.37	—	1 oz.
£	3	2	5¼.12	— 7 lb.	11 oz.

The remaining farthings are here reduced into 100 ths, and then divided; the amount, 112-100 ths, gives 1 farthing and 12 over.

EXERCISES.

Find the amount of

Ex.			£	s.	d.
1.	7 lb. 8 oz. at 11 d. per lb.	*Product* £	0	6	10½
2.	11 lb. 13 oz. at 14½ d. per lb.	—	0	14	3¼
3.	17 lb. 9 oz. at 1 s. 6 d. per lb.	—	1	6	4⅛
4.	10 oz. 11 dwts. at 5 s. 6 d. per oz.	—	2	18	0¼
5.	16 oz. 14 dwts. at 5 s. 1¾ d. per oz.	—	4	5	11⅛
6.	27 oz. 19 dwts. at 7 s. 6 d. per oz.	—	10	9	7½
7.	11 cwt. 3 qrs. 14 lb. at 24 s. 6 d. per cwt.	—	14	10	11¼
8.	25 cwt. 1 qr. 18 lb. at 31 s. 3 d. per cwt.	—	39	14	1
9.	63 cwt. 3 qrs. 7 lb. at 42 s. 0 d. per cwt.	—	134	0	1½

PRACTICE. PART II.

INDIRECT PRACTICE.

Case 1. When the given quantity is a simple quantity.

Rule. Assume the amount of the whole at an integral rate, and find the required amount in multiples or parts of the assumed amount.

Example 1.

Find the amount of 117 yards at 4 s. 6 d. per yard.

```
     £  s.              (or)          £
½ ... 5 17  amt. at 1 s.        ⅕ ... 117  amt. at 1 £
        4                             ———
    ———————                     ⅛ ...  23   8   0  . . 4 s.
                                        2  18   6  . . 6 d.
     23  8 . . . 4 s.                 ——————————
      2 18  6 . . 6 d.               £ 26   6   6
     ——————————                       ——————————
  £  26  6  6
     ——————————
```

Exercises.*

Find the amount of

Ex. 1. 167 lb. at ½ d.
2. 334 lb. at ¼ d.
3. 616 lb. at ¾ d.
4. 628 lb. at 1¼ d.
5. 444 lb. at 1½ d.
6. 627 lb. at 1¾ d.

Ex. 7. 444 yards at 2¾ d.
8. 815 yards at 3¼ d.
9. 626 yards at 3¾ d.
10. 489 yards at 4¼ d.
11. 626 yards at 4¾ d.
12. 814 yards at 5 d.

Products.

Ex. 1.	£ 0	6	11½	Ex. 2.	£ 0	6	11½	Ex. 3.	£ 1	18	6
4.	3	5	5	5.	2	15	6	6.	4	11	5¼
7.	5	1	9	8.	11	0	8¾	9.	9	15	7½
10.	8	13	2¼	11.	12	7	9½	12.	16	19	2

---

* In each of these Exercises the value may be assumed for the whole quantity at 1 d. per lb. or 1 s. per lb., and instead of putting it down in pence, it is shorter to

EXERCISES CONTINUED.

Assume the value at 1 s. each in the work, and at 1 d. each in the proof.

Find the value of

Ex. 13. 802 oz. at 6½ d.
14. 474 oz. at 6¾ d.
15. 964 oz. at 7½ d.
16. 824 oz. at 9¼ d.
17. 666 oz. at 9½ d.
18. 737 oz. at 9¾ d.

Ex. 19. 555 qr. at 10¼ d.
20. 444 qr. at 10½ d.
21. 333 qr. at 10¾ d.
22. 666 qr. at 11¼ d.
23. 789 qr. at 11½ d.
24. 897 qr. at 11½ d.

Assume the value at £ 1 each in the work, and at 1 s. each in the proof.

25. 438 lb. at 1 s. 10¼ d.
26. 626 lb. at 2 s. 3½ d.
27. 819 lb. at 3 s. 8½ d.
28. 864 lb. at 5 s. 8½ d.
29. 257 lb. at 7 s. 10¾ d.
30. 186 lb. at 8 s. 8½ d.

31. 804 cwt. at 10 s. 6¾ d.
32. 514 cwt. at 11 s. 7½ d.
33. 647 cwt. at 13 s. 8½ d.
34. 270 cwt. at 15 s. 11¾ d.
35. 445 cwt. at 16 s. 9¾ d.
36. 826 cwt. at 18 s. 4½ d.

PRODUCTS.

Ex.	£	s.	d.	Ex.	£	s.	d.	Ex.	£	s.	d.
13.	21	14	5	14.	13	6	7½	15.	30	2	6
16.	31	15	2	17.	26	7	3	18.	29	18	9¾
19.	23	14	0¾	20.	19	8	6	21.	14	18	3¾
22.	31	4	4½	23.	37	16	1½	24.	42	19	7½
25.	40	12	1½	26.	71	14	7	27.	151	17	1½
28.	246	12	0	29.	101	9	2¾	30.	80	19	9
31.	424	12	3	32.	298	15	3	33.	443	9	3½
34.	215	14	4½	35.	374	1	6¾	36.	758	17	9

---

express it at once in shillings; as, instead of putting down 167 d., to put down 13 s. 11 d., mentally performing the division by 12. In the same manner when the value is assumed at 1 s., the amount may be expressed in £, as in the first work of the above example.

Case 3. When the given quantity is a compound quantity, having denominations lower than the integer of the rate.

Rule. Separately find the value of the lower denominations, and add it to the value of the other part.*

EXAMPLE 6.

To find the amount of 73 cwt. 1 qr. 17 lb. at 47 s. 6 d. per cwt.

£
$\frac{1}{4}$ . . . 73
2
146 . . for 40 s.
$\frac{1}{8}$ . . . 18 5 0 . 5 s.
9 2 6 . 2 s. 6 d.
19 1 for 1 qr. 17 lb.
£ 174 6 7

s. d.
$\frac{1}{7}$ . . . $\frac{1}{4}$ . . . 47 6
11 10½
$\frac{1}{16}$ . . . 6 9½
5
s. 19 ·1

In the practice of business it is very common to unite the two parts in the following form; but it requires much caution to avoid confusion and inaccuracy.

s.
$\frac{1}{2}$ 73
$\frac{1}{7}$ . . . $\frac{1}{4}$ . . . 47 6
511
292
36 6 for 6 d.
11 10½ for 1 qr.
$\frac{1}{16}$ . . . . 6 9½ . 16 lb.
5 . 1 lb.
s. 3486 7
£ 174 6 7

We here first multiply 73 s. by 47 for 47 s., and take the half for 6 d.; then reckoning the multiplier as 47 s. 6 d., we take parts out of this value for the 1 qr. 17 lb.

---

* For various particular rules for Practice calculations, see the Appendix.

EXERCISES CONTINUED.

Assume the value at 1 s. each in the work, and at 1 d. each in the proof.

Find the value of

Ex. 13. 802 oz. at 6½ d.		Ex. 19. 555 qr. at 10¼ d.	
14. 474 oz. at 6¾ d.		20. 444 qr. at 10½ d.	
15. 964 oz. at 7½ d.		21. 333 qr. at 10¾ d.	
16. 824 oz. at 9¼ d.		22. 666 qr. at 11¼ d.	
17. 666 oz. at 9½ d.		23. 789 qr. at 11½ d.	
18. 737 oz. at 9¾ d.		24. 897 qr. at 11¼ d.	

Assume the value at £ 1 each in the work, and at 1 s. each in the proof.

25. 438 lb. at 1 s. 10¼ d.	31. 804 cwt. at 10 s. 6¾ d.
26. 626 lb. at 2 s. 3½ d.	32. 514 cwt. at 11 s. 7½ d.
27. 819 lb. at 3 s. 8½ d.	33. 647 cwt. at 13 s. 8½ d.
28. 864 lb. at 5 s. 8½ d.	34. 270 cwt. at 15 s. 11¾ d.
29. 257 lb. at 7 s. 10¾ d.	35. 445 cwt. at 16 s. 9¾ d.
30. 186 lb. at 8 s. 8½ d.	36. 826 cwt. at 18 s. 4½ d.

PRODUCTS.

Ex.	£	s.	d.	Ex.	£	s.	d.	Ex.	£	s.	d.
13.	21	14	5	14.	13	6	7½	15.	30	2	6
16.	31	15	2	17.	26	7	3	18.	29	18	9¾
19.	23	14	0¾	20.	19	8	6	21.	14	18	3¾
22.	31	4	4½	23.	37	16	1½	24.	42	19	7½
25.	40	12	[illegible]	[illegible]	71	14	7	27.	151	17	1½
28.	246	12	[illegible]	[illegible]	101	9	2¾	30.	80	19	9
31.	424	12	[illegible]	[illegible]	298	15	3	33.	443	9	3½
[illegible]	215	[illegible]	[illegible]	[illegible]	374	1	6¾	36.	758	17	9

[illegible] as, inste[illegible] d., to put down 13 s. 11 d.,
[illegible]vision [illegible]anner when the value is
[illegible]t may [illegible]e first work of the above

Promiscuous Exercises.

Find the amount of

Ex. 1. 17 cwt. 3 qrs. 11 lb. of Jordan Almonds at £ 14 per cwt.

2. 5 cwt. 1 qr. 17 lb. of Barbadoes Aloes at £ 7 10 per cwt.

3. 22 tons 10 cwt. 2 qrs. 14 lb. of Alum at £ 12 15 per ton.

4. 123 cwt. 3 qrs. 11 lb. of Canada Pot Ashes at 44 s. 6 d. per cwt.

5. 1375 gallons of Jamaica Rum at 2 s. 8½ d. per gallon.

6. 29 cwt. 0 qr. 22 lb. of Turkish Berries at 54 s. 6 d. per cwt.

7. 10 cwt. 1 qr. 18 lb. of Camphor at 2 s. 4 d. per lb.

8. 87 cwt. 2 qrs. 22 lb. of Capers at £ 8 10 per cwt.

9. 279¾ yards of Irish Linen at 17¾ d. per yard.

10. 306 cwt. 1 qr. 24 lb. of Cotton Wool at 11⅛ d. per lb.

11. 13 cwt. 1 qr. 8 lb. of Elephant's Teeth at £ 25 10 per cwt.

12. 282 cwt. 3 qrs. 22 lb. of Jamaica Sugar at 57 s. 6 d. per cwt.

13. 4 tons 10 cwt. 1 qr. 14 lb. of Pumice Stone at £ 14 per ton.

14. 24 cwt. 3 qrs. 24 lb. of Safflower at £ 7 10 per cwt.

15. 125 lb. 11 oz. of Milan Silk at 25 s. 6 d. per lb.

16. 16 cwt. 2 qrs. 7 lb. of Roman Vitriol at 36 s. per cwt.

17. 164 lb. 11 oz. 17 dwts. 15 grs. of Gold at 77 s. 9 d. per oz.

18. 868 gallons of Sicilian Wine at £ 14 10 per pipe of 93 gallons.

Products.

Ex. 1.	£ 249	17	6	Ex. 2.	£ 40	10	3
3.	287	5	6	4.	275	11	3
5.	186	4	0	6.	79	11	2
7.	136	0	8	8.	745	8	5
9.	20	13	10	10.	1591	1	3
11.	339	13	11	12.	813	9	5
13.	63	5	3	14.	187	4	8
15.	160	5	0	16.	29	16	3
17.	7696	15	9	18.	135	6	8

When lb. are to be valued at shillings per cwt., instead of reducing the lb. into cwt., the valuation may be made from an assumed amount at 112 s. per cwt., as in the following example.

EXAMPLE 7.

To find the amount of 628 lb. at 47 s. 6 d. per cwt.

		£	s.	d.		
	¼ ...	31	8	0* amt. at 112 s. per cwt.		
	½ ...	7	17	0	—	28 s.
1/7 ...	¼ ...	3	18	6	—	14
			19	7½	—	3 6
			11	2½	—	2
		£ 13	6	4	—	47 s. 6 d.

Observe. If the price per cwt. is multiplied by 3, and the product, reckoned as farthings, is divided by 7, the shillings in the quotient is the number of farthings per lb.:

thus, at 91 s. per cwt.

f. 91 × 3 ÷ 7 = 39 farthings = 9¾ d. per lb.

On the reverse. If the number of farthings per lb., reckoned as shillings, are multiplied by 7, and the product divided by 3, the quotient is the number of shillings per cwt.

thus, at 10¼ d. per lb. = 41 f.

s. 41 × 7 ÷ 3 = 95 s. 8 d. per cwt.

EXERCISES.

What is the cost of

			£	s.	d.
Ex. 1.	382 lb. at 21 s. per cwt.?	*Answer*	£ 3	11	8
2.	616 lb. at 32 s. per cwt.?	—	8	16	0
3.	733 lb. at 49 s. per cwt.?	—	16	0	8
4.	815 lb. at 58 s. per cwt.?	—	21	2	1
5.	214 lb. at 67 s. per cwt.?	—	6	8	0
6.	522 lb. at 81 s. per cwt.?	—	18	17	6

---

* We here use 628 s. as £ 31 8.

THE ALLOWANCES

OF

DRAFT, TARE, AND TRET,

UPON THE

GROSS WEIGHT.

The Gross weight is the whole weight of the hhd., chest, bag, &c., and of the goods they contain.

Tare is the weight of the hhd., &c., or an allowance made for that weight. It is called a *real* Tare, when it is the actual weight of the hhd., &c.; an *average* Tare, when it is the mean weight of two or more hhds., &c., selected by the buyer and seller; an estimated or customary Tare, when it is an allowance without weighing the hhd., &c.; and a *proportionate* Tare, when it is reckoned at a rate per cent., per cwt., &c.

Draft is an allowance made by the merchant to the buyer, to compensate for the small allowances or extra weights, that may arise from the goods being sold by retail.

Tret is an allowance, now restricted to a very few goods, for waste, &c.; it is always 4 lb. per 104 lb. or 1-26 th part of the weight, called the Suttle, which remains after all other allowances have been deducted.

Net is the weight after every allowance has been made.

Observe. Other allowances, as Extra-Tare, or Super-Tare, Shrinkage, Garbling, &c., are made under particular circumstances, and on particular goods; and formerly an allowance called Cloff was made for waste, but it has long since been abolished. Tret was also formerly a very general allowance, but it is now restricted to Pimento, and such Drugs as are not from the Levant or East Indies, and are sold by the pound Avoirdupois; and also upon Roman Vitriol, although sold by the cwt. Upon no other goods is it now allowed, in this manner or for this purpose, but there is an allowance of 1 lb. per bale upon Raw Silk, which is called Tret, but which is really allowed for *Bon Poids*, or *Draft.*

CASE 1.

When the real Tare is allowed.

Rule. Subtract the Tare from the Gross, and the remainder is the Net, if there is no other allowance made.

EXAMPLE.

To find the net weight of cwt. 17 2 23, Tare 1 3 21.

	cwt.	qrs.	lb.	
	17	2	23	Gross.
	1	3	21	Tare.
cwt.	15	3	2	Net.

Ex. 1. What is the net weight of 5 tubs of Butter, weighing Gross cwt. 8 2 11, real Tare 3 qrs. 23 lb.?

Ex. 2. What is the net weight of 2 hhds. of Sugar, weighing Gross cwt. 23 1 17, real Tare cwt. 3 1 22?

CASE 2.

When the Tare or Draft is a certain rate for each package.

Rule. Multiply the Tare or Draft upon each by the number of packages, and subtract the amount.

EXAMPLE.

To find the net weight of 2 bales of Jalap, weighing cwt. 3 2 2, Tare 13 lb. each, Draft 2 lb. each?

	cwt.	qrs.	lb.	
	3	2	2	Gross.
			26	Tare.
	3	1	4	
			4	Draft.
cwt.	3	1	0	Net.

N. B. When the Tare and Draft are both allowed, they may be added together, and the amount be subtracted.

EXERCISES.

Ex. 3. Find the net weight of 4 casks of Salep, weighing Gross cwt. 28 0 26, real Tare 4 1 5, Draft 4 lb. per cask.

Ex. 4. Find the net weight in lb. of ten bales of Surat Cotton, weighing Gross cwt. 30 2 15, Tare 19 lb. per bale, and Draft 1 lb. per bale.

Ex. 5. Find the net weight of 10 bags of British Plantation Coffee, weighing Gross cwt. 13 3 11, Tare 2 lb. per bag, Draft 2 lb. per bag.

CASE 3.

When the Tare is a given rate per cwt. or per cent.

Rule. Take parts for the rate out of 112 lb. or out of 100 lb., and when Draft is also allowed, first deduct the Draft, and calculate for the Tare upon the remainder.

EXAMPLE.

To find the net weight of 2 gourds of Barbadoes Aloes, weighing Gross cwt. 1 1 0, Tare and Draft 7 lb. per cwt.

	cwt.	qr.	lb.	
	1	1	0	Gross.
			$8\frac{3}{4}$	Tare and Draft.
cwt.	1	0	$19\frac{1}{4}$	Net.

EXERCISES.

Ex. 6. Find the net weight of 10 bales of Cotton Wool, weighing Gross cwt. 18 1 22, Draft 1 lb. per bale, Tare 4 lb. per cwt., and reduce the net weight into lb.

Ex. 7. Find the net weight of 5 casks of Soap Tallow, weighing Gross cwt. 47 1 22, Draft 2 lb. per cask, Tare 12 lb. per cwt.

Ex. 8. Find the net weight in lb. of 4 bales of Senna, weighing Gross cwt. 23 1 12, Draft 4 lb. per bale, Tare 6 lb. per cent.

CASE 4.

When Tret is allowed.

Rule. Reduce the Suttle weight in cwt. into lb., and divide by 26 for the Tret.

EXAMPLE.

To find the net weight of 1 chest of Bark, weighing Gross cwt. 2 0 12, Tare 22 lb., Draft 2 lb. Dust 3 per cent., and Tret 4 lb. per 104 lb.

	cwt.	qrs.	lb.
	2	0	12
Tare 22 lb. } Draft 2 lb. }			24
	1	3	16
	212 lb.		
Dust	6		
	206	Suttle.	
Tret	8		
	198	Net.	

EXERCISES.

Ex. 9. Find the net weight of 1 iron bottle of Quicksilver, weighing Gross cwt. 0 3 21, Tare and Draft 21 lb., Tret 4 lb. per 104 lb.

Ex. 10. Find the net weight in lb. of 30 bags of Pimento, weighing Gross cwt. 24 2 2, Draft 1 lb. per bag, Tare 4 lb. per cwt., Tret 4 lb. per 104 lb.

Ex. 11. Find the net weight in cwt. of 1 cask of Roman Vitriol, weighing Gross cwt. 3 1 24, Tare 50 lb., Draft 4 lb., Tret 4 lb. per 104 lb. ?*

PROMISCUOUS EXERCISES.

Ex. 12. Find the net weight in lb. of 10 hhds. of Maryland Tobacco, weighing Gross cwt. 87 3 23, Draft 4 lb. per hhd., Shrinkage 15 lb. per hhd., Tret 4 lb. per 104 lb.

Ex. 13. Find the net weight of 2 chests of Italian Juice, weighing Gross cwt. 3 3 25, Tare 3 qr. 8 lb., Draft 2 lb. per chest, Allowance for leaves 6 lb. per cwt.

---

* In this Exercise, the Suttle is not to be reduced into pounds.—See the note, page 84.

Ex. 14. Find the net weight in lb., and the number of gallons, at 9 lb. per gallon, of 25 jars of Salad Oil, weighing Gross cwt. 65 1 18 deducting 1-3rd of the whole weight for the weight of the jars, and 1-7th of the remainder for foot or sediment.

Ex. 15. Find the net weight in lb., and the number of barrels of 31½ gallons, at the rate of 9 lb. per gallon, in 10 barrels of American Pitch, weighing Gross cwt. 34 0 17 Tare 56 lb. per barrel.

Ex. 16. Find the net weight and the value at £ 275 per ton, of 7 bags of Orchilla Weed, weighing Gross cwt. 17 2 2, Draft 1 lb. per bag, Tare 4 lb. per cwt., and allowance for Damages 1 qr. 22 lb.

Ex. 17. Find the net weight and the value at 21½ d. per lb. of 60 bottles of Quicksilver, weighing as follows:

10 Bottles	cwt. 8	0	25	Draft	1 lb. each.
10 —	8	0	24	Tare	16 lb. each.
10 —	8	0	22	Tret	4 lb. per 104.
10 —	8	1	5		
10 —	8	0	20		
10 —	8	0	21		

Ex. 18. Find the net weight and value of 4 lots of Sapan Wood, at £ 29 per ton.

Lot 1.	cwt. 60	0	18	Draft 6 lb. per ton:
— 2.	60	0	18	
— 3.	60	0	18	
— 4.	59	0	7	

PRODUCTS.

Ex. 1.	cwt. 7 2 16	Ex. 7.	cwt. 42 1 4
2.	19 3 23	8.	lb. 2444
3.	23 3 5	9.	81
4.	lb. 3231	10.	2518
5.	cwt. 13 1 27	11.	cwt. 2 3 13
6.	lb. 1983	12.	lb. 9289

13. cwt. 2 3 22¼.

14.	lb. 4186 . . .	gallons	465
15.	gal. 362 . . .	barrels	11 15½ gallons
16.	cwt. 16 1 15 .	value	£ 225 5 7
17.	lb. 4328 . . .	—	387 14 4
18.	cwt. 238 3 17 .	—	346 8 2

COMMISSION AND BROKERAGE.

Commission and Brokerage are charges made by Agents and Brokers, for their skill and trouble in executing the business intrusted to their management.

A *Del-Credere* Commission is a charge made for insuring the responsibility of the parties to whom credit is given by the Agent.

These charges are usually per centages upon the amount of the goods. When there is a Discount, the Brokerage is usually charged upon the full amount, and the Commission upon the net amount, after the Discount has been deducted.

EXAMPLE.

To find the amount of the Commission at 2½ per cent., and the Brokerage at 1 per cent., upon the amount of 30 hhds. of Sugar weighing Gross cwt. 452 2 19, Tare and Draft cwt. 44 3 22; sold at Public Sale, at 54 s. 6 d. per cwt. Net, with a Discount of 2½ per cent.

	cwt.	qr.	lb.				
Gross	452	2	19				
Tare & Draft	44	3	22				
	407	2	25	at 54 s. 6 d.	£ 1111	0	11
				Discount	27	15	6
				Net Amount	£ 1083	5	5

Brokerage	£ 11	2	2	on full amount
Commission	27	1	7	on net amount.

EXERCISES.

Ex. 1. What is the Commission upon £ 845 16 7 at 10 per cent.?

Ex. 2. What is the Commission upon £ 2462 10 0 at 5 per cent.?

Ex. 3. What is the Commission upon £ 737 12 6 at 2½ per cent.?

Ex. 4. What is the Commission upon £ 459 19 10 at 1 per cent.?

Ex. 5. What is the Brokerage upon £ 1300 0 0 at ½ per cent.?

Ex. 6. What is the Brokerage upon £ 464 3 6 at ¼ per cent.?

Ex. 7. What is the Brokerage upon £ 1472 12 8 at ⅛ per cent.?

Ex. 8. What is the Brokerage upon £ 3100 0 0 at $\frac{1}{10}$ per cent.?

Ex. 9. What is the Commission at 2½ per cent. to be charged by an Agent, upon the following amounts of Goods and Charges; viz. Goods £ 867 14 10; Shipping Charges £ 17 14 6; Charges of Insurance £ 18 14 6?

Ex. 10. What is the amount to be paid for 5 puncheons of Rum, containing 503 gallons, at 2 s. 9 d. per gallon; including Brokerage at 5 s. per puncheon?

Ex. 11. What is the amount of the Commission at 2½ per cent. and the Brokerage at ½ per cent., for selling 20 hhds. of Sugar, weighing Gross cwt. 310 1 10, Tare and Draft cwt. 30 0 17, at 64 s. per cwt.?

Ex. 12. What am I to pay a Broker at ½ per cent., for buying upon my account at the East India Company's Sales 3 chests of Indigo, viz.

Lot 6125	cwt.	3	1	10	Tare	3 qrs.	20 lb.
6	-	2	3	21	-	3 -	17 -
7	-	3	0	19	-	3 -	16 -

Draft 2 lb. per chest; Selling Price 8 s. 7 d. per lb.

PRODUCTS.

Ex. 1.	£ 84	11	8	Ex. 5.	£ 6	10	0
2.	£ 123	2	6	6.	£ 1	3	3
3.	£ 18	8	10	7.	£ 1	16	10
4.	£ 4	12	0	8.	£ 3	2	0
9.	Amount £ 904	3	10	... Commission	£ 22	12	1
10.	Amount £ 69	3	3	... Commission	£ 1	5	0
11.	Net cwt. 280	0	21	... Value	£ 896	12	0
	Commission £ 22	8	4	... Brokerage	£ 4	9	8
12.	Net cwt. 6	2	19	= lb. 747			
	Value £ 320	11	9	... Commission	£ 1	12	1

BILLS OF PARCELS.

London, 25 March, 1833.

Mr. James Peacock,

Bought of John Jones & Son,

		s.	d.		£	s.	d.
Irish Linen . .	17 yds. at	2	9	per yd. . .			
Calico	24 - -		11	— . . .			
Jacconet Muslin .	13 - -	2	0	— . . .			
Mulled do. .	9½ - -	3	6	— . . .			
Welch Flannel .	15½ - -	2	4	— . . .			
Toweling . .	22 - -		7½	— . . .			
					£ 8	17	11

London, 1 May, 1833.

Mr. Thomas Browne,

Bought of Samuel Hart,

		s.	d.		£	s.	d.
Dorset Butter .	28 lb. at	1	1½	per lb. .			
Fresh do. . . .	3 - -	1	2	— . .			
Cheshire Cheese .	18 10 oz. -		9½	— . .			
Stilton do. .	5 7 oz. -	1	3	— . .			
Wiltshire Bacon .	14¾ lb. -		9¼	— . .			
Lard	7 3 oz. -		10	— . .			
					£ 3	13	11

London, 1 May, 1833.

Mr. Henry Gardener,

Bought of John Lewis,

		s.	d.		£	s.	d.
Souchong Tea .	6 lb. at	7	6	per lb.			
Hyson do. .	2½ - -	10	0	— . . .			
Loaf Sugar . .	17¾ - -		10½	— . . .			
Moist do. qr. 2	14 - -	63	0	per cwt. .			
Currants . . .	4 - -		11	per lb. . . .			
Rice	12 - -		6	— . . .			
					£ 6	14	7

London, April 7th, 1833.

Mr. George Smith,

Bought of F. Brett,

				£	s.	d.
2	dozen	Port Wine	at 54 s.			
2	do.	Sherry	- 45 s.			
1	do.	Bucellas	- 36 s.			
2	gall.	Brandy	- 28 s.			
2	do.	Rum	- 16 s.			
				£ 16	2	0

London, April 9th, 1833.

Messrs. Peacock and Bell,

Bought of J. Davey & Son,

				£	s.	d.
300	yds.	Cotton Knotted Fringe	at 7¼ d.			
300	-	Gimp Head	— - 10¼ d.			
144	-	White Net	— - 14¼ d.			
144	-	Fancy Ball	— - 14½ d.			
144	-	do. do.	— - 19½ d.			
	2	Trunks, Matting, &c.	at 14 s. each			
				£ 52	4	6

London, 20 April, 1833.

Messrs. Curtis and Mollett,

Bought of Danvers and Powell,

6 pipes Bronte Wine, in bond.

No. 1. 92 gallons.
2. 95 —
3. 89 —
4. 92 —
5. 96 —
6. 93 —

gall. at £ 14 per pipe of 93 gall. £ 83 17 0

London, April 7th, 1833.

Messrs. Pepys & Peacock,

Bought of Francis Lamb,

					s.	d.		£	s.	d.
6	Bales	Cinnamon	555 lb.	at	8	6	per lb.			
6	do.	do.	554 -	-	8	8	—			
5	Chests	do.	364 -	-	7	4	—			
5	do.	do.	352 -	-	7	4	—			
5	do.	do.	356 -	-	7	3	—			
5	do.	do.	367 -	-	7	3	—			
								£ 1000	11	3

Messrs. Pepys & Peacock,

Bought at E. I. Company's Sale,
By I. N. Rolls.

27 May, 1833.—6 Lots Coffee per Taunton Castle.

Lot	Bgs.	cwt.	qr.	lb.		s.	d.		£	s.	d.
3110	22	20	2	17	at	78	0	per cwt.			
3111	20	20	0	23	-	79	6	—			
3112	20	20	3	12	-	79	6	—			
3113	25	27	0	7	-	80	0	—			
3124	25	26	0	0	-	84	6	—			
3125	25	25	0	8	-	85	6	—			
									£ 569	0	9

London, May 28th, 1833.

Messrs. Pepys & Peacock,

Bought of Battley & Price,

						s.	d.		£	s.	d.
10	Ps.	Olive Velveteens	571½	yds.	at	2	5	per yd.			
10	-	Black —	576¾	-	-	2	5	—			
10	-	Olive —	574¼	-	-	2	6	—			
10	-	Black —	575½	-	-	2	6	—			
12	-	Olive —	662¾	-	-	2	7	—			
6	-	Black —	341¼	-	-	2	7	—			
									£ 412	2	11

AVERAGE PRICES.

An average price is the estimated price at which the whole quantity of several parcels of Goods, would amount to the same sum, as the several separate quantities.

EXAMPLE.

To find the average price for cwt. of the following quantities of Sugar, at the following net prices:

cwt.	138 3 16	at s. 34 6 d.	Net =	£ 239 11 10
	627 3 4	- 33 -	- =	1035 16 11
	824 1 14	- 32 6 d.	- =	1339 12 2
	1591 0 6			£ 2615 0 11

	cwt. qr. lb.		£ s. d.		cwt.
If	1591 0 6	———	2615 0 11	———	1 ?

Answer s. 32 10½ d.*

Instead of the above the quantities may be reckoned 139,628 and 824 cwt. and the result will be the same.

EXERCISES.

Ex. 1. What is the average price of the following quantities of Wheat, viz. 351 qrs. at 63 s., 217 qrs. at 65 s., and 333 qrs. at 67 s.?

Ex. 2. What is the average price of the following quantities of Sugar, sold at the following prices exclusive of the duty.

30	hhds. Net cwt.	451 2 17	sold at	36 s. per cwt.
150	- -	2270 3 14	-	40 s.
40	- -	623 1 25	-	43 s. 6 d.

PRODUCTS.

Ex. 1. s. 65

Ex. 2. 40 s. 2 d.

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* When the product of the proportional statement does not give an exact price the remainder should be reckoned as another farthing or penny, as otherwise the price will not yield the full amount.

BARTER.

Barter is the exchange of one sort of property for another estimated of equal value.

When the quantities of the articles exchanged are of similar denomination, they are inversely proportional to the prices.

Example 1.

To find how much Loaf Sugar, at 11 d. per lb., is equal in value to 3 qrs. 7 lb. of Moist Sugar, at 6½ d. per lb.

3 qrs. 7 lb. = 91 lb.

d. lb. d.
If 6½ ——— 91 ——— 11?
6½
11) 591½
lb. 53 $12\frac{4}{11}$ Answer.

Example 2.

To find how many gallons of Rum, at 13 s. 6 d. per gallon, are equal in value to 68½ yards of Irish Cloth, at 3 s. 6 d. per yard.

68½ yards at 3 s. 6 d. = 239 s. 9 d. value of the cloth.

s. d. g. s. d.
If 13 6 ——— 1 ——— 239 9?
27 479½

Gallons 479½ ÷ 27 = Gallons 17 $3\frac{1}{27}$ qts. Answer.

Or by Inverse Proportion.

s. d. g. s. d.
If 3 6 ——— 68½ ——— 13 6?
7
27) 479½
Gallons 17 $3\frac{1}{27}$ qts. Answer.

Exercises.

Ex. 1. How much Tea, at 8 s. per lb., should be given in exchange for 28 lb. of Cheshire Cheese, valued at 8½ d. per lb.?

Ex. 2. How many pounds' weight of Gum Benjamin, valued at 18 s. per lb., should be given in exchange for 23 lb. 10 oz. of Cochineal, valued at 11 s. 6 d. per lb.?

Ex. 3. A person who has 3 logs of Mahogany, measuring 209 cubic feet, valued at 18 s. per cubic foot, sells them for £ 50 14 11 in Cash, and the rest in Deals, valued at £ 32 10 s. per hundred of 120 Deals; how many Deals will he have to receive?

Ex. 4. I have sold the following Goods: viz.
cwt. 1 1 17 of Coffee, at 63 s. per cwt.
gallons 17 of Rum, at 13 s. 6 d. per gall.
cwt. 7 3 23 of Sugar, at 61 s. 6 d. per cwt.;
I have received in exchange,
25½ yds. of Cambric, at 11 s. 6 d. per yard,
133¼ - of Calico, at 7½ d. per yard,
63½ - of Irish Linen, at 2 s. 8 d. per yard;
and I am to receive the remainder of the amount of my goods in Calico, valued at 6¾ d. per yard; what quantity shall I have?

Ex. 5. How much 3½ per cent. Stock, at 97½ per cent., am I to receive in exchange for £ 3476, 3 per cent. Stock, at 81¼ per cent.

Ex. 6. How much 3 per cent. Stock, valued at 83⅛ per cent. am I to receive in exchange for £ 1000, 4 per cent. Stock, valued at 102¼ per cent.?

Products.

Ex. 1. lb. 2 7 10 Ex. 2. lb. 15 1½ oz.
3. Value of the Mahogany £ 188 2 . . . Number of Deals 509
4. Value of the Coffee, £ 4 8 4 . . . Rum £ 11 9 6
Sugar £ 24 9 3 . . . Cambric £ 14 13 3 . . . Calico £ 4 3 3
Irish Linen £ 8 9 4—Balance £ 13 1 3
yards 464 1 3
Ex. 5. £ 2896 13 4 Ex. 6. £ 1230 1 6

PROFIT AND LOSS.

Profit is the excess, and Loss is the deficiency, of the selling price in comparison with the cost price of Goods.

Per Centages of Profit and Loss are calculated by the Rule of Three, in which the first term is the cost price, the second term is the gain or loss, and £ 100 is the third term.

EXAMPLE.

To find the gain per cent. upon Sugar, bought at 54 s. 6 d. per cwt. and sold at 7 d. per lb.

	s.	d.	
112 lb. at 7 d. per lb. =	65	4	selling price
	54	6	cost
	s. 10	10	gain

s. d. — s. d. — £

If 54 6 ——— 10 10 ——— 100?[*]

654 d. — 130 d. — 130

654) 13000

£ 19 17 6 d.

Answer. £ 19 17 6, or 19⅞ per cent.

EXERCISES.

Ex. 1. If Sugar is bought at 63 s. 6 d. per cwt., and retailed at 7¼ d. per lb., how much is the amount of the gain per cwt. ?

Ex. 2. If the gain upon £ 82 10 s. is £ 11 4 6, what is the rate of the gain per cent. ?

Ex. 3. If Goods which cost £ 236 11 8 are sold for £ 273 9 7, what is the rate of the gain per cent. ?

Ex. 4. If Goods which cost £ 313 16 6, are sold for £ 292 14 5, what is the rate of the loss per cent. ?

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[*] When the first and second terms of these statements are not in pounds, the second term may be compared with the first, and the third may be made the producing term, instead of the second.

Ex. 5. How much must Goods which cost £ 135 10 6 sell for, to make a profit of 12½ per cent.?

Ex. 6. If Ginger is purchased at 98 s. 6 d. per cwt., and is sold so as to gain 15 per cent., what is the amount of the gain per cwt., and what rate is it to be sold at per lb.?

Ex. 7. If the Cost and Charges of 1 bale of Cinnamon, weighing net 3 qrs. 17 lb. are £ 42 5 4, at what rate per cent. would be the Gain or Loss upon this article, if the whole were retailed at 9 s. 3 d. per lb.?

Ex. 8. If Hops which cost £ 4 16 s. per cwt. are sold at the different rates of £ 5 7 6 and £ 5 10 s. per cwt., what is the difference in the rates per cent. of the Gains?

Ex. 9. At what rate must Coffee be sold per lb. to make a Profit of £ 16 per cent., when the Cost and Charges of cwt. 11 3 17 are £ 100 11 0?

Ex. 10. The prime Cost of 12 casks of Argol imported from Leghorn is £ 165 15 1; the Gross weight of the same is cwt. 74 1 7, the Tare cwt. 8 1 14, the Draft 2 qrs. 14 lb., the selling price is 65 s. 6 d. per cwt. with a Discount of 2½ per cent., and the amount of the importation Charges is £ 25 6 6;—It is required to find the amount of the Gain or Loss per cent.?

PRODUCTS.

Ex. 1. Gain 4 s. 2 d. per cwt.
2. Gain £ 13 12 1 per cent.
3. Gain £ 36 17 11—per cent. £ 15 11 11.
4. Loss £ 21 2 1—per cent. £ 6 14 6.
5. Selling Price 152 9 4.
6. Gain per cwt. £ 0 14 9—Selling Price per lb. 1 s.
7. Selling Price £ 46 14 3.
Gain £ 4 8 11—£ 10 10 5 per cent.
8. Difference £ 2 12 1 per cent.
9. Rate £ 0 1 9 per lb.
10. Net cwt. 65 1 7—Full Selling Price £ 213 17 11
Net Selling Price £ 208 11 0—Net Proceeds £ 183 4 6
Gain £ 10 10 9 per cent.

PARTNERSHIP OR FELLOWSHIP.

Partnership, Fellowship, or Distributive Proportion, is the distribution of a sum of money, &c. between the persons entitled to it according to the amount of their separate interests.

Calculations of Fellowship are usually divided into Simple Fellowship and Compound Fellowship, according to whether the causes upon which the results depend are simple or combined.

SIMPLE FELLOWSHIP.

The distribution of the proceeds of any concern in which there are separate interests is usually made according to one of the methods of the four following cases..

CASE 1.

When the proceeds are distributed into fractional shares.

Rule. Multiply the given amount by the fractions expressing the shares.

EXAMPLE.

Divide £ 64 10 6 between two persons, giving one $\frac{3}{8}$ and the other $\frac{5}{8}$.

	£	s.	d.	
	64	10	6	
1-8 th....	8	1	3¾	
3-8 ths..	24	3	11¼	... 1 st share
5-8 ths..	40	6	6¾	... 2 nd —
	£ 64	10	6	Proof.

EXERCISES.

Ex. 1. Divide £ 834 10 6 between two persons, giving one $\frac{5}{12}$ and the other $\frac{7}{12}$. *Product* £ 347 14 4½—£ 486 16 1½.

Ex. 2. Divide £ 526 14 8 between three persons, giving the first $\frac{7}{16}$, the second $\frac{5}{16}$, and the third $\frac{4}{16}$. *Product* £ 230 8 11 —£ 164 12 1—£ 131 13 8.

UofM

CASE 2.

When the proceeds are distributed according to the amount of each person's Stock, or Capital.

Rule. As the whole capital is to the amount to be distributed, so is each person's capital to his share of that amount.

EXAMPLE.

A's capital is £ 5000, B's capital is £ 3000, and C's capital is £ 2000, the gain is £ 800; how much is each person to receive according to his capital?

£ £ £
If 10,000 ——— 800 ——— 5,000?
5

Answer £ 400 0 0 A's share.

£ 5000
3000
2000

£ 10000

£ £ £
If 10,000 ——— 800 ——— 3,000?
3

Answer £ 240 0 0 B's share.

£ £ £
If 10,000 ——— 800 ——— 2,000?
2

Answer £ 160 0 0 C's share.
240 0 0 B's —
400 0 0 A's —

Proof £ 800 0 0 Whole Gain.

EXERCISES

Ex. 1. A and B have traded with a capital of £ 750; A's capital is £ 400, and B's £ 350, and they have gained £ 194 10 s., how much of the gain is each to receive?

Answer. A's share £ 103 14 8, B's £ 90 15 4.

Ex. 2. A and B traded with a capital of £ 1200; A's capital was £ 700, and B's £ 500, and they lost £ 264 16 s.; the partnership was dissolved, and it is required to find how much each person had to receive?

Answer. A's share £ 545 10 8, B s £ 389 13 4.

CASE 3.

When the proceeds are distributed by a rate per cent.

Rule. Find the rate per cent. by this proportion: as the whole capital is to the whole amount to be distributed, so is £ 100 to the rate per cent. Then find the amount of each person's capital at this rate per cent.

EXAMPLE.

Distribute £ 64 10 by a per centage amongst the following persons according to their capitals, viz. A £ 300, B £ 200, and C £ 150.

If 650 ——— £ 64 10 ——— £ 100?

£ 9 18 5½ Answer.

£ 300	at £ 9 18 5½ per cent.	£ 29 15 5
200	. . . — . . .	19 16 11
150	. . . — . . .	14 17 8
650	. . . — . . .	£ 64 10 0

N. B. When such distributions as the above are made upon *Policies of Insurances*, if there is a remainder to the quotient of the rate in pence, it is usually reckoned as another penny; thus in this case, the above rate would be reckoned £ 9 18 6, and the amount would be £ 64 10 3.—The second of the following Exercises is to be worked in this manner.

EXERCISES.

Ex. 1. Distribute £ 171 13 4 by a per centage amongst four persons according to their capitals, viz. A's £ 500, B's £ 500, C's £ 400, and D's £ 350.

Ex. 2. Upon a Policy of Insurance of £ 1182 a loss of £ 679 14 6 is to be distributed amongst the Underwriters—What is the rate per cent., and how much will each contribute, A's risk being £ 400, B's £ 200, C's £ 200, D's £ 100, E's £ 100, and F's £ 182.?

PRODUCTS.

Ex. 1. Rate £ 9 16 2—A's £ 49 0 11—B's £ 49 0 11
C's 39 4 9—D's 34 6 8
2. Rate £ 57 10 2—A's £ 230 0 8—B's £ 115 0 4
C's 115 0 4—D's 57 10 2—E's 57 10 2
F's 104 13 4

CASE 4.

When the proceeds are distributed by a rate in the pound.

Rule. Find the rate in the pound by this proportion :—as the whole capital is to the whole amount to be distributed, so is £ 1 to the rate in the pound.—Then find the amount of the given portions of the capital at this rate.

EXAMPLE.

A capital of £ 768 10 s. pays £ 347 16 4; at what rate is this to be distributed in the £, and what will be the amount to be received upon a portion of the capital amounting to £ 65 16 8 ?

	£	s.		£	s.	d.		£
If	768	10	———	347	16	4	———	1 ?

Answer s. 9 0½ d. rate in the Pound.

	£	s.	d.	
$\frac{1}{5}$ . . . $\frac{1}{4}$ . . .	65	16	8	
$\frac{1}{120}$ . . .	16	9	2	for 5 s.
	13	3	4	- 4 s.
		2	9	- ½ d.
	£ 29	15	3	

EXERCISES.

Ex. 1. A capital of £ 7000 pays £ 350, at what rate is this to be distributed in the £, and what shall I have to receive, my claim upon the capital being £ 500 ?

Ex. 2. The amount of a tradesman's debts is £ 4736 16 6, his assets are £ 2844 14 8; what will the amount pay in the £, and what will be paid upon a debt of £ 459 14 0 ?

PRODUCTS.

				£		
Ex. 1.	Rate	1 s.	Amount	£ 25	0	0
2.	—	12 s.	—	£ 275	16	4

COMPOUND FELLOWSHIP.

Rule. Multiply each capital by its number of years, months, days, &c., and take the sum of these products; then say, as the whole amount is to the whole proceeds, so is each separate product to each separate share of the proceeds.

Suppose A to have embarked in business with a capital of £ 75, and after a month to have been joined by B with a capital of £ 40, and suppose that at the end of 6 months from the commencement of the business, a profit of £ 70 has been realized, it is required to find how much of this belongs to each person in proportion to his capital and the time it has been employed.

$$\left.\begin{array}{r} £\,75 \times 6 = £\,450 \\ 40 \times 5 = \quad 200 \end{array}\right\} = £\,650$$

£ £ £
If 650 ——— 70 ——— 450?

£ 48 9 3

£ £ £
If 650 ——— 70 ——— 200?

£ 21 10 9

£ 70 0 0 Proof.

Exercises.

Ex. 1. If a profit of £ 364 7 be distributed between three persons according to their capitals, and the time each has been employed, how much will each have to receive, A having employed £ 500 for 11 months, B, 1300 for 9 months, and C, £ 2000 for 4 months?

Ex. 2. A had 97 oz. 10 dwts. of silver 10½ oz. fine, B, 68 oz. of silver 10¾ oz. fine, and C, 104 oz. 10 dwts. of silver 11 oz. fine; now supposing the whole to have been melted into a bar, and sold net for £ 64 10 6, how much would each person be entitled to receive?

Products.

Ex. 1.	A £ 79 10 5,	B £ 169 3 3,	C £ 115 13 4
2.	A £ 22 14 11,	B £ 16 4 10,	C £ 25 10 9

INTEREST.

Interest is the allowance or charge made for the use of a sum of money for a given time.

The Principal is the sum upon which the Interest is calculated.

The rate per cent. is the Interest upon £ 100, either for 1 year, or for some other particular given time, as 1 month, or 1 day.*

RULE 1. FOR A NUMBER OF YEARS.

Multiply the Principal by the number of the years, and that product by the rate, and divide the last product by 100.

Observe. The interest for one year at 5 per cent. is 1-20 th of the Principal, and at 4 per cent. it is 1-25 th of the Principal; consequently, for any number of years, it is so many of these parts.

Observe, also, that, *except for years*, the Principal is reckoned in the calculation at the nearest pound; 10 s. and above being reckoned another pound, and less than 10 s. being rejected.

EXAMPLE.

To find the Interest for 3 years on £ 575 at 5 per cent.

£ 575 × 3 ÷ 20 = £ 86 5 0.

N. B. At 5 per cent. per annum, the Interest for 1 year is 1 s. in the pound.

EXERCISES.

Ex. 1. What is the Interest on £ 800 for 2 years at 3 per cent. ?

Ex. 2. What is the Interest on £ 444 10 s. for 3½ years at 4 per cent. ?

Ex. 3. What is the Interest on £ 967 12 9 for 20 years at 5 per cent. ?

Ex. 4. What is the Interest on £ 1205 4 6 for 11¾ years at 6 per cent. ?

PRODUCTS.

Ex. 1.	£ 48	0	0	Ex. 3.	£ 967	12	9
2.	£ 57	15	8	4.	£ 849	13	8

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* In Interest calculations it is always understood, that the time in the rate is 1 year, unless otherwise expressed; consequently the words per year or per annum are seldom used in mentioning the rate of the Interest.

RULE 2. FOR A NUMBER OF MONTHS.

Multiply the Principal by the number of the months, and divide the product at 5 per cent. by 240, at 4 per cent. by 300, and at 3 per cent. by 400.

For any other rate per cent., multiply the Principal by the number of the months, then multiply this product by double the rate per cent., and cutting off the unit figure of the pounds in the last product, the other pounds will show the amount of the Interest in pence.

Ex. 2. To find the Interest on £ 854 10 s. for 7 months, at 4 per cent. and also at 5 per cent.

4 per cent.	5 per cent.
£	£
855	855
7	7
3) 5985	12) 5985*
100) 1995	20) 498 9
£ 19 19 0 (see page 50.)	£ 24 18 9

EXERCISES.

Ex. 1. What is the Interest upon £ 500 for 5 months at 5 per cent ?

Ex. 2. What is the Interest upon £ 84 10 6 for 7 months at 4 per cent ?

Ex. 3. What is the Interest upon £ 417 14 4 for 10 months at 3 per cent ?

Ex. 4. What is the Interest upon £ 1442 16 10 for 5½ months at 3½ per cent ?

PRODUCTS.

Ex. 1.	£ 10 8 4	Ex. 3.	£ 10 9 0
2.	£ 1 19 8	4.	£ 23 2 11

---

* Having multiplied by 7, we consider the pounds in the product as so many pence, and reduce them into pounds; because at 5 per cent. the Interest for 1 month is 1 d. in the pound.

Rule 3. For a number of Days.

At 5 per cent., multiply the Principal by the number of the days, and divide the product by 7300.*

At any other rate than 5 per cent., multiply the product of the Principal by the number of the days, by double the rate per cent., and divide the last product by 73000.

Or, proportion the amount at 5 per cent., to the amount at any other rate per cent.

Or, for 4 per cent., divide the product of the Principal multiplied by the number of the days, by 9125.

N. B. When the time is in months and days, find the Interest for the months by the last Rule, and for the days by this Rule.

Both these calculations should be then made at 5 per cent., and the amount should, if required, be reduced to any other rate per cent.

Instead of dividing by 9125, we may add 1-10 th, from the amount subtract 1-25th of this 1-10 th, and cut off and value the four lower figures of the remainder, in the same manner and with the same allowance as in calculating 5 per cent. It is to be observed, that the taking of the 25 th part may be performed by cutting off the two lower figures, and multiplying the other figures by 4.

Example 3.

To find the Interest on £ 864 12 6 for 35 days at 5 per cent., also at 4 per cent.

5 per cent.	4 per cent.			
£	£		£	
865	865	(or)	30275	
35	35		3027	1-10 th.
7300) 30275	9125) 30275		33302	
			120	1-25 th.
£ 4 2 11	£ 3 6 4		£ 3.3182	= £ 3 6 4

---

* To explain the reason of this Rule, it is to be observed, that at 5 per cent. the principal is equal to the amount of the interest for 20 years; and at 4 per cent., for 25 years; therefore, reckoning the year at 365 days, or 20 years, as 7300 days, and 25 years as 9125 days, we say by the Rule of Three,

If 7300 days — £ 865 — 35 days?
If 9125 days — £ 865 — 35 days?

And the reason why, at any other rate than 5 per cent., we may multiply by double the rate and divide by 10 times 7300, is evidently instead of using the single rate and dividing by 5.

As £ 4 2 11 × 4 ÷ 5 = £ 3 6 4
or £ 4 2 11 × 8 ÷ 10 = £ 3 6 4

EXAMPLE 4.

To find the Interest on £ 342 16 for 3 months 10 days, at 3½ per cent.

s.
¼ ... 343 for 1 year.

s. 85 9 - 3 mos.

£ 4 5 9

£
7300) 343

... 9 5 for 10 days.
4 5 9 - 3 mos.

4 15 2
7

10) 33 6 2

£ 3 6 7 - 3½ per cent.

N. B. After having multiplied the principal by the number of the days, instead of dividing by 7300 we may use this formula.

Take 1-3 rd of the product, 1-10 th of the 3 rd part, and 1-10 th of that 10 th part, add the whole together, and divide the amount by 10000 or 100000; that is, cut off the 4 right-hand figures for 7300, or 5 figures for 73000. (See Ex. 3.)

£
865
35

$\frac{1}{3}$ ... 30275
$\frac{1}{10}$ ... 10091
$\frac{1}{10}$ ... 1009
100

£ 4.1475 = £ 4 2 11.

Observe. The figures thus cut off are decimal parts of a £, the first three figures of which are valued according to the rule before given for dividing by 1000. (See page 50.)

For the principles upon which this substituted method for dividing by 7300 is founded, see the end of Decimal Division; but it may here be observed, that the result gives an excess above the true amount of 1 farthing in every £ 10 of Interest.

EXERCISES AT 5 PER CENT.

Ex. 1. Find the Interest on £ 415 16 10 for 65 days at 5 per cent. Interest £ 3 14 1.

Ex. 2. Find the Interest on £ 1464 7 6 for 37 days at 5 per cent. Interest £ 7 8 5.

Ex. 3. Find the Interest on £ 844 19 2 for 5 months 22 days at 5 per cent. Interest £ 20 3 0.

EXERCISES AT OTHER RATES.

Ex. 4. What is the Interest on £ 500 for 87 days at 4 per cent.? Interest £ 4 15 4.*

Ex. 5. What is the Interest on £ 845 10 for 111 days at 3½ per cent.? Interest £ 9 0 1.

Ex. 6. What is the Interest on £ 659 9 6 for 4 months 17 days at 3 per cent.?—Interest £ 7 10 3.

CASE 2.

When two or more sums are due upon the same day.

Rule. Take their amount, upon which find the Interest at the given rate.

EXAMPLE.

To find the Interest upon £ 61 17 6 and £ 151 12 2 for 30 days at 4 per cent.

£ 61	17	6
151	12	2
£ 213	9	8 reckoned £ 213.

£ 213 × 30 × 8 ÷ 73000 = £ 0 14 0.

EXERCISES.

Ex. 1. Find the Interest on £ 160 10 and £ 47 12 6 for 57 days at 5 per cent.—Interest £ 1 12 6.

Ex. 2. Find the Interest on £ 24 13 7, £ 69 15 2, and £ 64 14 4 from June 1 to July 27 at 5 per cent. Interest £ 1 4 5.

Ex. 3. Find the Interest on £ 66 16, £ 45 12 2, and £ 191 10 6 from May 18 to Aug. 4 at 4 per cent. Interest £ 2 12 0.*

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* See the Nota Bene to the calculation in the next page.

CASE 3

When various sums in the same account are due on different days.

Rule. Multiply each Principal by the number of the days belonging to it, and use the amount as for the product of a single Principal.

EXAMPLE.

To find the Interest upon the following sums for the following number of days, at 4 per cent.

£ 81 14 4 for 18 days.
260 10 0 for 25 days.
50 7 0 for 36 days.

£ 82 × 18 = 1476
261 × 25 = 6525
50 × 36 = 1800

£ s. d.
9125) 9801 (1 1 6
· · · 676 (.074
39

N. B. The best way of dividing by 9125 in such calculations, is, to work out the pounds, and then dropping in succession 1 figure in the divisor, work out the remainder, and value the result as directed in the division by 1000, after the pounds have been found. (See page 50, Compound Division.)

EXERCISES.

Ex. 1. Find the Interest on £ 42 16 4 for 25 days, £ 100 for 82 days, and £ 414 15 0 for 43 days, at 5 per cent.
Amount £ 3 14 4.

Ex. 2. Find the Interest on £ 204 15 2 for 61 days, £ 223 18 8 for 34 days, and £ 500 for 71 days, at 4 per cent.
Amount £ 6 1 11.

Ex. 3. Find the Interest on £ 500 from the 1 st of May to June 7 th, on £ 800 to June 23, and on £ 1224 16 0 to July 30, at 5 per cent.
Amount £ 23 8 11.

COMPOUND INTEREST.

Compound Interest is the amount of accumulated Interest, formed by the calculation of the Interest upon Interest due at a former period, as well as upon the Principal.

EXAMPLE.

To find the amount of the Compound Interest upon £ 500, calculated yearly, for 4 years, at 5 per cent. per annum?

	£	s.	d.	
$\frac{1}{20}$ ...	500	0	0	
	25	0	0	1 st Interest.
$\frac{1}{20}$ ...	525	0	0	
	26	5	0	2 nd do.
$\frac{1}{20}$ ...	551	5	0	
	27	11	3	3 rd do.
$\frac{1}{20}$ ...	578	16	3	
	28	18	10	4 th do.
	607	15	1	Total amt.
	500	0	0	Principal.
£	107	15	1	Compound Interest.

EXERCISES.

Ex. 1. What is the amount of the Compound Interest on £ 1000 for 4 years, at 5 per cent. per annum? Answer £ 215 10 2.

Ex. 2. What is the amount of the Compound Interest on £ 474 16 6, calculated half yearly, for 2½ years, at 5 per cent. per annum? Answer £ 62 7 11.

Note. It is not legal to charge Interest upon Interest, except upon the balances of accounts, including the balance or amount of the interest upon the account, but Compound Interest forms one of the principles upon which the calculations of Annuities, &c. are formed.

DISCOUNT.

Discount is a deduction allowed or charged for the present payment of a sum of money due at a future period.

The allowances of Discount are either,

1. A fixed sum agreed upon, without any consideration of either time or a rate per cent.
2. A per centage upon the Principal.
3. The Interest upon the Principal for the intervening time.
4. The Interest upon the sum actually paid, for the time it is paid in advance.

As the first species of Discounts requires no calculation, and as the calculations of the second and third sorts have been shown under the Rules of Per Centages and Interest, we shall give here only the Rule for the fourth sort of Discounts, observing, that in this country, this plan is rarely ever practised except in the calculations of the present worth of Life Insurances and Annuities.

Rule.

For calculating Discount as the Interest upon the Present Worth.

Find the Interest upon £ 100 for the given time, and add it to £ 100; then say, as this amount is to the Interest, so is the Principal to the Discount, or, as this amount is to £ 100 so is the Principal to the Present Worth.

Example.

To find the Present Worth and Discount on £ 500 for ½ year at 5 per cent. per annum.

The Interest on £ 100 for ½ year is £ 2½.

If £ 102½ ———— £ 100 ———— £ 500?

	£	s.	d.	
Answer	487	16	1	Present Worth.
	500	0	0	Principal.
£	12	3	11	Discount.

EXERCISES.

Ex. 1. Find the net amount of £ 639 14 5, allowing for Discount 7½ per cent.

Ex. 2. Find the net amount of 30 bales of Cotton, Gross weight 84 cwt. 2 qrs. 17 lb., Draft 1 lb. per bale, Tare 4 lb. per cwt., the Net weight being sold at 11¼ d. per lb., with a Discount of 1½ per cent. for Cash in 1 month.

Ex. 3. Find the net amount of Spanish Wool, which was sold for £ 1164 17 6, allowing a Discount for 8 months at 5 per cent.

Ex. 4. Find the net proceeds of a Bill of Exchange for £ 600, due the 21 st of May, and Discounted the 17 th of April, at 5 per cent.

Ex. 5. Find the net proceeds of the following Bills of Exchange, Discounted the 1 st of March at 4 per cent., allowing also a Brokerage of ¼ per cent.

2 Bills each	for	£ 747	11	6	due	April	17.
1 Bill	for	500			due	May	4.
3 Bills	for	600			due	May	11.

Ex. 6. Find the Present Worth of £ 750 for 1 year at 3 per cent. per annum, allowing for Discount the Interest upon the Present Worth.

Ex. 7. If I buy a lot of Goods for £ 153 12 on the 1 st of March, how much must I sell them for on 27 th of May to clear the Interest upon them at 5 per cent. per annum, and make a profit of 7½ per cent. upon the Principal and Interest, clear of a Discount of 10 per cent.?

PRODUCTS.

Ex. 1. £ 591 14 11 Ex. 2. Net weight 9113 lb.
3. 1126 0 10 4. £ 597 4 1.
5. Bills £ 3795 3 0—Days 47, 64, and 71 days.
Discount £ 25 4 3—Net amount £ 3765 3 10
6. Present Worth £ 728 3 1—Discount £ 21 16 11
7. Interest £ 1 16 9—Profit £ 11 13 2
Discount £ 18 11 4—Selling price £ 185 13 3

EQUATION OF PAYMENTS,

OR

AVERAGE TIMES OF PAYMENT.

An equated or average time of payment, is a time at which the Interest upon the whole of several sums of money is equal to the amount of the Discounts upon the separate sums for the separate times they have to run.

Rule.

Multiply each payment by the number of years, months, or days, which that payment has to run. Then say, by the Rule of Three, If the amount of all the payments require 1 year, &c., what will the amount of the products require? The answer to which statement will be the equated time required.

N. B. When particular days are specified, it is usual to reckon the time of each payment from the day on which the first payment becomes due; then the first payment will not require any multiplication, and the whole amount will be reckoned due at the end of the number of days in the equated time, calculated from the day of the first payment.

Example 1.

On March 2 nd, £ 300 is due; on March 18 th, £ 350 is due; and on April 17 th, £ 550 is due.—It is required to find an average time of payment for the whole sum.

From March 2 to March 18 is 16 days
to April 17 is 46 days

$$\begin{array}{rrcccr}
 & \pounds\ 300 & & & & \pounds \\
 & 350 & \times & 16 & = & 5600 \\
 & 550 & \times & 46 & = & 25300 \\
 & \overline{} & & \text{day} & & \overline{} \\
\text{If} & 1200 & \text{———} & 1 & \text{———} & 30900\,?
\end{array}$$

Answer 26 days.
March 2

March 28 Equated day of payment.

EXAMPLE 2.

To find an average time of payment for £ 600 due in 2 months, £ 300 due in 3 months, and £ 1100 due in 6 months.

£				£
600	×	2	=	1200
300	×	3	=	900
1100	×	6	=	6600
2000				8700

£		m		£
If 2000	———	1	———	8700?

Answer, months 4 11 days.

EXERCISES.

Ex. 1. What is the average time of payment for the following sums due in the following manner? viz.

£ 450 due in 5 months,
300 — 4 months,
400 — 4 months.

Ex. 2. On what day are the following sums due on an average?

£ 432 12 due April 1.
388 16 — April 18.
566 0 — May 17.

Ex. 3. 200 bags of Pernambucco Cotton Wool were sold on the following days, at three months credit, on what day were the amounts due upon an average?

	bags		cwt.	qr.	lb.		d.	
June 1.	50	Gross	74	2	17	at	9½	per lb.
27.	50	—	72	3	1	-	10	—
Aug. 21.	100	—	148	1	10	-	10½	—

Draft 1 lb. per bag, Tare 4 lb. per cwt.

ANSWERS.

Ex. 1. 4 months 12 days. Ex. 2. April 25.
3. Amounts due Sept. 1, Sept. 27, and Nov. 21.
Average day, October 19.

THE STOCKS.

The Stocks, or public Funds, are the Capitals on which certain Annuities payable by Government are calculated.

The valuations of Stock are made per centages, in which the given rates, reckoned in pounds and fractions of a pound, are the prices of £ 100 Stock; varying according to the rate of the annuity, (or, as is it called, the Interest,) and to the supply of the market.

Stock is bought or sold through the medium of a broker, whose usual charge is 1-8 th, or 2 s. 6 d., per cent.; but it is to be observed, that if the 1-8 th per cent. on the quantity of the Stock does not amount to 1 s., it is charged at 1 s., as it is the custom of Stock Brokers not to take less than that sum.

EXAMPLE 1.

To find the cost of £ 680 Stock, at 83½ per cent.

	£	s.	d.	
	83	10	-	
		7		
	584	10	-	for £ 700
	16	14	-	— 20
Value of the Stock	567	16	-	— £ 680
Brokerage		17	-	
Whole Cost	£ 568	13	-	

EXAMPLE 2.

To find the net proceeds of the sale of £ 564 13 10 Stock at 85 per cent.

	£	s.	d.			
	564	13	10	amount at	100	per cent.
	56	9	4.6	—	10	—
	28	4	8.3	—	5	—
Value of the Stock	479	19	9	—	85	—
Brokerage		14	1			
Net proceeds	£ 479	5	8			

EXAMPLE 3.

To find the cost of £ 672 10 4 Stock at 88⅝.

	£	s.	d.	
	672	10	4	
			88⅝	
100)	59601	·15	9	
	596	0	4	Value of the Stock.
		16	10	Brokerage.
	£ 596	17	2	Amount.

N. B. For the Brokerage, reckon 2 s. 6 d. for every £ 100 Stock, 3 d. for each £ 10 Stock, and 1 d. for any quantity of Stock from £ 1 13 4 to £ 5, 2 d. from £ 5 to £ 8 6 8, and 3 d. above that sum.

EXERCISES TO EXAMPLE 1.

	Products.		
Find the cost of £ 700 Stock at 81⅜. . .	£ 570	10	0
Find the cost of £ 450 Stock at 92½. . .	416	16	3
Find the cost of £ 580 Stock at 85¾. . .	498	1	6

EXERCISES TO EXAMPLE 2.

Find the net proceeds of £ 857 Stock at 80.	£ 684	10	7
Find the net proceeds of £ 113 Stock at 95.	107	4	2
Find the net proceeds of £ 926 Stock at 102½.	947	19	10

EXERCISES TO EXAMPLE 3.

Find the cost of £ 6172 10 0 Stock at 84⅛.	£ 5200	6	8
Find the cost of £ 3018 16 2 Stock at 77⅛.	2339	11	7
Find the cost of £ 4479 15 5 Stock at 91⅜.	4098	19	10

INSURANCES.

Fire and Sea Insurances are contracts by which the insurers agree to pay to the assured the value of the property that may be destroyed or damaged by accidents, against which the Insurances are made.

Life Insurances are contracts to pay the assured a specified sum of money, upon the death of the person or persons named in the contract.

The deed of contract is called a Policy, and it is required by law that every Policy should be upon stamped paper or parchment.

The amount of the remuneration paid to the insurers is called the Premium, and it is reckoned by a per centage upon the amount insured.

Ex. 1. To find the amount of a Fire Insurance on £ 1200 at 2 s. 6 d. per cent., with the Duty on the same at 3 s. per cent.

£ 1200 at 2 s. 6 d. = £ 1 10 0 Premium.
at 3 s. 0 = 1 16 0 Duty.

£ 3 6 0 Amount.

N. B. It is not usual for the offices to charge for the Policy Stamp on Fire Insurances, when the Insurance amounts to £ 300 and upwards.

Ex. 2. To find the amount of a Sea Insurance on £ 578, at 12 s. per cent., with the Policy Stamp at 1 s. 3 d. per cent., on £ 600.

s. 5.78 × 12 = s. 69.36*

£ 3 9 4 Premium.
7 6 Policy.

£ 3 16 10 Amount.

N. B. The Stamp or Policy Duty for any part of £ 100 is the same as for £ 100, and the rate varies according to the rate of the Premium, and to whether the Insurance is foreign or from one part of the united kingdom to another.

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* Multiply 36 by 12 and cut off the two lower figures in the product.

Ex. 3. To find the amount of the yearly payment for the whole life, for insuring £ 2000 at £ 4 15 3 per cent., and the amount of the first payment, including £ 3 for the Policy Stamp.

	£	s.	d.	
	4	15	3	
			20	
	95	5	0	Yearly payment.
	3	0	0	Policy.
£	98	5	0	First payment.

N. B. The rates of the premium for Life Insurances vary at different offices according to the age of the life insured, and to whether the Premium is engaged to be paid for only one year, for a specified term of years, or for the whole life.

The rate of the Policy Stamp is according to the amount insured, and is charged to the insured on the first payment.

Exercises to Ex. 1. Fire Insurances.

What is the amount to be paid for premium and duty,

£ 1000 at 2 s. 6 d. per cent ?	Answer	£ 2	15	-
£ 1300 at 1 s. 6 d. per cent ?	—	£ 2	18	6
£ 1800 at 4 s. 6 d. per cent ?	—	£ 6	15	-

Exercises to Ex. 2. Sea Insurances.*

What is the amount to be paid for insuring

£ 1000 at 5 s. per ct., Policy 1 s. 3 d. per ct. ?	Ans.	£ 3	2	6
£ 1450 at 30 s. per ct., Policy 2 s. 6 d. per ct. ?	—	£ 23	12	6
£ 2356 at 2 gs. per ct., Policy 5 s. per cent. ?	—	£ 55	9	6

Exercises to Ex. 3. Life Insurances.

What is the amount to be paid yearly for insuring

£ 600 at £ 2 12 9 per cent. ?	Answer	£ 15	16	6
£ 1000 at £ 3 4 7 per cent. ?	—	£ 32	5	10
£ 2500 at £ 4 18 6 per cent. ?	—	£ 123	2	6

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* For further Exercises and much further information on the subject of Sea or Marine Insurances, see the Appendix.

FOREIGN EXCHANGES.

The calculation of a foreign exchange, is the finding of what sum in the money of one country is equal to a given sum in the money of another country.

A rate of exchange is the price at which the valuation is made.

A course of exchange is a list of the prices or rates of exchange, at which bills of exchange are negociated at the given place.

A bill of exchange is a written order for the payment either of the money it directs, or the amount of that money at a given rate of exchange.

Explanation of a London Course of Exchange.

London, March 26 th, 18—

London receives for 1 £ Sterling from

Amsterdam .	12	9	Florins and Stivers.
Hamburg . .	13	4	Marks and Schillings.
Paris . . .	25	85	Francs and Centimes.
Frankfort . .	155		Batzen.
Vienna . . .	10	16	Florins and Kreuzers.
Genoa . . .	26	10	Lire Nuove and Centisimi.
Berlin . . .	7	2	Dollars and Silver Groschen.

London gives in Pence Sterling to

	d.			
St. Petersburg .	10¼	for	1	Banco Rouble.
Madrid . . .	36	-	1	Dollar of Plate.
Gibraltar . . .	47½	-	1	Hard Dollar.
Leghorn . . .	47¼	-	1	Pezza.
Naples	39½	-	1	Ducat.
Palermo . . .	118½	-	1	Onza.
Venice	47½	-	6	Lire Austriache.
Lisbon	44	-	1	Milreis.
Rio de Janeiro .	35	-	1	Milreis.

N. B. Of the two terms forming the rate of exchange, the variable term is in foreign money in the rates of the first places in this course, and in Sterling money of the rest.

GENERAL RULE

FOR THE CALCULATION OF FOREIGN EXCHANGES.

Make a statement of a proportional question, placing the sum to be exchanged as the third term of the question, that part of the rate of exchange which is of the same money as the first term, and the other part of the rate as the second term; then when the fixed term of the rate is the first term, the calculation may be worked by Multiplication or by Practice; but when the variable term of the rate is the first term, it can be only made by the Rule of Three.

N. B. In quotations of exchange rates it is usual to give only the variable prices.

FRANCE.

100 Centimes make 1 Franc.

The value of the Franc is about 9½ d. Sterling, making the par of exchange* about 25.26 Frs. per £ Sterling.

Ex. To exchange Francs 20280.00 into Sterling at 25.35.

If Frs. 2535 ——— £ 1 ——— Frs. 20280.00?

£ 20280.00 ÷ 25.35 = £ 800 Sterling.

Reverse.

To exchange £ 800 into Francs at 25.35.

If £ 1 ——— Frs. 25.35 ——— £ 800?

Frs. 25.35 × 800 = Frs. 20280.00.

Ex. 1. Exchange £ 250 into Francs at 25.70.
2. Exchange £ 96 10 6 into Frs. at 25.55.
3. Exchange Frs. 8000 00 cts. into Sterling at 25.30.
4. Exchange Frs. 7138 64 cts. into Sterling at 25.65.

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* A par of exchange is the rate of exchange produced from the valuation foreign coins.

For the products of the Exercises see page 129.

THE NETHERLANDS.

AMSTERDAM, ROTTERDAM, ANTWERP.

100 Cents make 1 Florin.

In the rate of exchange of London on the Netherlands, the Florin is divided into 20 Stivers, each of which is worth 5 Cents.

The value of the Florin is about 20 d. Sterling, making the par of exchange about 12 Florins per £ Sterling.

N. B. The Netherland rate of exchange on London is frequently given in Schillings and Grotes Flemish, reckoning 10 Schillings for 3 Florins, and 2 Grotes for 5 Cents.—47¼ Florins, or 157½ Schillings Flemish, are reckoned equal to 100 French Francs.

Ex. 5. Exchange £ 500 into Florins at 12 Fl. 10 Cts.
6. Exchange £ 375 into Florins at 11 Fl. 15 Stiv.
7. Exchange Fl. 10000 into Sterling at 12 Fl. 5 Cts.
8. Exchange Fl. 8745 50 Cts. into Sterl. at 11 Fl. 19 Stiv.

GERMANY.

The exchange places of London on Germany are, Hamburg, Berlin, Frankfort, Vienna, and Trieste.

HAMBURG.

12 Pfennings	make	1	Schilling.
16 Schillings	—	1	Mark Banco.

The Mark Banco is worth about 17½ d. Sterling, making the par of exchange about 13 Mks. 10½ Sch. per £ Sterling.—The exchange on Altona, in Denmark, is similar to that of Hamburg.

Ex. 9. Exchange £ 100 Sterling into Marks at 13.10.
10. Exchange £ 475 15 Sterling into Marks at 13.12.
11. Exchange Bmf. 10000 into Sterling at 14.0.
12. Exchange Bmf. 6754.10 into Sterling at 13.15.

BERLIN.

30 Silver Groschen make 1 Prussian Dollar.

The value of the Dollar is about 2 s. 10¾ d. Sterling, making the par of exchange about 6 Dol. 27 Silver Gro. per £ Sterling.

Ex. 13. Exchange £ 500 into P. Dollars at 7.3.
14. Exchange P. D. 1000 Sterling at 6.27.

FRANKFORT.

90 Kreuzers make 1 Rixdollar.

Also, 4 Kreuzers make 1 Batzen, 60 Kreuzers, or 15 Batzen, make 1 Florin, and 22½ Batzen make 1 Rixdollar, or 90 Batzen make 4 Rixdollars.

The Rixdollar of exchange is worth about 36 d. Sterling, making the par of exchange about 150 Batzen per £ Sterling.

Ex. 15. Exchange £ 500 into Rixdollars at 150.
16. Exchange Florins 3000 into Sterling at 154.

VIENNA AND TRIESTE.

60 Kreuzers make 1 Florin.

The value of the Florin is about 2 s. Sterling, making the par of exchange about 10 Florins per £ Sterling.

Ex. 17. Exchange £ 100 into Florins at 10.2.
18. Exchange Fl. 4156 30 kr. into Sterling at 10.5.

ITALY.

The chief exchange places of Italy are Genoa, Venice, Leghorn, Naples, and Palermo, to which may be added, but of less importance, Milan, Rome, Ancona, and Florence.

GENOA.

100 Centisimi make 1 Lira Nuova.

The Lira Nuova of Genoa and the Sardinian States, is equal in value to the French Franc, or about 9½ d. Sterling, making the par of Exchange about 25 L. N. 26 Cti. per £ Sterling.

Ex. 19. Exchange £ 84 12 6 into Lire N. at 26.10.
20. Exchange Lire N. 8397 52 Cti. into Sterl. at 25.85.

VENICE AND MILAN.

100 Centisimi make 1 Lira Austriacha.

The Lira Aust. is worth the third part of an Austrian Florin, or about 8 d. Sterling, and therefore the pars of exchange are, for London on Venice 6 Lire A. for about 48 d. Sterling; and for Milan on London about 30 Lire A. per £ Sterling.

Ex. 21. Exchange £ 300 into Lire A. at 48 d. for 6 Lire.
22. Exchange Lire A. 4000 into Sterling at 30 L. A.

ROME AND ANCONA.

10 Paoli or 100 Bajocchi make 1 Scudo or Crown.

The Roman Crown is worth about 50 d. Sterling, making the par of Exchange of Rome or Ancona on London, 48 Paoli per £ Sterling.

Ex. 23. Exchange £ 100 into Scudi at 48½ P.
24. Exchange Scudi 500 into Sterling at 50¼ P.

NAPLES.

100 Grani make 1 Ducat

The value of the Neapolitan Ducat, called the Ducato del Regno, is about 40 d. Sterling, making the par of exchange of Naples on London about 600 Grani per £ Sterling.

Ex. 25. Exchange £ 500 Sterling into Ducats at 39½ d.
26. Exchange Ducats 4176 34 Gr. into Sterling at 39⅞.

PALERMO.

20 Grani make 1 Taro.
30 Tari — 1 Onza.

The value of the Onza is about 120 d. Sterling, making the par of exchange at Palermo on London, about 60 Tari per £ Sterling.

Ex. 27. Exchange £ 100 Sterling into Onze at 118¾
28. Exchange 1000 Onze into Sterling at 119⅝.

LEGHORN AND FLORENCE.

12 Denari make 1 Soldo.
20 Soldi — 1 Pezza.

The Pezza, in gold valuation, called Pezza d'oro, or in the money of exchange of Leghorn, is worth about 48 d. Sterling; but in silver valuation, or in the money of account, (and also the money of exchange of Florence,) the Pezza is worth about 45 d. Sterling; 100 Pezze in Gold are valued at 107 Pezze in Silver.

Ex. 29. Exchange £ 300 Sterling into Pezze d'oro at 47¾.
30. Exchange 1000 Pezze d'oro into Sterling at 48⅜.

SPAIN.

Plate Money.

2⅛ Maravedis	make	1	Quarto.
16 Quartos	—	1	Real of Plate.
8 Reals of Plate	—	1	Dollar of Plate.

Vellon Money.

34 Maravedis	make	1	Real Vellon.
20 Reals Vellon	—	1	Hard Dollar.

The value of the Dollar of Plate is nearly 3-4 ths of a Spanish or Hard Dollar, 85 Dollars of Plate being equal to 64 Hard Dollars.*

The Hard Dollar is valued at 50 d. Sterling, and the value of the Dollar of Plate, which is the Dollar of Exchange, is therefore about 37½ d. Sterling.

Ex. 31. Exchange £ 400 into Dollars of Plate at 36 d.

Ex. 32. Exchange Dollars P. 3472 5 rls. 8 qtos. into Sterling at 36¾ d.

PORTUGAL.

1000 Reis make 1 Milreis.

The value of the Milreis of Exchange is uncertain, because payments of Bills and Accounts in Portugal are made in legal money, or one half in Cash and the other half in paper money, which is at a large and fluctuating Discount.

Ex. 33. Exchange £ 837 17 10 Sterling into Milreis at 48 d.

34. Exchange Mil. 4606 456 Reis into Sterling at 50 d.

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* Vellon money is employed in the public accounts, but plate money is generally used in exchanges, and it is also the money of accounts in some parts of Spain.

The Dollar of plate is generally reckoned at 15 reals 2 maravedis vellon, but at Malaga it is reckoned at 15 reals vellon.

In Alicant and all Valencia, as also in Barcelona and all Catalonia, the money of account is in Libras of 20 sueldos each of 12 dineros: in the former the Libra is equal to the Dollar of plate; in the latter 7 Libras are equal to 5 Dollars of plate.

TURKEY.

40 Paras make 1 Piastre.

The Turkish Piastre bears only a nominal value, at present it may be reckoned about 3 d. Sterling, as about 80 Piastres make the rate of exchange for the £ Sterling.

Ex. 35. Exchange £ 400 into Piastres at 78½.
36. Exchange Piastres 10,000 into Sterling at 75.

RUSSIA.

100 Copecs make 1 Rouble.

The Rouble of exchange and account is generally the Bank or Paper Rouble; and as the common rate of exchange between this and the Silver Rouble is about 3¾ Paper Roubles for 1 Silver Rouble, which is worth about 37½ d. Sterling, the Bank Rouble may be reckoned at 10 d. Sterling, or the £ Sterling at 24 Roubles.

N. B. The rate of exchange of Odessa on London is in Roubles per £ Sterling.

Ex. 37. Exchange £ 500 Sterling into Roubles at 10¼ d.
38. Exchange Roubles 10,000 into Sterling at 10½.

SWEDEN.

48 Skillings make 1 Rixdollar.

The Rixdollar of exchange and account being in Paper, has only a nominal value; according to the present rate of exchange the value is about 1 s. 9 d. Sterling, making the £ Sterling worth about 11 RD. 21 sk.

Ex. 39. Exchange £ 500 Sterling into Rixdollars at 12.
40. Exchange 5000 Rixdollars into Sterling at 11.24.

DENMARK.

96 Skillings make 1 Rigsbank Dollar.

The value of the Danish Rigsbank Dollar is equal to that of 1½ Marks Banco, or about 2 s. 2¼ d. Sterling, making the value of the £ Sterling about 9 RB. D. 13 sk.

Ex. 41. Exchange £ 500 Sterling into RB. D. at 9.35.
42. Exchange RB. D. 1000 into Sterling at 9.14.

NORTH AMERICA.

United States.

100 Cents make 1 Dollar.

The value of the United States Dollar is about 4 s. 1½ d. Sterling, but the fixed exchange value of this Dollar, called the par value, is 4 s. 6 d. Sterling, or rather Currency. Hence at present Sterling bears a premium upon this valuation of about 10 per cent., £ 100 Sterling being worth about £ 110 U. S. money, at 4 s. 6 d. per Dollar.

Ex. 43. Exchange £ 100 Sterling into U. S. Dollars at par, and also at 10 per cent.

Ex. 44. Exchange 100 U. S. Dollars into Sterling at par, and also at 10 per cent.

BRITISH COLONIES.

Accounts are kept in pounds, shillings, and pence, Currency. In Canada, £ 100 Currency are, at par, equal to £ 90 Sterling; in Jamaica, £ 140 Currency are equal to £ 100 Sterling; and exchange is made with England by a premium or discount per cent. on the Sterling money. In other places exchange is made by giving a variable number of pounds Currency for £ 100 Sterling.

Ex. 45. Exchange £ 500 Sterling into Canada Currency at par.
46. Exchange £ 300 Sterling into Jamaica Currency at par.
47. Exchange £ 400 Sterl. into Barbadoes Currency at 137½.
48. Exchange £ 200 Sterling into Grenada Currency at 225.

SOUTH AMERICA.

Spanish Settlements.

16 Quarts make 1 Real.
8 Reals — 1 Dollar.

The Dollar is either Currency or the Hard Spanish Dollar; the value of the Currency Dollar is nominal.

Ex. 49. Exchange £ 100 Sterling into Hard Dollars at 45½ d.
50. Exchange £ 100 Sterling into Paper Dollars at 7½ d.

Portuguese Settlements.

1000 Reis make 1 Milreis.

The Milreis of exchange is in Paper, and is nominal in value.

Ex. 51. Exchange £ 100 Sterling into Milreis at 26½ d.

THE EAST INDIES.

Bengal and Madras.

12 Pice make 1 Anna.
16 Annas — 1 Rupee.

Bombay.

100 Rees make 1 Quarter.
4 Quarters — 1 Rupee.

The value of the Bengal Rupee is about 2 s. Sterling, and of the Madras and Bombay Rupee about 1 s. 10¼ Sterling.

Ex. 52. Exchange £ 100 Sterling into Bengal Rupees at 2 s. 0½ d.

Ex. 53. Exchange £ 300 Sterling into Madras Rupees at 1 s. 10½ d.

Ex. 54. Exchange £ 500 Sterling into Bombay Rupees at 1 s. 10¼ d.

PRODUCTS.

1. Frs. 6425 00 cts.
2. Frs. 2466 21 cts.
3. £ 316 4 1
4. £ 278 6 2
5. Flor. 6050 00 cts.
6. Flor. 4406 25 cts.
7. £ 829 17 6
8. £ 731 16 10
9. Mks. 1362 8 sch.
10. Mks. 6541 9 sch.
11. £ 714 5 9
12. £ 484 12 9
13. P. D. 3550
14. £ 144 18 6
15. R. D. 3333 30 kr.
16. £ 292 4 2
17. Flor. 1003 20 kr.
18. £ 412 4 3
19. L. N. 2208 71 cts.
20. £ 324 17 1
21. L. A. 9000
22. £ 133 6 8
23. Scudi 485
24. £ 99 10 1
25. Duc. 3037 97 gr.
26. £ 693 17 8
27. Onze 202 3 3
28. £ 498 8 9
29. P. d'oro 1507 17
30. £ 201 11 3
31. D. P. 2666 5 5
32. £ 531 15 1
33. Reis 4 189 458
34. £ 959 13 7
35. Piastres 31400
36. £ 133 6 8
37. Roubles 11707 31 cop.
38. £ 437 10
39. Rixdollars 6000
40. £ 434 15 8.
41. RB. D. 4682 28 sch.
42. £ 109 6 9
43. Doll. 488 88 cts.
44. £ 204 10 11
45. £ 555 11 1
46. £ 420
47. £ 550
48. £ 450
49. Doll. 527 3 12
50. P. Dollars 3200
51. Reis 905 660
52. Rup. 979 9 5
53. Rup. 3200
54. Rup. 5393 1 3

FRACTIONS.

A Fraction is a number expressing one or more of the equal parts into which a unit or an integer is divided.

NUMERATION.

A Fraction is expressed by two numbers; one, which is called the denominator, shows how many of parts make the unit or integer, and the other, which is called the Numerator, shows how many of those parts are employed in the Fraction:—thus, $\frac{5}{8}$ is a fraction; the denominator (the lower number) shows that 8 of these parts make the unit or integer; and the numerator (the upper number) shows that five of these parts are here used.

A Fraction may also be considered as expressing the result of a division, whether the quotient can or cannot be expressed by a whole number; thus 5-8 ths may be considered as the quotient of 5 divided by 8, and may be expressed as the 8 th part of 5.

A Fraction having 1 for its Numerator is called a part, or, to more particularly mark it, an aliquot part; but a fractional expression having 1 for the denominator, is merely the fractional formula for a whole number; as $\frac{11}{1}$ for 11.

When the numerator of a fraction is less than the denominator, it is called a proper fraction; as $\frac{5}{8}$, the value of which is less than unity.

When the numerator of a fraction is not less than the denominator, it is called an improper fraction; as $\frac{8}{8}$, and $\frac{11}{8}$, the value of the first being equal to unity, and of the second greater than unity.

A number composed of a whole number and a fraction, is called a mixed number; as $3\frac{5}{8}$.

The fractional part of any other number than unity is called a compound fraction; as $\frac{3}{4}$ of $\frac{7}{8}$; or $\frac{3}{8}$ of 11.

When either the numerator or the denominator of a fraction contains a fraction, the expression is called a complex fraction, as,

$$\frac{3\frac{1}{4}}{5} \qquad \frac{\frac{3}{8}}{15} \qquad \frac{7}{11\frac{1}{4}} \qquad \frac{13}{\frac{11}{12}} \qquad \frac{3\frac{7}{8}}{5\frac{1}{4}} \qquad \frac{\frac{5}{8}}{\frac{11}{12}}$$

FRACTIONAL REDUCTION.

CASE 1.

To reduce an improper fraction to its equivalent whole or mixed number.

Rule. Divide the numerator by the denominator; the quotient will be the equivalent whole number, and if there is a remainder, annex it as the numerator of a fraction having the given denominator.

Example. To reduce $\frac{537}{25}$ to a mixed number.

$$\frac{537}{25} = 537 \div 25 = 21\frac{12}{25}$$

Ex. 1. Reduce $\frac{32}{8}$, $\frac{25}{5}$, and $\frac{112}{4}$ to equivalent whole numbers.

2. Reduce $\frac{25}{7}$, $\frac{33}{5}$, and $\frac{617}{11}$ to equivalent mixed numbers.

3. Reduce $\frac{321}{17}$, $\frac{8461}{513}$, and $\frac{10001}{382}$ to equivalent mixed numbers.

Products. Ex. 1. 4, 5, 28.

Ex. 2. $3\frac{4}{7}$, $6\frac{3}{5}$, $56\frac{1}{11}$. Ex. 3. $18\frac{15}{17}$, $16\frac{253}{513}$, $26\frac{69}{382}$.

CASE 2.

To reduce a mixed number to an improper fraction.

Multiply the whole number by the denominator of the fraction, and add in the numerator; then make this amount the numerator to the required fraction, having for its denominator the denominator of the given fraction.

Example. To reduce $7\frac{3}{5}$ to an improper fraction.

$$7\frac{3}{5} = \frac{7 \times 5 + 3}{5} = \frac{38}{5}$$

Ex. 1. Reduce $4\frac{1}{2}$, $5\frac{3}{4}$, and $11\frac{7}{8}$ to improper fractions.

2. Reduce $7\frac{5}{112}$, $23\frac{14}{23}$, and $54\frac{131}{157}$ to improper fractions.

PRODUCTS.

Ex. 1. $\frac{9}{2}$, $\frac{23}{4}$, $\frac{65}{8}$. Ex. 2. $\frac{789}{112}$, $\frac{543}{23}$, $\frac{8609}{157}$.

CASE 3.

To reduce a complex fraction to its simple terms.

Rule. When only one of the terms contains a fraction, multiply both terms by the denominator of the annexed fraction.*

When both terms contain fractions, multiply them by the product of the denominators of the annexed fractions, or by their least multiple.

Example 1. To reduce $\frac{6}{7\frac{1}{5}}$ and $\frac{11\frac{1}{4}}{15}$ to simple fractions.

$$\frac{6}{7\frac{1}{5}} = \frac{30}{36} = \frac{5}{6} \qquad \frac{11\frac{1}{4}}{15} = \frac{45}{60} = \frac{3}{4}$$

Example 2. To reduce $\frac{11\frac{3}{4}}{16\frac{5}{8}}$ to a simple fraction.

$$\frac{11\frac{3}{4}}{16\frac{5}{8}} \begin{matrix}\times\\ \times\end{matrix} \frac{8}{8} = \frac{94}{133}$$

Example 3. To reduce $\frac{5\frac{7}{11}}{14\frac{3}{5}}$ to a simple fraction.

$$\frac{5\frac{7}{11}}{14\frac{3}{5}} \times \frac{11}{5} \begin{matrix}\times\\ \times\end{matrix} \frac{5}{11} = \frac{310}{803}$$

EXERCISES.

Ex. 1. Reduce $\frac{1}{3\frac{1}{4}}$ and $\frac{5\frac{3}{7}}{10}$ to simple fractions.

2. Reduce $\frac{3}{\frac{7}{8}}$ and $\frac{\frac{5}{16}}{12}$ to simple fractions.

3. Reduce $\frac{\frac{5}{8}}{\frac{11}{12}}$ and $\frac{6\frac{2}{3}}{7\frac{5}{6}}$ to simple fractions.

PRODUCTS.

Ex. 1. $\frac{4}{13}$, $\frac{19}{35}$. Ex. 2. $\frac{24}{7}$, $\frac{5}{192}$. Ex. 3. $\frac{15}{22}$, $\frac{40}{47}$.

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* For the multiplication of a fraction by a whole number see Obs. 3. Fractional Multiplication.

The least multiple is the smallest number or product in which the given numbers are contained.

CASE 4.

To reduce a compound fraction to its equivalent simple fraction.

Rule. Mixed numbers and whole numbers being expressed as improper fractions, and complex fractions being reduced to simple fractions, multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.

Observe, that if the same number occurs as both a numerator and a denominator, it may be cancelled; and if one numerator and one denominator will divide by the same number, each may be divided, and its quotient used in its place.

N. B. Only one numerator and one denominator may be taken at a time, but the cancelment or division may be repeated as often as possible.

EXAMPLE.

To reduce $\frac{3}{7}$ of $2\frac{4}{5}$ of $\frac{5}{8}$ of 9 to a simple fraction.

$$2\tfrac{4}{5} = \tfrac{14}{5} \qquad\qquad 9 = \tfrac{9}{1}$$

$$\underset{1}{\frac{3}{7}} \times \overset{\overset{1}{2}}{\underset{1}{\frac{14}{5}}} \times \overset{1}{\underset{4}{\frac{5}{8}}} \times \frac{9}{1} = \frac{27}{4} = 6\tfrac{3}{4}$$

The mixed number and the whole number being expressed fractionally, and the numbers being arranged for multiplication; beginning with the 1 st numerator 3, we find no denominator that will divide it, and therefore we pass on to the 14, which, with the 7, divides by 7, cancelling the 7 in the denominator, and making the 14 into 2;—this 2 divides with 8, cancelling the former and making the latter 4; and then the two fives cancel one upon the other;—there then remains 4 in the denominator which will not divide with either the 3 or the 9, and therefore compounding these latter numbers we get 27-4 ths or $6\frac{3}{4}$.

N. B. We have here placed a 1 above or below the numbers to be cancelled, but this is not necessary in the performance of these calculations.

EXERCISES.

Reduce $\frac{3}{4}$ of $\frac{5}{7}$ of 11 to a simple fraction. *Product* $5\frac{25}{28}$
Reduce $\frac{1}{5}$ of $\frac{10}{11}$ of $\frac{6}{7}$ of $3\frac{1}{3}$ to a simple fraction. — $\frac{40}{77}$
Reduce $\frac{3}{14}$ of $\frac{7}{12}$ of $\frac{16}{23}$ of $\frac{1}{7}$ of $8\frac{5}{8}$ to a simple fraction. — $\frac{69}{644}$

CASE 5.

To find the greatest common measure to two numbers.

Rule. Divide the greater number by the less; then if there is a remainder make it the divisor of another operation, in which the preceding divisor is made the dividend, and thus continue the calculation until there is not any remainder, and the last divisor will be the greatest common measure.

EXAMPLE.

Find the greatest common measure to 612 and 828.

612) 828 (1
612
216) 612 (2
432
180) 216 (1
180
36) 180 (5
180

The greatest common measure is 36, being the greatest number that will divide both 612 and 828.

EXERCISES.

Ex. 1. Find the greatest common measure to 171 and 180.
2. Find the greatest common measure to 4536 and 4872.
3. Find the greatest common measure to 1785, 3255, and 7140.

PRODUCTS.

Ex. 1. 9. Ex. 2. 168. Ex. 3. 105.

N. B. The principle of this rule may be easily explained from the above example; for as 36 is the exact measure of 180, it must also exactly measure 36 and 180, or 216; again, as 36 measures 216 and 180, it must measure twice 216 and 180, or 612; lastly, as 36 measures 216 and 612, it must measure the sum, or 828; hence 36 is the common measure of 612 and 828.

CASE 6.

To reduce a fraction to its lowest terms.

Rule. Divide both terms of the fraction by their greatest common measure.

Note 1. Instead of finding the greatest common measure, divide the numerator and denominator by any number that will exactly divide them; repeat the calculation, if possible, with the new fraction produced, and so continue the operation until the fraction is reduced into its lowest terms.

Note 2. Numbers ending in 0 are divisible by 10, and in two ciphers by 100, &c. Numbers ending in 5 are divisible by 5. If the sum of the figure is divisible by 9 or 3, they are divisible by those numbers. Even numbers are divisible by 2. If the units and tens are divisible by 4 the numbers are divisible by 4. If the units, tens, and hundreds are divisible by 8, the numbers are divisible by 8.

EXAMPLE.

To reduce $\frac{3780}{88200}$ to its lowest terms

3780) 88200 (23
7560

12600
11340

1260) 3780 (3
3780

$$1260\ )\ \frac{3780}{88200} = \frac{3}{70}$$

$$\text{or } \overset{(10)}{\frac{3780}{88200}} = \overset{(2)}{\frac{378}{8820}} = \overset{(9)}{\frac{189}{4410}} = \overset{(7)}{\frac{21}{490}} = \frac{3}{70}$$

EXERCISES.

Ex. 1. Reduce $\frac{27}{36}$ to its lowest terms. *Product* $\frac{3}{4}$
2. Reduce $\frac{2520}{3960}$ to its lowest terms. — $\frac{7}{11}$
3. Reduce $\frac{315}{1155}$ to its lowest terms. — $\frac{3}{11}$
4. Reduce $\frac{11883}{13281}$ to its lowest terms. — $\frac{17}{19}$

CASE 7.

To find the least common multiple of two or more numbers.

Rule. When there are only two numbers—Divide one of the given numbers by the greatest common measure of the two numbers, and multiply the other number by the quotient.

When there are more than two numbers—Divide any two or more of the given numbers by their greatest common divisor. Use their quotients in their places, find again the greatest common divisor of two or more of these numbers, divide as before, and so continue the divisions until there are not two numbers that have a common measure; then multiply the common measures and the remaining numbers together, and the product will be the least common multiple.

N. B. When one of the given numbers is a multiple of the other, or of the rest, it is evidently the least common multiple.

When any of the less numbers are contained in a greater, they may be rejected from the calculation.

The least common multiple can frequently be found by taking in succession the multiples of the greatest number, and using the lowest multiple that contains the other or the rest.

Example 1. To find the least common multiple of 12 and 16.

4 is the greatest common measure.

$$12 \div 4 \times 16 = 48 \text{ the least common multiple.}$$

Example 2. To find the least common multiple of 9, 8, 7, 6, 4.

$$\begin{array}{r|ccccc} 4 & 9 & 8 & 7 & 6 & 4 \\ \hline 3 & 9 & 2 & 7 & 6 & 1 \\ \hline 2 & 3 & 2 & 7 & 2 & 1 \\ \hline & 3 & 1 & 7 & 1 & 1 \\ \hline \end{array}$$

$$4 \times 3 \times 2 \times 3 \times 7 = 504 \text{ the least common multiple.}$$

EXERCISES.

Find the least common multiple,

Ex. 1.	Of 12, 24, and 48.	*Product*	48
2.	- 4, 7, and 12.	—	84
3.	- 5, 15, and 25.	—	75
4.	- 10, 11, 15, 25, and 33.	—	1650
5.	- 5, 7, 9, and 11.	—	3465

CASE 8.

To reduce fractions of different denominators to similar fractions, or fractions having the same denominator.

Rule 1. Multiply both terms of every fraction by the denominators of the other fractions.

Rule 2. Find the least common multiple of the denominators, and multiply both terms of each fraction by the quotient produced by dividing the common multiple by the denominator of that fraction.

Observe, that when the same denominator occurs twice or more it may be used only once in finding the least common multiple.

EXAMPLE 1.

To reduce $\frac{5}{6}$, $\frac{2}{7}$, and $\frac{3}{11}$ to similar fractions.

$$\begin{array}{lllllll} \frac{5}{6} & \times & 7 & \times & 11 & = & \frac{385}{462} \\ \frac{2}{7} & \times & 6 & \times & 11 & = & \frac{132}{462} \\ \frac{3}{11} & \times & 7 & \times & 6 & = & \frac{126}{462} \end{array}$$

EXAMPLE 2.

To reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{16}$ to similar fractions.

$$\begin{array}{lllll} \frac{3}{4} & \times & 4 & = & \frac{12}{16} \\ \frac{5}{8} & \times & 2 & = & \frac{10}{16} \\ & & & \text{and} & \frac{11}{16} \end{array}$$

EXERCISES.

Ex. 1. Reduce $\frac{3}{5}$, $\frac{2}{3}$, and $\frac{5}{11}$ to similar fractions.
2. Reduce $\frac{5}{9}$, $\frac{3}{10}$, and $\frac{4}{13}$ to similar fractions.
3. Reduce $\frac{2}{3}$, $\frac{9}{10}$, and $\frac{7}{12}$ to similar fractions.
4. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{16}$ to similar fractions.
5. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{13}{18}$ to similar fractions.

PRODUCTS.

Ex. 1. $\frac{99}{165}$, $\frac{110}{165}$, $\frac{75}{165}$. Ex. 2. $\frac{650}{1170}$, $\frac{351}{1170}$, $\frac{360}{1170}$.
3. $\frac{40}{60}$, $\frac{54}{60}$, $\frac{35}{60}$. 4. $\frac{8}{16}$, $\frac{12}{16}$, $\frac{10}{16}$, $\frac{11}{16}$,
Ex. 5. $\frac{162}{216}$, $\frac{180}{216}$, $\frac{189}{216}$, $\frac{198}{216}$, $\frac{156}{216}$.

CASE 9.

To find the value of a fractional quantity.

Rule. Consider the numerator of the fraction as so many integers, and divide it by the denominator, reducing it into the lower denominations, as in Simple Division of quantities.

EXAMPLE.

To find the value of $\frac{5}{11}$ £.

$$\begin{array}{r|ccc} & \text{£} & \text{s.} & \text{d.} \\ 11\,) & 5 & 0 & 0 \\ \hline & \text{s.} & 9 & 1\frac{1}{11} \\ \hline \end{array}$$

* $\frac{5}{11}$ £ = s. 9 $1\frac{1}{11}$

EXERCISES.

Ex. 1. Find the value of $\frac{1}{4}$ £, $\frac{1}{11}$ £, and $\frac{5}{7}$ £.
2. Find the value of $\frac{4}{7}$ s. $\frac{5}{16}$ s. and $\frac{17}{22}$ s.
3. Find the value of $\frac{1}{7}$ cwt. $\frac{4}{9}$ qr. and $\frac{11}{12}$ lb.
4. Find the value of $\frac{3}{5}$ yd. $\frac{5}{7}$ ell, and $\frac{6}{11}$ mile.

PRODUCTS.

Ex. 1. 5 s., 1 s. $9\frac{3}{4}$ $\frac{3}{11}$ d., s. 14 $3\frac{1}{4}$ $\frac{5}{7}$ d.
2. $6\frac{3}{4}$ $\frac{3}{7}$ d., $3\frac{3}{4}$ d., $9\frac{1}{4}$ $\frac{1}{11}$ d.
3. 16 lb., 12 lb. $7\frac{1}{9}$ oz., 14 oz. $10\frac{2}{3}$ drs.
4. 2 qr. $1\frac{3}{5}$ n., 3 qr. $2\frac{2}{7}$ n., 960 yds.

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* This is all that is usually necessary to be expressed, but the work may be expressed thus:

$$\frac{5}{11}\ \text{£} = 20\ \text{s.} \times 5 \div 11 = 9\ \text{s.}\ 1\frac{1}{11}\ \text{d.}$$

CASE 10.

To reduce a simple or compound quantity into the fraction of another quantity.

Rule. Reduce if necessary the two quantities into the lowest denomination contained in either; then make the number of the quantity to be reduced, the numerator, and the other number the denominator, of the fraction required.

Ex. To reduce 5 s. 9 d. to the fraction of a pound sterling.

$$5\text{ s. }9\text{ d.} = 69\text{ d.}\quad £\,1 = 240\text{ d.}$$
$$5\text{ s. }9\text{ d.} = \tfrac{69}{240}\,£ = \tfrac{23}{80}\,£.$$

Ex. 1. Reduce 2 s. 6 d., 7 s. 10 d., and 9 s. $7\frac{1}{2}$ d. to fractions of a £.

Ex. 2. Reduce 2 qr., 3 qr. 7 lb., and 1 qr. 24 lb. to fractions of a cwt.

PRODUCTS.

Ex. 1. $\frac{1}{8}$ £, $\frac{47}{120}$ £, $\frac{77}{160}$ £. Ex. 2. $\frac{1}{2}$ cwt., $\frac{13}{16}$ cwt., $\frac{13}{2}$ cwt.

CASE 11.

To reduce the fraction of one integer into the fraction of another integer.

Rule. Express the integer of which the fraction is given as either the fractional part or the multiple of the integer in which the fraction is to be found; then prefix to it the given fraction in the form of a compound fraction, and find its value as a simple fraction.

Ex. To reduce $\frac{3}{8}$ of a shilling into the fraction of a £, and a penny.

$$\tfrac{3}{8}\text{ s.} = \tfrac{3}{8}\text{ of }\tfrac{1}{20}\,£ = \tfrac{3}{160}\,£.$$
$$\tfrac{3}{8}\text{ s.} = \tfrac{3}{8}\text{ of }\tfrac{12}{1}\text{ d.} = \tfrac{36}{8}\text{ d.} = 4\tfrac{1}{2}\text{ d.}$$

Here, as 1 shilling is 1-20 th of a £, and 12 times 1 penny, 3-8 ths of a shilling are 3-8 ths of 1-20 th of a pound, or 3-8 ths of 12 times 1 penny.

Ex. 1. Reduce $\frac{1}{4}$ s., $\frac{3}{5}$ s., and $\frac{5}{7}$ s. to the fraction of a penny.
2. Reduce $\frac{1}{7}$ s., $\frac{1}{11}$ s., and $4\frac{3}{5}$ s. to the fraction of a £.

PRODUCTS.

Ex. 1. $\frac{3}{1}$ d., $\frac{36}{5}$ d., $\frac{60}{7}$ d. Ex. 2. $\frac{1}{140}$ £, $\frac{1}{220}$ £, $\frac{23}{100}$ £.

FRACTIONAL ADDITION.

Rule. Reduce the given fractions, if they require it, to similar fractions, or fractions having the same denominator; then add the numerators together for the numerator of the fraction of the amount, having for its denominator the common denominator.

Observe. If the fractions are mixed with whole numbers, find the amount of the fractions, and if the amount is equal to or greater than unity, reduce the fraction to its equivalent whole or mixed number, and add the integers to the given whole numbers.

If the given fractions are either of different integral quantities, or if an integer that can be separated into lower denominations, the values of the given fractions may be used for addition.

Ex. 1. Add $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{8}$ together.

$$\begin{array}{l}
\frac{2}{3} \times 5 \times 8 = \frac{80}{120} \\
\frac{3}{5} \times 8 \times 3 = \frac{72}{120} \\
\frac{5}{8} \times 5 \times 3 = \frac{75}{120} \\
\hline
\frac{227}{120} = 1\frac{107}{120}
\end{array}$$

Ex. 2. Add $4\frac{1}{3}$ £, $6\frac{1}{4}$ £, and $7\frac{5}{6}$ £, together.

$$\begin{array}{lcl}
\begin{array}{rcl}
\text{£} & & \text{£} \\
{}^{*}4\frac{1}{3} & = & 4\frac{4}{12} \\
{}^{*}6\frac{1}{4} & = & 6\frac{3}{12} \\
7\frac{5}{6} & = & 7\frac{10}{12} \\
\hline
 & & 18\frac{5}{12}
\end{array}
& \text{(or)} &
\begin{array}{rcrrr}
\text{£} & & \text{£} & \text{s.} & \text{d.} \\
4\frac{1}{3} & = & 4 & 6 & 8 \\
6\frac{1}{4} & = & 6 & 5 & 0 \\
7\frac{5}{6} & = & 7 & 16 & 8 \\
\hline
 & \text{£} & 18 & 8 & 4
\end{array}
\end{array}$$

Ex. 1. Find the amount of $\frac{1}{5}$ and $\frac{1}{11}$.*

2. Find the amount of $\frac{5}{7}$ of $\frac{3}{5}$, and $\frac{3}{11}$ of $5\frac{1}{2}$.

3. Find the amount of $\frac{4}{7}$ of $\frac{13}{18}$, and $\frac{9}{14}$.

4. Find the amount of $4\frac{1}{4}$ £, $5\frac{3}{8}$ £, and $11\frac{11}{16}$ £.

AMOUNTS.

Ex. 1. $\frac{16}{55}$.	Ex. 2. $1\frac{13}{14}$.
3. $1\frac{7}{126}$.	4. $21\frac{5}{16}$ £, or £ 21 6 3.

---

* In the addition of two aliquot parts it is to be noticed, that the numerator of their sum is the *amount* of their denominators; and the denominator, is the *product* of the same.

FRACTIONAL SUBTRACTION.

Rule. Prepare the fractions as for Addition, and find the difference of the numerators.

Observe. If, when the given fractions are mixed with whole numbers, the fraction belonging to the less whole number is greater than the other fraction, an integer of the greater whole number must be taken from it, and reduced into the same sort of parts, which are then to be added to the less fraction.

Also, as in Addition, the values of the given fractions may be used in the subtraction.

Subtract $\frac{1}{5}$ from $\frac{2}{3}$ of $\frac{1}{2}$.

$$\begin{array}{rcccc} \frac{2}{3} \text{ of } \frac{1}{2} & = & \frac{1}{3}^{*} & = & \frac{5}{15} \\ & & \frac{1}{5} & = & \frac{3}{15} \\ \hline & & & \text{Difference} & \frac{2}{15} \end{array}$$

Subtract $7\frac{5}{8}$ s. from $9\frac{5}{12}$ s.

$$\begin{array}{lcl} \text{s.} & & \text{s.} \\ 9\frac{5}{12} & = & 9\frac{10}{24} \\ 7\frac{5}{8} & = & 7\frac{15}{24} \\ \hline \text{Difference} & & 1\frac{10}{24} \end{array} \qquad \text{(or)} \qquad \begin{array}{lcrr} \text{s.} & & \text{s.} & \text{d.} \\ 9\frac{5}{12} & = & 9 & 5 \\ 7\frac{5}{8} & = & 7 & 7\frac{1}{2} \\ \hline & \text{s.} & 1 & 9\frac{1}{2} \end{array}$$

Exercises.

Ex. 1. Subtract $\frac{1}{4}$ from $\frac{1}{3}$, and $\frac{1}{11}$ from $\frac{1}{9}$.*

2. Subtract $\frac{13}{17}$ from $\frac{4}{7}$ of $2\frac{3}{11}$.

3. Subtract $\frac{4}{15}$ of $3\frac{3}{4}$ from $2\frac{5}{7}$.

4. Subtract $3\frac{1}{4}$ from $4\frac{1}{2}$.

5. Subtract $1\frac{3}{7}$ from $3\frac{8}{11}$.

6. Subtract $2\frac{3}{4}$ s. from $6\frac{5}{8}$ s.

Differences.

Ex. 1. $\frac{1}{12}$, $\frac{2}{99}$. Ex. 2. $\frac{699}{1309}$ Ex. 3. $1\frac{5}{7}$.

4. $1\frac{1}{4}$. 5. $2\frac{23}{77}$ 6. $3\frac{7}{8}$ s. or s. 3 $10\frac{1}{2}$.

---

* The numerator of the difference between two aliquot parts is equal to the difference between the denominators, and the denominator is their product.

FRACTIONAL MULTIPLICATION.

Rule. Multiply the numerators together for the numerator of the product, and the denominators together for the denominator of the product.

Observe 1. To use this rule, mixed numbers and whole numbers should be reduced or expressed as improper fractions, but compound fractions need not be reduced.

Observe 2. The same directions apply to the division or cancelment of the terms of the fractions, as have been given for the reduction of compound fractions.

Observe 3. When the multiplier is a whole number, the calculation is performed by multiplying the numerator by that whole number, but when the multiplier is exactly contained in the denominator, the calculation may be performed by dividing the denominator by the whole number.

Ex. 1. To multiply $3\frac{3}{8}$ by $5\frac{1}{3}$.

$$3\tfrac{3}{8} = \tfrac{27}{8} \qquad 5\tfrac{1}{3} = \tfrac{16}{3}$$

$$\frac{\overset{9}{\cancel{27}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{1}{\cancel{3}}} = \frac{18}{1} = 18.$$

Ex. 2. To multiply $\frac{11}{16}$ by 4.

$$\tfrac{11}{16} \times \tfrac{4}{1} = \tfrac{44}{16} = \tfrac{11}{4} = 2\tfrac{3}{4}$$

$$\text{or, } \tfrac{11}{16} \div 4 = \tfrac{11}{4} = 2\tfrac{3}{4}.$$

Exercises.

Ex. 1. Multiply $\frac{3}{4}$ by $\frac{5}{7}$ *Product* $\frac{15}{28}$

2. Multiply $\frac{1}{2}$ by $\frac{1}{4}$ of $\frac{1}{3}$ — $\frac{1}{24}$

3. Multiply $\frac{3}{4}$ of $\frac{5}{6}$ by $\frac{4}{5}$ of $\frac{6}{11}$ — $\frac{3}{11}$

4. Multiply $2\frac{1}{3}$ by $\frac{3}{16}$ — $\frac{7}{16}$

5. Multiply $6\frac{1}{4}$ by $3\frac{1}{2}$ — $21\frac{7}{8}$

6. Multiply $\frac{3}{4}$ of $6\frac{2}{5}$ by $\frac{3}{11}$ of $7\frac{1}{2}$ — $9\frac{9}{11}$

7. Multiply $\frac{4}{7}$ by 4 and by 7 — $2\frac{2}{7}$ and 4

8. Multiply $\frac{4}{21}$ by 7 — $\frac{4}{3} = 1\frac{1}{3}$

FRACTIONAL DIVISION.

Rule. Invert the divisor and use it as a multiplier.*

Observe. The reduction of whole and mixed numbers (as well as the division or cancelment of the numerators and denominators, after the divisor has been inverted) is to be practised as in Fractional Multiplication.

Also, When the divisor is a whole number, it may be used either as a divisor to the numerator, or as a multiplier to the denominator.

When both terms of the fractional divisor will act as exact divisors upon the corresponding terms of the dividend, they may be used as such.

When only one term is an exact divisor, it may be used as a divisor, and the quotient is then to be multiplied by the other term of the divisor.

When both terms have the same denominator, the quotient will be formed of the division of the numerator of the dividend by the numerator of the divisor.

Ex. 1. Divide $1\frac{3}{4}$ by $\frac{3}{5}$.

$$\frac{7}{4} \div \frac{3}{5} = \frac{7}{4} \times \frac{5}{3} = \frac{35}{12} = 2\frac{11}{12}.$$

Ex. 2. Divide $1\frac{3}{4}$ by 7 and by $3\frac{1}{2}$.

$$\frac{7}{4} \div 7 = \frac{1}{4}, \text{ and } \frac{7}{4} \div \frac{7}{2} = \frac{1}{2}.$$

Ex. 3. Divide $3\frac{3}{4}$ by 4, by 5, and by $\frac{5}{7}$.

$$3\frac{3}{4} \div 4 = \frac{15}{4} \div \frac{4}{1} = \frac{15}{4} \times \frac{1}{4} = \frac{15}{16}$$

$$\frac{15}{4} \div 5 = \frac{3}{4} \qquad \frac{15}{4} \div \frac{5}{7} = \frac{15 \div 5 \times 7}{4} = \frac{21}{4} = 5\frac{1}{4}.$$

Exercises.

Ex. 1.	Divide $\frac{5}{6}$ by $\frac{2}{3}$	*Product*	$1\frac{1}{4}$
2.	Divide $2\frac{1}{4}$ by $\frac{1}{3}$ of $\frac{7}{9}$	—	$8\frac{19}{28}$
3.	Divide $\frac{3}{5}$ of $\frac{10}{11}$ by $\frac{3}{4}$ of $\frac{1}{5}$	—	$3\frac{7}{11}$
4.	Divide $\frac{10}{13}$ by 3	—	$\frac{10}{39}$
5.	Divide $\frac{12}{17}$ by 6	—	$\frac{2}{17}$
6.	Divide $\frac{9}{10}$ by $\frac{3}{5}$	—	$1\frac{1}{2}$
7:	Divide $\frac{9}{10}$ by $\frac{3}{7}$	—	$2\frac{1}{10}$
8.	Divide $2\frac{3}{4}$ by $\frac{3}{4}$	—	$3\frac{2}{3}$

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* For the multiplication and division of integral quantities by fractional numbers, see pages 52, 53, and 54.

FRACTIONAL PROPORTION.

Rule. Reduce, if requisite, the three terms into simple fractions, and the first and third terms into fractions of the same integer.

Then, for Direct Proportion, multiply the second term by the number of the third, and divide the product by the number of the first, or invert the number of the first term, and multiply the three numbers together.

For Inverse Proportion, invert the number of the third term, and multiply the numbers of the three terms together.

Example 1.

If $2\frac{3}{4}$ of a yard cost $\frac{5}{7}$ £ what will $1\frac{3}{4}$ of an ell of 5 quarters of a yard cost?

$2\frac{3}{4}$ yd. $=$ $\frac{11}{4}$ yd. $1\frac{3}{4}$ ell $=$ $\frac{7}{4}$ of $\frac{5}{4}$ yd. $=$ $\frac{35}{16}$ yd.

If $\frac{11}{4}$ yd. ——— $\frac{5}{7}$ £ ——— $\frac{35}{16}$ yd.?

$$£\,\frac{5}{\underset{1}{7}} \times \frac{\overset{5}{35}}{\underset{4}{16}} \times \frac{\overset{1}{4}}{11} = \frac{25}{44}\,£ = \text{s. } 11\ 4\tfrac{4}{11}\ \text{d}$$

When all the terms are not fractionally expressed, it is not particularly necessary to use this form, as in the following Example.

Example 2.

If $2\frac{3}{4}$ yds. of Cloth cost £ 1 7 6 what will be the cost of $11\frac{5}{8}$ yds.?

yds.		£	s.	d.		yds.
If $2\frac{3}{4}$	———	1	7	6	———	$11\frac{5}{8}$?
				$11\frac{5}{8}$		
		15	2	6		
			13	9	for 4-8 ths	
			3	$5\frac{1}{4}$	- 1-8 th	
$2\frac{3}{4}$)		15	19	$8\frac{1}{4}$ *		
11)		63	18	9		
		£ 5	16	3		

Or, we may reduce the first and third terms into the same sort of parts, and then rejecting the denominators, use the numerators as whole numbers: thus,

---

* The divisor and dividend are here multiplied by 4, to get rid of the fraction in the divisor.

$$\begin{array}{lcrrrcr}
 & \text{yds.} & \pounds & \text{s.} & \text{d.} & & \text{yds.} \\
\text{If} & 2\frac{3}{4} \text{ ———} & 1 & 7 & 6 & \text{———} & 11\frac{5}{8}\,? \\
 & & & & 93 & & \\
 & \text{22-8 ths} & & & & & \text{93-8 ths.} \\
 & & 22\,)\;127 & 17 & 6 & & \\
 & & \pounds\;5 & 16 & 3 & \text{Answer.} &
\end{array}$$

Or, as 93-22 nds are 4 and 5-22 nds, we may work thus,

$$\begin{array}{lrrrl}
 & \pounds & \text{s.} & \text{d.} & \\
 & 1 & 7 & 6 & \\
 & & & 4 & \\
\frac{1}{11}\ldots & 5 & 10 & 0 & \\
\frac{1}{4}\ldots\ldots & & 5 & 0 & \text{for 4-22 nds} \\
 & & 1 & 3 & \text{- 1-22 nd} \\
 & \pounds\;5 & 16 & 3 &
\end{array}$$

Lastly, to use the simple fractional form, we have,

$$\begin{array}{lcccc}
 & \text{yds.} & & \pounds & & \text{yds.} \\
\text{If} & 2\frac{3}{4} & \text{———} & 1\frac{3}{8} & \text{———} & 11\frac{5}{8}\,? \\
 & \frac{11}{4} & \text{———} & \frac{11}{8} & \text{———} & \frac{93}{8}
\end{array}$$

$$\pounds\,\frac{11}{8} \times \frac{93}{8} \times \frac{4}{11} = \frac{93}{16}\,\pounds = \pounds\;5\;16\;3.$$

EXERCISES IN DIRECT PROPORTION.

Ex. 1. What is the cost of $5\frac{7}{8}$ oz. of Gold, when $1\frac{3}{5}$ oz. cost £ $6\frac{1}{5}$? Answer £ 22 15 $3\frac{3}{4}$.

Ex. 2. What is the worth of a Ship, when $\frac{3}{4}$ of $\frac{5}{16}$ cost £ $382\frac{1}{5}$? Answer £ 1630 14 5.

Ex. 3. If $\frac{5}{7}$ lb. cost $\frac{11}{16}$ of a shilling, what will $\frac{7}{9}$ of an cwt. cost? Answer £ 4 3 $10\frac{6}{45}$.

EXERCISES IN INVERSE PROPORTION.

Ex. 4. If 12 inches broad require 12 inches long to make a foot square, what length will $5\frac{3}{4}$ inches broad require?

Answer $2\frac{2}{23}$ inches.

Ex. 5. If Calico $\frac{3}{4}$ yard wide require $13\frac{3}{4}$ yards long, what length will Calico $\frac{7}{8}$ yard require?

Answer $11\frac{11}{14}$ yards.

Ex. 6. If by working $13\frac{3}{4}$ hours per day a work requires $17\frac{1}{5}$ days, how many days will be required, if only $10\frac{1}{6}$ hours per day be employed. Answer $23\frac{16}{61}$ days.

When fractions occur in the Statements of Compound and Continued Proportions, it is generally most convenient to use the fraction in the third or second terms, in whatever manner may be the best for the multiplication; and for a fraction in a first term, to multiply it and one of the other terms by the denominator of the fraction.

The whole may however be used fractionally in the manner of a Simple Proportion, as in the following operation worked in a fractional form.

EXAMPLE OF A COMPOUND PROPORTION.

If the carriage of $3\frac{1}{4}$ cwt. of Goods for 147 miles amounts to $67\frac{3}{5}$ s., what will be charged for the carriage of $11\frac{5}{6}$ cwt. for 132 miles ?

$$\textit{Statement}\quad \left.\begin{array}{l}\text{If } 3\frac{1}{4}\text{ cwt.}\\ 147\ \ \text{miles}\end{array}\right\}\text{ require } 67\frac{3}{5}\text{ s. }\left\{\begin{array}{l}11\frac{5}{6}\text{ cwt.}\\ 132\ \ \text{miles ?}\end{array}\right.$$

$$3\tfrac{1}{4} = \tfrac{13}{4} \qquad 67\tfrac{3}{5} = \tfrac{338}{5} \qquad 11\tfrac{5}{6} = \tfrac{71}{6}$$

$$\text{s. } \tfrac{338}{5} \times \tfrac{71}{6} \times \tfrac{132}{1} \times \tfrac{4}{13} \times \tfrac{1}{147} = 221\tfrac{13}{735}\text{ s.} = \pounds\,11\ \ 1\ \ 0\tfrac{52}{245}.$$

EXAMPLE OF A CONTINUED PROPORTION,

worked with the multiplication of the opposite terms to cancel the fractions in the antecedents.

To find the value of 12000 Hamburg Marks Banco, in Spanish Dollars of Plate, at the exchange of $13\frac{5}{8}$ Marks per £ Sterling, and $36\frac{1}{2}$ d. Sterling per Dollar of Plate.

$$\begin{array}{rcrl} & & 12000 & \text{Marks Bo ?}\\ 13\frac{5}{8} & = & 1 & \pounds\text{ Sterling}\\ 1 & = & 240 & \text{d.}\\ 36\frac{1}{2} & = & 1 & \text{Dollar.}\end{array}$$

Allowing for the fractions, we have,

$$\begin{array}{rcrl} & & 12000 & \text{Marks Bo ?}\\ 109 & = & 8 & \pounds\\ 1 & = & 240 & \text{d.}\\ 73 & = & 2 & \text{Dollars}\end{array}$$

$$\frac{2 \times 240 \times 8 \times 12000}{109 \times 73} = 5791\tfrac{1013}{7957}\text{ Dollars.}$$

DECIMALS.

A Decimal is a number expressing a portion of a unit or an integer in tenths, hundredths, thousandths, &c.

NUMERATION.

The value of a decimal is shown without employing a denominator, by the position which each figure has relative to the place of units: thus, a point being used to separate decimal parts from units, tenths occupy the first place on the right, hundredths the second place, thousandths the third place, &c. as with the figures in the following table.

tens	units		tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
4	6	.	7	6	3	5	8	7

Which number, supposing it to be applied to Pounds, may be read as 46 Pounds, and 763587 millionths of a pound; but when there are more than two or three places of decimals it is usual to read it thus, forty-six Pounds, decimal, seven, six, three, five, eight, seven.

It is to be observed, that separation of the greater parts into those which are less, in order to be decimally made, must always be into tenths, or the larger part into 10 smaller parts. The integer is also said to be decimally separated when it is done upon the same principle, whatever name may be given to the parts produced: thus, if 10 shillings made 1 pound, 10 pence made one shilling, 10 farthings made 1 penny, and 10 parts made 1 farthing, &c., the pound would be decimally separated.

When this mode of separation is adopted, the reductions are made by merely altering the place of the decimal point: thus, 46 pounds, 7 decimal shillings, 6 decimal pence, and 3 decimal farthings, would be expressed by £ 46.763, and the number of decimal shillings would be 467.63 s., or the decimal pence 4676.3 d., and the decimal farthings 46763 f.

In the same manner as with whole numbers, ciphers are employed to keep the figures in their proper position, by filling up any vacancy that may occur: a cipher is also used when there is not an integer; thus, to express 3 units and 37 ten thousandths, or 3 units, no tenths, no hundredths, 3 thousandths, and 7 ten thousandths, we write 3.0037; and for no units 37 hundredths, or 3 tenths and 7 hundredths, we write 0.37. Ciphers are seldom affixed to the right of the decimal, except when the decimal point is in any operation to be removed beyond the given figure, or when a decimal is to be reduced or expanded into a lower sort of parts; thus, to express 0.37 hundredths as thousandths, we write 0.370, or, as ten thousandths, 0.3700.

When a decimal figure continually repeats,* as 0.4333, &c. the part which repeats may be expressed by placing a dot over the repeating figure; thus,

$$0.4\dot{3}.$$

When a decimal number circulates,* as, 0.7925925, &c. the first circulate only is necessary to be expressed, if a dot be placed over the first and last of the circulating figures; as,

$$0.7\dot{9}2\dot{5}.$$

The complement of a decimal is the difference between it and unity, as the complement of 0.792563 is 0.207437, the amount of the two being equal to unity.

Exercises upon the Decimal Notation.

Ex. 1. Value £ 674.373 as in the preceding table.

Ex. 2. Value £ 83.406703 as in the preceding table.

Ex. 3. Express by figures, one hundred and twenty-seven pounds and twenty-seven hundredths of a pound.

Ex. 4. Express by figures, fifty-four shillings and thirty-three thousandths of a shilling.

Ex. 5. Express by figures, no pounds and seven millionths of a pound.

Ex. 6. Express by figures, one, decimal, nought nought two seven.

Ex. 7. Express by figures, no units, decimal, nought nought nought nought nought seven.

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* For the operations of repeating and circulating decimals, see the end of this Section.

DECIMAL REDUCTION.

CASE 1.

To reduce a decimal into a fraction.

Rule. If the decimal does not repeat or circulate, make the given number the numerator, and 1, with as many ciphers as there are decimal places, the denominator.

If the decimal repeats or circulates, make the given number as before the numerator, but make the part of it that repeats or circulates a fraction, having as many nines for its denominator as there are places of decimals in the part that repeats; the denominator is to be formed of 1, with as many ciphers as there are places of decimals in the part that does not repeat.

EXAMPLE 1.

To express 0.046875 as a fraction.

$$0.046875 = \frac{46875}{1000000} = \frac{1875}{40000} = \frac{75}{1600} = \frac{3}{64}.$$

There being 6 places of decimals in the given number, there are 6 ciphers annexed to the 1 in the denominator; both terms are then successively divided by 25.

EXAMPLE 2.

To express 64.423 as a fraction.

$$64.42\dot{3} = 64\frac{42\frac{3}{9}}{100} = 64\frac{381}{900} = \frac{57981}{900} = \frac{19327}{300}.$$

EXERCISES.

Ex. 1. Express 0.45, 0.125, and 0.475 as fractions.
2. Express 4.36, 8.927, and 67.54 as fractions.

PRODUCTS.

Ex. 1. $\frac{9}{20}$, $\frac{1}{8}$, $\frac{19}{40}$. Ex. 2. $4\frac{9}{25}$, $8\frac{927}{1000}$, $67\frac{27}{50}$.

N. B. The performance of the calculations in Decimal Reduction may be postponed until some of the calculations in Addition, &c. have been practised. If they are taken as here placed, some reference will occasionally be necessary to the rules of Decimal Multiplication and Division.

CASE 2.

To reduce a fraction into a decimal.

Rule. Divide the numerator by the denominator, reducing the numerator into tenths, hundredths, &c. by annexing ciphers, as far as may be necessary.

Ex. 1. To reduce $\frac{2}{3}$ into a decimal.

$$\begin{array}{r} 3\ )\ 2.000 \\ \hline 0.6666,\ \&c. \\ \hline \end{array}$$

This may be expressed by $0.\dot{6}$. In general, when the division has been carried as far as necessary, if the last remainder is greater than half the divisor, the last quotient figure may be reckoned 1 more; as the above may be called 0.6667. Sometimes the exact value may be required, and then when there is a remainder it must be expressed as a fraction, as $0.6666\frac{2}{3}$.

Ex. 2. To reduce $\frac{1}{7300}$ into a decimal.

$$\begin{array}{r} 7300\ )\ 1.00000000 \\ \hline 0.00013698 \\ \hline \end{array}$$

Or very nearly 0.000137.

EXERCISES.

Ex. 1. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{7}{16}$, and $\frac{7}{20}$ into decimals.
2. Reduce $\frac{3}{2}$, $\frac{7}{4}$, $\frac{11}{6}$, and $\frac{527}{20}$ into decimals.
3. Reduce $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{125}$, and $\frac{1}{365}$ of $\frac{1}{20}$ into decimals.

PRODUCTS.

Ex. 1. 0.5, 0.75, 0.3333, &c., 0.375, 0.4375, 0.35
2. 1.5, 1.75, 1.8333, &c., 26.35
3. 0.25, 0.125, 0.0625, 0.2, 0.04, 0.008, 0.00013698

Case 3.

To find the value of a decimal quantity.

Rule. Reduce the decimal into the next lower denomination, and separate the integers from the decimal; then reduce this decimal into the next lower denomination, again separate the integers, and so continue the operation, until either there is not any decimal, or it is reduced into the lowest denominator required; then collect the integers produced by each reduction, and they will express the required value.

Ex. 1. To find the value of £ 0.875.

$$\begin{array}{lr} & £ \\ & 0.875 \\ \hline \text{s.} & 17.500 \\ \hline \text{d.} & 6.000 \\ \hline \end{array} \qquad £\ 0.875 = 17 \text{ s. } 6 \text{ d.}$$

This decimal £ 0.875 is 875-1000 ths of a £, or the 1000 th part of £ 875.

Ex. 2. To find the value in lb. &c. of 0.22714 of an cwt.

$$\begin{array}{lr} & \text{cwt.} \\ & 0.22714 \\ & 272568 \\ \hline \text{lb.} & 25.43968 \\ \hline \text{oz.} & 7.03483 \\ \hline \end{array} \qquad \text{cwt. } 0.22714 = 25 \text{ lb. } 7 \text{ oz.}$$

We here multiply by 112 in the manner shown in page 11, and as there are 5 places of decimals in the given quantity, we cut off 5 places in the product; then we multiply the 0.43968 by 16, and cut off 5 places as before.

Exercises.

Ex. 1. Find the value of 0.04575 of a £.
2. Find the value of 0.22723 of a £.
3. Find the value of 0.37267 of a £.
4. Find the value of 0.67131 of a lb. troy.
5. Find the value of 0.72376 of an oz. troy.
6. Find the value of 0.61764 of an cwt. in qrs. lb. and oz.

Products.

Ex.	Ex.	Ex.
1. 11 d. nearly.	2. 4 s. 6½ d.	3. 7 s. 5¼ d.
4 8 oz. 1 dwt. 2 grs.	5. 14 dwts. 11 grs.	6. 2 qrs. 13 lb. 2 oz.

CASE 4.

To reduce any quantity expressed in the lower denominations of an integer into the decimal of that integer.

Rule. To reduce a simple quantity, consider it changed into the denomination required, and divide it by the number of that denomination which is contained in the integer.

To reduce a compound quantity, reduce each of the quantities in the lower denominations, beginning with the lowest, into the decimal of the next higher denomination, and reduce the highest denomination into the decimal of the integer; or express the given quantity as a fraction of the integer into which it is to be reduced, and find the decimal value of this fraction.

EXAMPLE.

To reduce 17 s. 6¾ d. into the decimal of a £.

4) 3. farthings
12) 6.75 pence
20) 17.5625 shillings
£ 0.878125

Three farthings are the 4 th of 3 d., or the decimal 0.75 d.; sixpence and this decimal are so many 12 ths of 1 s., or the decimal 0.5625 s.; and lastly, seventeen shillings and this decimal are so many 20 ths of a £, which are equal to the decimal £ 0.878125.

Otherwise 17 s. 6¾ d. reduced into farthings are 843 farthings, or 843-960 ths of a £, and the value may be found as such, by dividing £ 843 by 960.

EXERCISES.

Ex. 1. Reduce 15 s. to the decimal of a £.
2. Reduce 7¾ d. to the decimal of a s.
3. Reduce 11 oz. to the decimal of a pound Troy.
4. Reduce 13 dwts. to the decimal of an oz. Troy.
5. Reduce 11 dwts. 13 grs. to the decimal of a pound Troy.
6. Reduce 3 qrs. to the decimal of a cwt.
7. Reduce 17 lb. to the decimal of a cwt.
8. Reduce 3 qrs. 17 lb. to the decimal of a cwt.

PRODUCTS.

Ex. 1. £ 0.75 Ex. 2. s. 0.64583 Ex. 3. lb. 0.9166
4. oz. 0.65 5. lb. 0.04809 6. cwt. 0.75
Ex. 7. cwt. 0.15178 Ex. 8. cwt. 0.90178.

CASE 5.

To reduce by inspection the decimal parts of a £ into shillings, pence, and farthings.

Rule. Reckon every tenth as 2 shillings, 5 hundredths as 1 shilling, and every thousandth as 1 farthing, abating 1 when they exceed 12, or 2 when they exceed 37; or divide the tenths and hundredths by 5 to find the shillings, and reckon the remaining thousandths as so many farthings, abating 1 when they exceed 12, or 2 when they exceed 37.

N. B. In general, when there are more than three places of decimals, the third place may be reckoned one more when the fourth figure is above 5, or when it is 5 with any other figures.

EXAMPLE 1.

To value £ 0.2, and £ 0.4.

£ 0.2 = 4 s.
0.4 = 8 s.

The tenth of a £ being 2 s., it is evident that 2-10 ths are 4 s., and that 4-10 ths are 8 s.

EXAMPLE 2.

To value £ 0.3, £ 0.35, £ 0.325, and £ 375.

£		s.	d.
0.3	=	6	0
0.35	=	7	0
0.325	=	6	6
0.375	=	7	6

3-10 ths of a £ are 6 s.; 5-100 ths or 50-1000 ths of a £ are the same in value as 1-20 th of a £ or 1 s.; 25-1000 ths are the half of a 20 th, or the 40 th of a £; and are equal to 6 d.,* consequently 75-1000 ths are 1 s. 6 d.

---

* N. B. It is from this approximation of 1000 ths of a £ to farthings, that the latter part of the rule is formed; for as 25 of these parts are 24 farthings or 6 d., by making the allowance for the excess to the nearest farthing, and reckoning half way between 0 and 25 on the one part, and between 25 and 50 on the other, we allow 1 part from 12 to 37 and 2 parts from 37 to 50.

EXAMPLE 3.

To value £ 0.742 and £ 0.773.

£ 0.742 = 14 s. 10 d.
£ 0.773 = 15 s. 5½ d.

7-10 ths are 14 s., and 42-1000 ths are 40 farthings, or 10 d.; 7-10 ths and 5-100 ths, or 7-10 ths and 50-1000 ths, are 15 s., and the remaining 23-1000 ths are 22 farthings, or 5½ d.

EXAMPLE 4.

To value £ 0.74275 and £ 0.71463.

£ 0.74275 reckoned £ 0.743 = 14 s. 10¼ d.
£ 0.71463 reckoned £ 0.714 = 14 s. 3½ d.

In the first of these calculations we can reckon the thousandths as 43, because the deduction 2, being too great, allows with correctness a little excess in the valuation; but in the second, as the proper deduction at 14 is only a little more than ½ of 1000 ths, it is only about equal to the excess or the remaining 63-100000 ths, and the latter are therefore not reckoned as 1 part more.

EXERCISES.

Find the value of the following decimals of a £.

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.
£ 0.2	£ 0.15	£ 0.025	£ 0.213
0.5	0.35	0.075	0.627
0.7	0.45	0.225	0.839
0.8	0.85	0.575	0.848
0.9	0.95	0.875	0.999

Ex. 5. £ 0.004 £ 0.011 £ 0.21 £ 0.373 £ 0.037 £ 2.348 £ 0.588

PRODUCTS.

See the Exercises of the following Case.

CASE 6.

To reduce by inspection shillings, pence, and farthings into decimal parts of a £.

RULE 1. FOR 3 DECIMAL PLACES.

Take half the number of shillings for the number of the 10 ths, and if there is an odd shilling reserve it as 5-100 ths or 50-1000 ths.

Reduce the pence into farthings, consider them, with the farthings, if any, as so many 1000 ths, with the addition of 1 part if above 12, or 2 if above 37, and also of 50-1000 ths if there be a shilling reserved.

Ex. 1. To reduce 14 s. and 15 s. into decimals of a £.

14 s. = 0.7 £.
15 s. = 0.75 £.

Ex. 2. To reduce 14 s. 6 d. and 15 s. 6 d. into decimals of a £.

14 s. 6 d. = 0.725 £.
15 s. 6 d. = 0.775 £.

Ex. 3. To reduce 14 s. 10 d. and 15 s. 5½ d. into decimals of a £.

14 s. 10 d. = 0.742 £.
15 s. 5½ d. = 0.773 £.

As the above are merely the converse of the examples to Case 5, we shall explain only how the performance of the last may be read.

The half of 15 is 7 and 1 over; put down 7-10 ths and reserve 50-1000 ths; 5½ d. are 22 farthings, or 23 parts, and 50 reserved are 73 parts.

EXERCISES.

Reduce into decimal parts of a pound,

Ex. 1. 4 s., 10 s., 14 s., 16 s., 18 s.
2. 3 s., 7 s., 9 s., 17 s., 19 s.
3. 6 d., 1 s. 6 d., 4 s. 6 d., 11 s. 6 d., 17 s. 6 d.
4. 4 s. 3 d., 12 s. 6½ d., 16 s. 9¼ d., 16 s. 11½ d., 19 s. 11¾ d.
5. 1 d., 2¾ d., 4 s. 2½ d., 7 s. 5½ d., 9 d., 6 s. 11½ d., 11 s. 9 d.

Rule 2. For an unlimited number of places.

Take half the number of shillings for the number of tenths, and reserve 5 hundredths, or 50 thousandths, if there is an odd shilling.

Reduce the pence into farthings, consider them as so many thousandths, with the addition of 1 thousandth if the pence are 6 or above, and 50 thousandths if there is an odd shilling.

Then to continue the decimal parts, divide the given pence, or the pence above 6, by 6, annexing as many ciphers as may be necessary to the number of the pence.*

N. B. If there are farthings apply their decimal values to the pence, before applying the ciphers; that is, annex 25 for 1 farthing, 5 for ½ d., or 75 for 3 farthings.

Example 1.

To value 18 s. 4 d. and 17 s. 11 d. as decimals of a £.

s.	d.		£
18	4	=	0.916666, &c.
17	11	=	0.895833, &c.

18 s. being 9-10 ths and 4 d. being 16 farthings, we obtain 916 for the first 3 figures in the first calculation; then the operation is continued by dividing 4000, &c. by 6 for the remaining figures 666, &c.

17 s. in a like manner give 850 thousandths, and 11 d. or 44 farthings give 45-1000 ths, making together 895-1000 ths; then as the pence above 6 are 5, we divide 5000, &c. by 6, for the remaining figures 833, &c.

Example 2.

To value 14 s. 4½ d. and 17 s. 7¾ d. as decimals of a £.

s.	d.		£
14	4½	=	0.71875
17	7¾	=	0.88229166

14 s. 4½ d. produce as before 0.718 thousandths; and the 4½, that is 4.50, divided by 6 produce the exact additional parts 75.

17 s. 7¾ d. produce 0.882 thousandths, and the 1¾ d. above 6 d., that is 1.75, divided by 6, produces the infinite decimal 29166, &c.

---

* The reason of this part of the rule evidently is from 1-1000 th being the exact addition to the number of the farthings when the pence are 6, and consequently for any other number of pence the additional parts are the 6 th part of so many pence.

EXAMPLE 3.

To value 18 s. 0¼ d. and 17 s. 6½ d. as decimals of a £

s.	d.		£	
18	0¼	=	0.9010416	&c.
17	6½	=	0.8770833	&c.

18 s. 0¼ d. produce 901 thousandths; then ¼ d., or 0.25 pence, divided by 6 produces 0416, &c.

17 s. 6½ d. produce 877 thousandths, and ½ d., or 0.5 pence, divided by 6 produces 0833.

N. B. We see from this example, that when there is only ¼ d. or ½ d. above 1 s. or 6 d., there will be a cipher in the fourth place of decimals: also from the preceding calculations we observe, that when the excess above 1 s. or 6 d. is 3 farthings, or any multiple of 3 farthings, as 1½ d., 2¼ d., 3 d., 3¾ d., 4½ d., or 5¼ d., the decimal will terminate at the 5 th place with halfpence, or at the 6 th place with farthings; but in other cases, the decimal 3 or 6 repeats, at or before the 7 th place of decimals.

EXERCISE.

Find the decimal value of the following sums to 7 places of decimals, when the number of places is unlimited:*

4 s. 1 d., 6 s. 7 d., 8 s. 4 d., 9 s. 10 d., 11 s. 3 d.
11 s. 9 d., 2 s. 2¼ d., 13 s. 8¼ d., 17 s. 4½ d., 16 s. 1¾ d.
17 s. 0½ d., 11 s. 3¼ d., 16 s. 7¾ d., 0 s. 3¼ d., 19 s. 2¾ d.

---

* N. B. It forms a good proof to this Exercise to add together the decimal values of these sums, and see that the amount is the decimal value of the amount of these sums. To perform this addition with accuracy, we must divide the amount of the column of the 7 th place of decimals by 9, and putting down the remainder carry the quotient to the 6 th column, which with the rest of the columns, is added in the common manner. See Addition of Circulating Decimals.

DECIMAL ADDITION.

Rule. Arrange the given numbers so that the similar parts may be in the same columns, and find the amount as in the Addition of whole numbers.

Example. To find the amount of £ 36.047, £ 4.67, £ 83.4045, and £ 516.2.

$$\begin{array}{r@{.}l} \multicolumn{2}{c}{\pounds} \\ 36 & 047 \\ 4 & 67 \\ 83 & 4045 \\ 516 & 2 \\ \hline \pounds\ 640 & 3215 \\ \hline \end{array}$$

When the numbers are properly arranged, the decimal points will stand one under another; and as 10 in each lower denomination make 1 of the next higher, the addition is necessarily performed as in the addition of whole numbers.

For the practice of beginners the columns of figures may be headed as in the Numeration Table, page 147, or the fractional value of each may be expressed thus:

$$36.047 = 36\tfrac{47}{1000}$$
$$4.67 = 4\tfrac{67}{100},\ \&c.$$

Exercises.

Ex. 1. Find the amount of £ 48.6, £ 27.3, and £ 101.7.
2. Add together s. 11.28, s. 7.145, s. 8.036, and s. 14.0007.
3. Add together lb. 6.07, lb. 100.082, and lb. 1.600766.
4. Add together £ 0.366, £ 0.58, and £ 0.00455.

Amounts.

Ex. 1. £ 177 12 0. Ex. 2. £ 2 0 5½ Ex. 3. lb. 107.12.
Ex. 4. 0 19 0.

In the addition and subtraction of decimal quantities, if they are not in the same denomination they must be reduced into either the higher or the lower denominations according to the Rule, page 151; thus in Ex. 2. of Subtraction, if the 3.72 was £ instead of s., then either the £ 3.72 should be multiplied by 20 to bring them into s., making 74.4 s., or the 2.30706 should be divided by 20 to bring them into the decimal of a £, making £ 0.115353; the difference would then be either s. 72.09294, or £ 3.604647.

DECIMAL SUBTRACTION.

Rule. After having arranged the numbers as in Decimal Addition, find the difference as in the Subtraction of whole numbers.

EXAMPLE 1.

To subtract £ 60.437 from £ 82.141.

£
82.141
60.437

£ 21.704

EXAMPLE 2.

To subtract s. 2.30706 from s. 3.72, and also from s. 5.0.

s.	s.
3.72	5.0
2.30706	2.30706
s. 1.41294	s. 2.69294

In subtractions like these, as many ciphers may be annexed to the greater quantity, as are required to make the number of the figures equal to those in the lower quantity; and in the performance of such calculations it will be seen, that the right hand figure of the remainder and the one above it are equal to ten, and that the sum of the other parts or figures, until we come to the figure in the upper line, is each equal to 9.

EXERCISES.

Ex. 1. From £ 26.365 take £ 1.234.
2. From £ 11.111 take £ 4.6782.
3. From s. 31.81 take s. 14.51689.
4. From oz. 7 take oz. 3.672682.

REMAINDERS.

Ex. 1. £ 2.131. Ex. 2. £ 6.4328.
3. s. 17.29311. 4. oz. 3.327318.

DECIMAL MULTIPLICATION.

Rule. Multiply as in the Multiplication of integral numbers, without regarding the difference between whole numbers and decimals.

Then, if the multiplier is a whole number, there will be as many places of decimals in the product as there are decimal places in the multiplicand; and if the multiplier is also a decimal number, there will be as many more places of decimals in the product as there are decimal places in the multiplier.*

Example 1.

To multiply £ 37.31 by 10, by 100, and by 1000.

£ 373.1 - 1 st product
£ 3731.0 - 2 nd —
£ 37310.0 - 3 rd —

In multiplications of decimals by 10, 100, 1000, &c. all that there is to be done, is to remove the decimal point one, two, three, &c. places to the right.

Example 2.

To multiply £ 84761 by 3 and by 31.

£ 8.4761	£ 8.4761 by 1
3	254.283
£ 25.4283	£ 262.7591

These are worked upon the same principles as common multiplication.

Example 3.

To multiply £ 4263 by 0.25.

£ 4263
.25
£ 1065.75

As there are not any places of decimals in this multiplicand, but two places in the multiplier, we cut off two places from the right of the product.

---

* The second part of this rule may be expressed thus — then from the right of the product cut off as many places for decimals, as there are decimal places in both the multiplier and the multiplicand.

The reason of this may be easily explained—The decimal 0.25 is 25-100 ths, for which we take first 25 times, and then the 100 th part of the product; hence we derive the directions of the latter part of the rule, because to allow for the decimal places in the multiplier, is actually to divide by 10, by 100, &c. according to the number of those places, or the denominator of the decimal fraction.

EXAMPLE 4.

To multiply £ 42.641 by 3.27.

$$\begin{array}{r}
£\ 42.641 \\
3.27 \\
\hline
298487 \\
85282 \\
127923 \\
\hline
£\ 139.43607 \\
\hline
\end{array}$$

There being 3 places of decimals in the multiplicand, and 2 in the multiplier, we cut off 5 places in the product; we cut off 3 places, because if the multiplier were a whole number 13943.607 would be the value of the product, on the principle of Addition; and we cut off 2 places more, because by using the multiplier as a whole number, viz. 327, while its real value is only the 100 th part, or 3.27, we have the product, at first, 100 times its proper amount; and therefore to correct it, we take the 100 th part of the 1 st product by removing the decimal point 2 places more to the left.

EXAMPLE 5.

To multiply £ 0.3 by 0.2.

$$\begin{array}{r}
£\ 0.3 \\
0.2 \\
\hline
£\ 0.06 \\
\hline
\end{array}$$

There being 1 place of decimals in the multiplicand, and 1 in the multiplier, we must have 2 places of decimals in the product; which is effected by placing one 0 before the 6, and then prefixing the decimal point.

The operation is to take 2-10 ths of 3-10 ths of a £; now twice 3 tenths are 6 tenths, and the tenth part of 6 tenths is 6-100 ths.

EXAMPLE 6.

To multiply £ 0.0024 by 0.24.

$$\begin{array}{r} \pounds \\ 0.0024 \\ 0.24 \\ \hline \pounds\ 0.000576 \\ \hline \end{array}$$

There being 6 places of decimals in the two factors, we make 6 places of decimals in the product, by prefixing 3 ciphers to the 576 before we place the decimal point.

EXERCISES.

Multiply

Ex.	£			Ex.	£		
1.	6734.9	by	10	11.	327046	by	4.06
2.	416.42	-	100	12.	814965	-	83.7
3.	837.42	-	1000	13.	537627	-	6.24
4.	91.625	-	100	14.	448070	-	0.473
5.	0.7371	-	10	15.	500000	-	0.051
6.	445.932	-	4	16.	83.7273	-	34.64
7.	7.41626	-	21	17.	4.16934	-	4.172
8.	9153.71	-	117	18.	0.38222	-	0.645
9.	8.27953	-	112	19.	0.00137	-	0.0073
10.	5436.25	-	404	20.	0.03003	-	0.0003

PRODUCTS.

Ex.	£	Ex.	£	Ex.	£
1.	67349.	2.	41642.	3.	837420.
4.	9162.5	5.	7.371	6.	1783.728
7.	155.74146	8.	1070984.07	9.	927.30736
10.	2196245.	11.	1327806.76	12.	68212570.5
13.	3354792.48	14.	211937.110	15.	25500.
16.	2900.313672	17.	17.39448648	18.	0.2465319

Ex. 19. £ 0.000010001 Ex. 20. £ 0.000009009

DECIMAL DIVISION.

Rule. Divide as in Division of whole numbers, paying no regard in the operation to any distinction between whole numbers and decimals.

For the valuation of the quotient, it is to be observed, that when the divisor is a whole number, the value of each quotient figure will be the same as the lowest dividend figure used to procure that quotient figure; and when the divisor is not a whole number, it may be made such, by cancelling the decimal point in the divisor, and correcting the dividend by removing the decimal point so many places to the right, as there are places in the given divisor.*

Example 1.

To divide £ 3764 by 10, by 100, and by 1000.

£ 3.764
0.3764
0.03764

To divide by 10 we remove the decimal point one place to the left, by 100 two places, and by 1000 three places.

Example 2.

To divide 37.64 by 7.

7) 37.64

5.37714 &c.

The number of places of decimals to which the quotient should extend must depend upon the purpose for which it is made.—Three places are sufficient in the division of Pounds of money, when the quotient is to be valued to the nearest penny or farthing; for other purposes, and particularly when there are whole numbers, 5 places are the most that are usually necessary.

The ciphers for continuing the division may be used, but they are not necessary.

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* The quotient may otherwise be valued by cutting off so many places on the right of the quotient, as the decimal places in the dividend, including the ciphers used to continue the operation, exceed those in the divisor.

EXAMPLE 3.

To divide £ 37.64 by 843.

£ £
843) 37.64 (0.0446
3920
5480
422

The divisor not being contained in the 37 units, there are not any units in the quotient; and the same being the case with the 376 tenths, there are not any tenths in the quotient; the first quotient figure, 4 hundredths, is produced from the 3764 hundredths.

It may here be observed, that in the division of a decimal quantity by a whole number, after the whole number in the quotient has been obtained, instead of extending the quotient decimally, the remainder is sometimes required to be reduced and divided according to the ordinary form of division; and care must then be taken, that the whole of the remainder, including the decimal, is multiplied by the reducing multiplier.

EXAMPLE 4.

To divide £ 872 by 0.4.

£
4.) 8720.
£ 2180.

As there is 1 decimal place in the divisor, and none in the dividend, the dividend is corrected by annexing a cipher, and then removing the decimal place (which in decimal calculations is always considered to be on the right of all whole numbers) one place to the right; the divisor is then considered a whole number.

The reason of this removal of the decimal place in the dividend and then calling the divisor a whole number, is, evidently, to multiply both terms by the denominator of the decimal, and thus to substitute an equivalent but easier operation, producing the same result; thus, as in the above, instead of dividing £ 872 by 4-10 ths we divide £ 8720 by 4; and, as in the next example, instead of dividing £ 837.64 by 6.17 we take 100 times each term, and divide £ 83764 by 617.

EXAMPLE 5.

To divide £ 837.64 by 6.17.

£ £
617) 83764 (135.7601, &c.
2206
3554
4690
3710
800

When both the divisor and the dividend contain the same number of decimal places, both are taken as whole numbers.

In general it is not necessary to work the last quotient figure.

EXERCISES.

£
Divide 36.74 by 10
Divide 3.147 by 10
Divide 4.006 by 100
Divide 0.047 by 100
Divide 0.164 by 1000

Divide 34.26 by 8
Divide 4.413 by 12
Divide 67.34 by 31
Divide 4.206 by 84
Divide 555.1 by 167

£
Divide 64273 by 0.7
Divide 47541 by 0.64
Divide 83265 by 0.315
Divide 41472 by 0.046
Divide 62000 by 0.0073

Divide 816.42 by 31.3
Divide 4475.6 by 8.473
Divide 0.6734 by 0.249
Divide 0.4444 by 5.132
Divide 623.45 by 87.42

QUOTIENTS.

£ £ £
Ex. 1. 3.674 Ex. 2. 0.3147 Ex. 3. 0.04006
4. 0.00047 5. 0.000164 6. 4.2825
7. 0.36775 8. 2.17225 9. 0.05007
10. 3.32395 11. 91818.571 12. 7.4282,8125
13. 26433.333 14. 901565,217 15. 8.493150,684
16. 26.083 17. 528.219 18. 2.7044
Ex. 19. £ 0.086593 Ex. 20. £ 7.1316

CONTRACTED MODES OF MULTIPLICATION.

Long Multiplications may sometimes be contracted.

1 st. By contracting the partial products, when the whole of the figures are not required in the result.

2 nd. By the use of reciprocal divisors.

3 rd. By proportioning the result of an approximate multiplier.

METHOD 1.

Rule. Mark that figure of the multiplicand which corresponds with the place or value of the lowest figure to be retained in the product.

Place the unit figure of the multiplier under the marked figure of the multiplicand, and keeping this figure in this place, invert the figures in the multiplier.

Then multiply by each of the given figures, beginning each operation with the figure immediately above it in the multiplicand, and placing the right hand figure of each product in the same column, and find the amount of the whole.

Obs. 1. For the sake of greater exactness, it may be proper to mark the multiplicand one place lower than the lowest figure to be retained.

Obs. 2. If the multiplicand does not contain a sufficient number of decimal places, annex as many ciphers as may be necessary.

Obs. 3. If there is not any unit figure in the multiplier, put a cipher in its place.

Obs. 4. In taking each partial product, such a number must be added to the lowest figure, as is equal to the number of the tens in the product of the highest figure rejected.

Obs. 5. Each multiplying figure must be separately used.

Ex. 1. To multiply 47.3485 by 5.164 retaining 4 places of decimals in the product.

$$\begin{array}{r l r}
47.348\dot{5} & & \\
4615 & & \\
\hline
236.7425 & 5 \text{ times} & 47.3485 \\
4.7348 & \text{once} & 47348 \\
2.8409 & 6 \text{ times} & 4734 \\
1894 & 4 \text{ times} & 473 \\
\hline
244.5076 & & \\
\hline
\end{array}$$

Four places of decimals being required to be retained, the 5 is marked, and the unit figure, 5, of the multiplier is placed under it.

To the third product we add 5 for 6 times the 8 rejected, and to the 4 th product we add 2 for 4 times the 4 rejected.

Example 2. To multiply 62.807 by 13.486, and by 0.13486, retaining 4 places of decimals in the product.

$$\begin{array}{r} 62.807\dot{0}0 \\ 68431 \\ \hline 628.0700 \\ 188.4210 \\ 85.1228 \\ 5.0246 \\ .3768 \\ \hline 847.0152 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 62.807\dot{0}0 \\ 68431.0 \\ \hline 628070 \\ 188421 \\ 25123 \\ 5024 \\ 377 \\ \hline 8.4701 \\ \hline \end{array}$$

In the second operation 5 places are used, (that is, 1 more than the 4 places required in the product,) according to the first of the preceding observations.

METHOD 2.

By the use of a reciprocal divisor.

A reciprocal divisor is the decimal value of a fractional part, the denominator of which is the same number as the multiplier; thus 0.5, which is equal to $\frac{1}{2}$, is the reciprocal divisor for 2 as a multiplier.

Instead of using these decimal values, we may consider the reciprocal as a fraction of which the denominator is equal to the division of the numerator by the given number; as

$\frac{10}{5}$ for 2, $\frac{100}{4}$ for 25, $\frac{1000}{8}$ for 125, $\frac{10000}{16}$ for 625,

which are the chief numbers for which these substitutions can conveniently be made.

But for 125 we may also take 100 times and add 1-4 th, for 625 we may take $6\frac{1}{4}$ times 100 times, and for 3125 we may take $3\frac{1}{8}$ times 100 times.

Thus to multiply 4.37611 by 125.

with 125 as 1000 ths.

8) 4376.11	for 1000
547.01375	for 125.

or with 125 as 1¼ times 100.

¼ . . .	437.611
	109.40275
	547.01375

Method 3.

Examples of Approximate products.

To multiply 249.68 by 612½ and by 587½.

⅛ . . .	24968.	100 times	24968.	
	6		6	
	149808.	600 times	149808.	
	3121.	12½ —	3121.	*subtract.*
	152929.		146687.	

To multiply 126.372 by 1005¼, and by 994¾

126372.	1000 times	126372.	
631.860*	5 —	631.860*	*subtract.*
31.593	¼ —	31.593	
127035.453		125708.547	

To multiply 43.708 by 99.9, and by 99.7.

4370.8	for 100	4370.8	
4.3708	0.1	13.1124	3 times.
4366.4292		4357.6876	

---

* We here multiply the 1000 times by 5, placing each of the figures in the result 3 places to the right of the figure multiplied.

CONTRACTED MODES OF DIVISION.

The contractions that may be practised for shortening the operations of Long Divisions, are,

1 st. The cancelling of the dividing figures, or the contraction of the partial products and remainders.

2 nd. The use of reciprocal multipliers.

3 rd. The use of approximate divisors.

METHOD 1.

Rule. Correct the dividend if necessary, for the decimal in the divisor; then find how many places of whole numbers there will be in the quotient, and add to the number as many places of decimals as the quotient may be required to contain.

Then, if the divisor contains as many figures, instead of bringing down ciphers in the lower dividend figures, drop or cancel one figure of the divisor at each step in the operation.

If the divisor contains a greater number of figures, drop, at the commencement, all the superfluous figures.

If the divisor contains a less number of figures, commence and continue the division by the ordinary method, as many steps as there are places deficient.

Obs. As each of the figures is dropped it may be cancelled or marked. To the first figure of each product, is to be added the number of tens produced from the cancelled figure, and for the sake of greater precision, one more place may be reckoned than the number of places required to be retained.

Ex. 1. To divide 62.873654 by 3.17646 to retain 4 places of decimals.

```
317646 ) 6287365.4 ( 19.7937
           311090
            25209
             2974
              116
               21
```

When this dividend has been corrected for the 5 places of decimals in the given divisor, it appears that there will be 2 places of whole numbers in the quotient; and as there are to be 4 places of decimals, there will be 6 figures in all, the same as the number of figures in the given divisor.

Therefore after having found the first quotient figure, we drop the 6 in the divisor in the next step of the division, instead of bringing down the 5 from the dividend; but in taking 9 times the 31764, we add 5, for the tens in 9 times 6; saying, 9 times 6 are 54, carry 5; 9 times 4 are 36 and 5 are 41 from 50, there remains 9, and carry 5; 9 times 6 are 54 and 5 are 59, &c.

The 4 is dropped in the 3 rd step, and 3 is carried for 4 times 7; the 6 is dropped in the 4 th step, and 5 is carried for the tens; the 7 is dropped in the 5 th step, and 2 is carried for the tens; and in the 6 th step, the quotient figure is taken at 7, as giving the nearest product.

Ex. 2. Divide 67854.34 by 2784 to 4 places of decimals.

2784) 67854.34 (24.3729
12174
10383
2031
82
26

As there are here to be 6 figures in the quotient, and there are only 4 in the divisor, the contraction does not begin until 2 figures have been brought from the dividend.

Ex. 3. Divide 63.84715 by 436.27827 to 5 places of decimals.

43627827) 6384715.0 (0.14634
20219
2768
151
20

As there are not any places of whole numbers in the quotient, and only 5 places of decimals are required, we begin the operation with only the five highest figures in the divisor, and drop 1 figure at each step.

METHOD 2.

The use of reciprocal multipliers.

A reciprocal multiplier is the fractional or decimal value of the divisor, expressed as the denominator of a fractional part: thus the divisor 2 is expressed as a multiplier by $\frac{1}{2}$ or by 0.5, and 5 as a divisor is equal to $\frac{1}{5}$ or 0.2 as a multiplier.

As with multiplications, the chief of these numbers that can so be reversed are 5, 25, 125, 625, &c., for which the reciprocal multipliers are 0.2, 0.04, 0.008, 0.0016, &c.

Ex. 1. Divide 626.443 by 5 and by 125.

626.443		626.443	
0.2	for 5	0.008	for 125
125.2886		5.011544	

Sometimes a very near decimal value may be found for the fractional form of the divisor, which, with a correction upon the result, can be used with advantage as a multiplier; as in the following example.

Ex. 2. Divide 64876.1 by 7300.

$$\tfrac{1}{7300} = 0.000137 \text{ very nearly.}$$

$$64876.1 \times 0.000137 = 8.8880257.$$

To rectify this result which should be 8.8871369, we must subtract the 10000 th part of it, that is, we must subtract the same figures, commencing to put them under the 5 th place.

$$\begin{array}{r} 8.8880257 \\ 8888 \\ \hline 8.8871369 \end{array}$$

This substitution becomes more useful, if, for this multiplication by 1 ten-thousandth and 37-100 ths of 1 ten-thousandth, we take parts for the latter, saying $33\frac{1}{3}$ are the 3 rd, $3\frac{1}{3}$ are the 10 th of $33\frac{1}{3}$, and $\frac{1}{3}$ is the 10 th of $3\frac{1}{3}$; in the manner of the calculation in the following formula for the calculation of Simple Interest by Decimals, page 179.

METHOD 3.

The use of approximate Divisors.

The chief use that can be made of approximate divisors, is when the given number is either near 100, 1000, &c. or when it differs from a number that can be easily used as a divisor, by a number that can also be conveniently used as a part of the assumed number.

Rule. Divide by the assumed divisor, and out of the quotient take a series of products, according to the part which the difference between the given and assumed divisors is of the assumed divisor, extending the series for the sake of precision to one figure more than the required number of figures.

Then if the assumed divisor is *greater* than the given divisor, take the amount of the quotient and of all the terms of the series; but if it is *less*, take the amount of the quotient and of the 2 nd, 4 th, 6 th, &c. terms of the series, and subtract from it the amount of the 1 st, 3 rd, 5 th, &c. terms.

Case 1. When the assumed divisor is greater than the given divisor.

Ex. 1. Divide 6485.92 by 99 and by 98 to 5 places of decimals.

$$\begin{array}{r} 64.8592 \\ 648592 \\ 6485 \\ 64 \\ \hline 65.51434 \end{array} \qquad\qquad \begin{array}{r} 64.8592 \\ 1.297184 \\ 25943 \\ 518 \\ 10 \\ \hline 66.18285 \end{array}$$

In each operation 100 is substituted as the approximate divisor, and in the first, for the excess 1, the 100 th part is taken of the given number, and then the 100 th part of this quotient, and so on, until the series works out beyond the required limits, which, for the sake of precision, are taken at one place more than the required number of decimals.

In the second calculation, for the excess 2, we either take 1-50 th or multiply by 2-100 ths; that is, multiply by 2, and place each product 2 figures to the right.

Ex. 2. Divide 483.467 by 999 and by 992 to 6 places of decimals.

0.483467	0.483467
4834	38676
4	309
	2
0.483951	0.487366

In these calculations after having divided by 1000 in the first calculation we take 1-1000 th for the difference 1, and in the 2 nd, 8-1000 ths for the difference 8.

Thus 4834 × 8 adding 4 for tens from the back figure.
38 × 8 — 5 —
0 × 8 — 2 —

Ex. 3. Divide 367054 by 119 and by 396 to 6 figures:

12,0) 36705.4	4,00) 3670.54
$\frac{1}{120}$ . . . 3058.783	$\frac{1}{100}$ . . 917.635
— 25.489	— 9.1763
— 212	— 915
2	9
3084.49	926.904

For 119 we divide by 120, and add, for 1 the difference, a series of 120 ths.

For 396 we divide by 400, and add, for 4 the difference, a series of 100 ths.

Ex. 4. Divide 439571 by $97\frac{1}{2}$ and by $99\frac{7}{8}$ to 7 figures.

$\frac{1}{40}$ . . . 4395.71	$\frac{1}{800}$ . . . 4395.71
— 109.8927	— 5.4946
— 2.7473	68
— 686	4401.211
17	
4508.420	

For $97\frac{1}{2}$ we divide by 100, and add, for the difference $2\frac{1}{2}$, a series of 40 ths.

For $99\frac{7}{8}$ we divide by 100, and add, for the difference $\frac{1}{8}$, a series of 800 ths.

Case 2. When the assumed divisor is less than the given divisor.

EXAMPLE 5.

To divide 483.467 by 1001 and by 1008 to 6 figures.

$$\begin{array}{rlcrl} 0.483467 & & \qquad & 0.483467 & \\ 4834 & - & & 3867 & - \\ 4 & + & & 30 & + \\ \hline 0.483984 & & & 0.479630 & \\ \hline \end{array}$$

When the series converges rapidly, that is, when the difference between the given and assumed divisors is comparatively small, the result is obtained with great facility ; but when the difference is great, the series converges so slowly, that the common method of division is generally preferable. The divisors to this example are the same as those to Example 2.

EXAMPLE 6.

To divide 562887 by 1002½ and by 1000½ to 6 figures.

$$\begin{array}{lrlclr} \frac{1}{400} \ldots & 562.887 & & \qquad & \frac{1}{2000} \ldots & 562.887 \\ & 1.407 & - & & & .281 \\ & 3 & + & & & \\ \hline & 561.483 & & & & 562.606 \\ \hline \end{array}$$

Here, having taken, in each, the 1000 th part by the removal of the decimal point, the divisor to the first is 400, for the difference 2½, and the divisor to the second is 2000, for the difference ½.

EXAMPLE 7.

To divide 427563 by 100⅛ to 6 figures.

$$\begin{array}{lrl} \frac{1}{800} \ldots & 4275{,}63 & \\ & 5{,}34 & - \\ \hline & 4270{,}29 & \\ \hline \end{array}$$

EXAMPLE 8.

Of the German method of contracting a long Division.

To divide 37552382 by 4576.

```
4576 ) 37552382 ( 8206
       --944126
         -297
          -1
```

In this division we begin with the 37552 in the dividend, in which the divisor is contained 8 times with the remainder of 944; then taking in the next figure 3 in the dividend, but without bringing it down, the divisor is contained twice, and in subtracting the product we begin with this 3, and then go down to the 4 and 94. The remainder is 291, in which, with the 8, the divisor is not contained; we therefore bring in the 2, in the whole of which the divisor is contained 6 times, and in making the subtraction we begin with the 82, then go down to the 1, and lastly to the 29, the remainder being the figures without others under them, or 1726. Some persons place a point, and some a nought, under the figures in the dividend, successively taken into consideration, and then write over it when they put down the first figure belonging to the remainder; and it must also be remarked, that foreigners more generally place the divisor over the quotient, than on the left of the dividend.

It will be seen, that the only advantage which this method has over the usual form of an English Division is, that it saves the bringing down of the dividend figures to the remainders; but it is more troublesome and difficult to revise, and it is not brought into use when we employ the method of contracting the divisor and rejecting the lower dividend figures.

If the remainder is to be multiplied, for the purpose of reduction, it is also inconvenient to use this method; and the same remark also applies to the decimal continuation of the division, unless space is allowed on the right of the dividend, for the insertion of all the ciphers that may be wanted for this purpose.

DECIMAL PROPORTION.

Rule. Arrange the given terms according to the directions of the Rule of Three, and reduce, if necessary, the first and third terms into similar decimal quantities.

Then, for Direct Proportion, multiply the second term by the number of the third, and divide the product by the number of the first.

Or, for Inverse Proportion, multiply the second term by the number of the first, and divide the product by the number of the third.

Observe. When the second term is a compound quantity, it will in general be best to reduce the lower denominations into the decimal of the highest integer; but when either the second or third terms of a direct Proportion is a fractional number, it may be used as a multiplier without being reduced into a decimal.

Ex. 1. To find the amount of $13\frac{1}{4}$ per cent. upon £ 837 16 2.

£ 837 16 2 = £ 837.808

$$\begin{array}{lcccr} & \text{£} & & \text{£} & \text{£} \\ \text{If} & 100 & \text{———} & 13\frac{1}{4} & \text{———}\ 837.808\,? \\ & & & & 13\frac{1}{4} \\ \hline & & & & 10891.504 \\ & & & & 209.452 \\ \hline & & & & \text{£}\ 111.00956 = \text{£}\ 111\ 0\ 2\frac{1}{2} \end{array}$$

The 16 s. 2 d. is valued as a decimal by the Rule to Case 6. Page 155.

The three terms being similar, we make use of the third as the producing term, and multiplying it by the number of the second, viz. $13\frac{1}{4}$, we divide the product by 100, by cutting off two more places of decimals. See Ex. 1. page 163.

Then valuing the result by the Rule to Case 5, page 153, and reckoning the 9-1000 ths as 10 on account of the remainder, we obtain the amount to the nearest farthing; but as farthings are for such purposes not generally used, the proper result is, in pence, only 2 d.

Example 2. To find the per centage profit produced by a gain of £ 64 13 3 upon £ 382 16 0.

£ 64 13 3 = £ 64.662 £ 382 16 = £ 382.8

£ £ £
If 382.8 ——— 64.662 ——— 100?

£ £ £ s. d.
3828) 64662 (16.892 = 16 17 10.
26382
3414
352
8

We value the given shillings and pence, and also the decimal in the result, by the rules quoted in the last page.

To multiply by 100 we remove the decimal place in the 2 nd term 2 places to the right, and to correct for 1 place in the divisor, we remove it another place, in all 3 places, to the right, which makes the dividend a whole number.

After the first two quotient figures are obtained, we contract the division, because we want only 5 figures in the quotient, and there are 4 in the divisor.

Example 3. In order to include £ 90 6 6, it is required to insure £ 100, how much must be insured to include £ 835 17 2?

£ 90 6 6 = £ 90.325 £ 835 17 2 = £ 835.858

£ £ £
If 90.325 ——— 100 ——— 835.858?

£ £
90325) 8358580 (925.389
229330
48680 Answer £ 925 7 9
3518
808
86

Both the first and third terms containing the same number of decimal places, they may be both considered as whole numbers.

EXERCISES.

Ex. 1. What is the amount of 17⅜ per cent. upon £ 456 7 8 ?
Answer £ 79 5 11½.

Ex. 2. What is the amount of 23¼ per cent upon £ 527 11 9 ?
Answer £ 122 13 3½.

Ex. 3. What is the amount of £ 629 14 5 Stock at 92⅜ per cent. ?

Answer £ 581 14 1.

Ex. 4. What is the amount of £ 1343 18 7 Stock at 93¼ per cent. ?

Answer £ 1253 4 3.

Ex. 5. If 91⅝ in money will buy £ 100 Stock, how much Stock may be bought with £ 1138 14 5 ?
Answer £ 1242 16 1.

N. B. *After the third term has been expressed as a decimal, both the* 1 *st and* 3 *rd terms may be multiplied by* 8 *to get rid of the fraction.*

Ex. 6. If £ 100 produce a profit of £ 16 5 8, what will be the profit upon £ 520 17 11 ?
Answer £ 84 16 4.

Ex. 7. If goods which cost £ 1711 14 4 produce a profit of £ 273 17 7, what is the rate of this gain per cent. ?
Answer £ 16 0 0. .

Ex. 8. If goods which cost £ 821 15 9 had been sold for £ 784 16 2, what is the rate of this loss per cent. ?
Answer £ 4 10 0.

Ex. 9. What is the rate per Pound at which the distribution of effects must be made, when the amount of the claims is £ 37634 16 0, and the assets are only £ 8316 14 2 ?
Answer £ 0 4 5.

Ex. 10. What is the rate per Pound at which the distribution of effects must be made, when the amount of the claims is £ 11208 4 4 and the assets are only £ 2603 8 1 ?
Answer £ 0 4 8.

FORMULA FOR THE CALCULATION OF

SIMPLE INTEREST.

To find the amount of the Interest on £ 864 12 6 for 35 days at 5 per cent. per annum.

£ 864 12 6 = £ 864.625
35

4323.125
25938.75

$\frac{1}{3}$ ... 30262
$\frac{1}{10}$ ... 10087
$\frac{1}{10}$ ... 1008
100

£ 4.1457 = £ 4 2 11 very nearly.

For the Rule see page 106.

As it is not usual to calculate the Interest upon the shillings and pence in the Principal, the above calculation would commonly be worked as for £ 865; (see page 104;) but it is sometimes required to calculate for the lower denominations, and then the best method is to value them decimally.

The third, tenth, and tenth of the 10.000 th is an approximation for the division by 7300, as shown page 171.

The valuation is made upon £ 4.1457 reckoned £ 4.146.

EXERCISES.

Ex. 1. What is the amount of the Interest upon £ 348 13 6 for 101 days at 5 per cent. per annum.

Answer £ 4 16 6.

Ex. 2. What is the amount of the Interest upon £ 654 for 234 days at 5 per cent. per annum.

Answer £ 20 19 4.

Ex. 3. What is the amount of the Interest on £ 1635 17 4 for 29 days at 3½ per cent. per annum.

Answer £ 4 11 0.

FORMULA OF THE CALCULATION OF

COMPOUND INTEREST BY DECIMALS.

To find the amount of the Compound Interest on £ 6781 11 4 half yearly for 2 years at 4 per cent. per annum.

	£	
	6781.5660	
Interest	135.6313	
Amount	6917.1973	for 1 st ½ year.
—	138.3439	
—	7055.5413	- 2 nd.
—	141.1108	
—	7196.6521	- 3 rd.
—	143.9330	
—	7340.5851	- 4 th.
Principal	6781.5660	
Compound Interest £	559.0191	= £ 559 0 5.

At 4 per cent. per annum the half yearly Interest is 2 per cent., which is calculated for, either by multiplying by 2, and placing the figures 2 places to the right of their regular situation, or by dividing by 50.

Another method of calculating Compound Interest, is, to take the amount of £ 1 for 1 period of Interest, and involve it as many times, less one, as the compound Interest is to be taken; then to multiply the given sum by this product, and subtract the original sum from the product.

thus 1.02 × 1.02 × 1.02 × 1.02 = 1.08243216

£ 6781.566 × 1.08243216 = £ 7340.585

Subtract the Principal 6781.566

Compound Interest £ 559.019 = £ 559 0 5

In general, however, it will be found that the common way is best.

N. B. The Exercises given page 111 may be recalculated by decimals.

REPEATING AND CIRCULATING

DECIMALS.

NUMERATION.

A Decimal is said to repeat or circulate when it may be carried to any extent by the repetition of the same figure or figures: as, 0.333, &c. is called a repeating decimal; 0.372372, &c. is called a circulating decimal; and they are generally expressed by placing a point over the repeating figure of a repetend, and the same over the first and last repeating figures of a circulate. Thus, we may express

0.333, &c. by $0.\dot{3}$, and 0.372372 by $0.\dot{3}7\dot{2}$.

A pure repetend or circulate is a number which contains only the repeating figure or figures, as the above.

The mixed repetend or circulate is a number in which other figures, either integral or decimal, précede the repeating figure or figures;

as, $3.70\dot{3}$ $30.46\dot{3}7\dot{2}$.

Similar repetends or circulates are those which commence at the same place, either of integers or decimals: as,

$4.2\dot{3}$ and $0.3\dot{5}$, or $5.\dot{1}6\dot{4}$ and $4.\dot{2}7\dot{2}$.

Conterminous circulates are those which contain the same number of figures; as,

$2.\dot{8}2\dot{6}$ and $4.\dot{0}6\dot{4}$, or $0.\dot{3}7\dot{8}$ and $0.4\dot{2}6\dot{5}$.

Therefore, similar and conterminous circulates are those which commence at the same place, and contain the same number of figures; as,

$3.\dot{1}7\dot{6}$ and $4.\dot{8}6\dot{2}$, or $0.36\dot{3}4\dot{2}$ and $0.01\dot{5}2\dot{7}$.

All repeating and circulating decimals are produced, either from fractions having as many nines in the denominator as there are places in the repetend or circulate, or from the equivalent lowest terms of such fractions.

REDUCTION.

To reduce repeating and circulating decimals to fractions.

CASE 1.

When the repetend or circulate is pure.

Rule. Express it as the numerator of a fraction, and place as many nines for the denominator as the repetend or circulate contains figures.

Observe. When the highest place of figures in the repetend or circulate does not commence in the place of 10 ths, so many ciphers must be annexed to the numerator when higher, or to the denominator when lower, as that place is distant from the place of 10 ths.

EXAMPLE.

$0.\dot{4} = \frac{4}{9}$ $\quad 0.0\dot{5} = \frac{5}{90}$ $\quad 0.00\dot{7} = \frac{7}{900}$

$0.\dot{3}\dot{5} = \frac{35}{99}$ $\quad 0.0\dot{4}7\dot{2} = \frac{472}{9990}$ $\quad 0.00\dot{3}14\dot{6} = \frac{3146}{999900}$

$5.\dot{7} = \frac{570}{99} = \frac{190}{33}$ $\quad 51.5\dot{7} = \frac{515700}{9999} = \frac{57300}{1111}$

$0.\dot{0}\dot{9} = \frac{9}{99} = \frac{1}{11}$ $\quad 0.\dot{0}0\dot{9} = \frac{9}{999} = \frac{1}{111}$.

CASE 2.

When the repetend or circulate is mixed.

Rule. Express the fractional value of the figures preceding the repetend or circulate, and to the numerator express the fractional value of the repetend or circulate, and reduce the whole to a simple fraction.

N. B. The numerator of the result may be formed by subtracting the finite figures from the whole of the given figures, and the denominator will contain as many nines as the repetend or circulate contains figures, placing ciphers as before to either the numerator or denominator when the repetend or circulate does not commence in the place of 10 ths.

EXAMPLES.

$$5.4\dot{3} = 5.4\tfrac{3}{9} = \frac{54\frac{3}{9}}{10} = \tfrac{489}{90} = \tfrac{163}{30}$$

$$0.07\dot{4} = 0.07\tfrac{4}{9} = \frac{7\frac{4}{9}}{100} = \tfrac{67}{900}$$

$$3\dot{4}.1\dot{2} = 30\tfrac{4120}{999} = \tfrac{34090}{999}$$

$$0.04\dot{7}\dot{3} = \frac{4\frac{73}{99}}{100} = \tfrac{469}{9900}$$

or the numerators may be found thus,

$$\begin{array}{rcrcr} 543 & - & 54 & = & 489 \\ 74 & - & 7 & = & 67 \\ 34120 & - & 30 & = & 34090 \\ 473 & - & 4 & = & 469 \end{array}$$

EXERCISES.

Reduce to Fractions.

Ex. 1. $0.\dot{3}$, $0.\dot{7}$, $0.\dot{9}$, and $0.0\dot{7}$.

2. $5.\dot{1}$, $3.1\dot{2}$, $4.0\dot{6}$, and $7.30\dot{7}$.

3. $0.\dot{1}2\dot{3}$, $0.\dot{7}\dot{4}$, $0.00\dot{3}\dot{7}$, and $\dot{5}2.380\dot{9}$.

4. $4.\dot{2}1\dot{6}$, $4.2\dot{1}\dot{6}$, $53.\dot{6}2\dot{7}$, and $5\dot{3}.62\dot{7}$.

N. B. These Exercises are to be proved by reducing the fractions to decimals.

To make dissimilar repetends similar and conterminous.

Rule. Make the first repeating or circulating figure in each commence at the lowest place in which the given repetends or circulates commence; then, with circulates, find the least common multiple of the number of figures, and continue each circulate to that number of places.

When finite numbers are, for the purposes of Addition or Subtraction, required to be expressed as similar and conterminous circulates, they must be continued with ciphers, with the first of which the common circulate must commence, unless the finite parts of the other number extend to a lower place of decimals.

MULTIPLICATION.

Rule. Reduce the two terms to their equivalent fractions, find their product, and reduce it to a decimal.

N. B. When one term is a finite number and the other is a repeating decimal, the latter may be made the multiplier, and parts may be taken for the repetend.

EXAMPLES.

To multiply 7.46 by $3.\dot{4}$.

$$7.46 = \tfrac{746}{100} \text{ and } 3.\dot{4} = 3\tfrac{4}{9} = \tfrac{31}{9}$$

$$7.46 \times 3\tfrac{4}{9} = \frac{746 \times 31}{100 \times 9} = \frac{23126}{900} = 25.69\dot{5}$$

To multiply $2\dot{3}.1\dot{5}$ by $0.0\dot{1}\dot{8}$.

$$2\dot{3}.1\dot{5} = \tfrac{23130}{999} = \tfrac{2570}{111}, \text{ and } 0.0\dot{1}\dot{8} = \tfrac{18}{990} = \tfrac{2}{110}$$

$$\tfrac{2570}{111} \times \tfrac{2}{110} = \tfrac{5140}{12210} = 0.\dot{4}2096\dot{6}$$

EXERCISES.

Ex. 15. Multiply 23.25 by $4.\dot{2}$ and by $0.\dot{5}$.
16. — $6.34\dot{5}$ by 21 and by $4.0\dot{4}$.
17. — 4.26 by $4.\dot{1}\dot{6}$ and by $\dot{3}.0\dot{2}$.
18. — $\dot{7}.\dot{1}$ by 2.02 and by $5.\dot{3}\dot{6}$.

DIVISION.

Rule. Reduce the two terms to their equivalent fractions, find their quotient, and reduce it to a decimal.

N. B. If the divisor is finite, extend the dividend as far as may be required, and use the divisor as in common Division.

If the divisor is a repetend, multiply both terms by 9 and proceed as above.

If the divisor and dividend contain the same number of places of circulates, the division is performed by the division of the numerators of their equivalent fractions ; but if the number of the places is unequal, we may extend each to their common multiple, and proceed as before.

EXAMPLES.

To divide $63.\dot{7}3\dot{2}$ by 7.

$$\begin{array}{r} 7\,)\,63.\dot{7}3\dot{2}732 \\ \hline 9.\dot{1}0467\dot{6} \\ \hline \end{array}$$

To divide $6.4\dot{3}$ by $5.\dot{7}$.

$$\begin{array}{rcl} 5.7 & & 6.4\dot{3} \\ 9 & & 9 \\ \hline 52 &) & 57.9 \\ \hline & & 1.11\dot{3}46153\dot{8} \\ \hline \end{array}$$

To divide $13.\dot{1}\dot{7}$ by $4.2\dot{1}\dot{5}$.

$$13.\dot{1}\dot{7} = \frac{1304}{99} \qquad 4.2\dot{1}\dot{5} = \frac{4173}{990}$$

$$\begin{array}{r} 4173\,)\,13040 \\ \hline 3.1248502, \text{ \&c.} \\ \hline \end{array}$$

EXERCISES.

Ex. 19. Divide $4.1\dot{8}3\dot{2}$ by 12.
20. — $168.2\dot{7}$ by 4.
21. — $440.\dot{4}\dot{3}$ by 43.
22. — $51.\dot{0}2\dot{7}$ by $\dot{7}.\dot{1}$
23. — $9.39\dot{2}\dot{9}$ by $\dot{5}.\dot{7}$
24. — 89.165 by $0.\dot{0}\dot{6}$
25. — 57363 by $0.0\dot{6}$
26. — $49\dot{8}.5\dot{9}$ by $\dot{4}.1\dot{7}$

PRODUCTS.

Ex. 1. $\frac{1}{3}$, $\frac{7}{9}$, 1, $\frac{7}{90}$.
2. $5\frac{1}{9}$, $3\frac{11}{90}$, $4\frac{1}{15}$, $7\frac{277}{900}$.
3. $\frac{41}{333}$, $\frac{74}{99}$, $52\frac{8}{21}$.
4. $4\frac{8}{37}$, $4\frac{107}{495}$, $53\frac{209}{333}$, $53\frac{697}{1111}$.
5. $3.111\dot{1}$, $42.766\dot{6}$, $0.135\dot{7}$.
6. $67.3\dot{4}5454\dot{5}$, $8.6\dot{2}1621\dot{6}$, $0.2\dot{4}2424\dot{2}$, $0.888888\dot{8}$.
7. $69.194\dot{6}$. (8.) ’$81.01\dot{4}9\dot{2}$.
9. $66.048\dot{8}6395\dot{4}$. (10.) $71.06321\dot{8}63\dot{9}$.
11. $3.\dot{7}$ and $4.6\dot{2}$. (12.) $3.\dot{5}$ and $3.5\dot{2}$.
13. $50.7\dot{5}\dot{2}$ and $47.31\dot{5}\dot{2}$.
14. $9.\dot{1}9\dot{4}$ and $19.98\dot{3}7901\dot{5}$.
15. $98.1\dot{6}$ and $12.91\dot{6}$.
16. $133.25\dot{6}$ and $25.66\dot{4}246913580\dot{}$ —
17. $17.72\dot{8}\dot{4}$ and $12.8\dot{7}8\dot{0}$.
18. $14.4\dot{8}\dot{6}$ and $3\dot{8}.46648301193755739210\dot{2}$.
19. $0.348\dot{6}0694027\dot{3}$. (20.) $42.069\dot{4}$.
21. $101.\dot{6}3869\dot{4}$. (22.) 7.1150361629, &c.
23. 1.631403508, &c. (24.) 1471.2225.
25. 86.0445. (26.) 119.448441247, &c.

DUODECIMALS.

Duodecimals are the parts resulting from the continued separation of the integer, and of the parts produced, into 12 equal parts: thus, the separation of a foot into 12 inches, and of each inch into 12 parts, is a duodecimal separation.

The first parts produced from the integer are called primes, and the next seconds, &c.

The most common, if not the only, application* of duodecimals, is in the measurement of surfaces and solids, when the dimensions are taken in feet and inches.

DUODECIMAL MULTIPLICATION.

Rule 1. Multiply by the number of the units or feet, and take parts out of unity or 1 foot for the lower denominations.

Rule 2. Multiply the whole of the multiplicand, 1 st, by the number of the units or feet, as in common multiplication; 2 ndly, by the number of the 12 ths or inches, and consider each product as one step lower in value; or the product of inches consider as 12 ths of an inch, and the product of feet as inches; 3 rdly, by the number of 12 ths of inches, if any, and consider each product as 2 steps lower in value; and so on, if there are any lower denominations employed in the multiplier.

Rule 3. called Cross Multiplication. Multiply each term in the multiplicand separately by each term in the multiplier.

And observe, as above, that when the multiplier is units, or a number of feet, the product will be of the same value as its multiplicand; when the multiplier is primes, or a number of inches, each product will be one step lower in value; when seconds, or twelfths of inches, 2 steps, &c., and that each product in a less denomination than feet, is, if it will admit of it, to be reduced into a higher denomination.

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* In consequence of this application, in speaking of the multiplication resulting from these dimensions, instead of the proper mode of expressing the multiplier by times and parts, it is usual to call them feet and inches; as for taking 3 times and 5-12 ths, to direct the multiplication by 3 feet and 5 inches.

EXAMPLE.

To multiply 17 feet 10 in. by 9 and 7-12 ths, or 9 feet 7 inches.*

1 st		2 nd		3 rd
ft. in.		ft. in.		ft. in.
½ . . . 17 10	(or)	17 10	(or)	17 10
9		9 7		9 7
160 6		160 6		153 -
⅙ . . . 8 11		10 4 10		7 6
1 5 10				9 11
		170 10 10		5 10
170 10 10				
				170 10 10

In the first method, we multiply 17 feet 10 in. by 9, and take parts for 7-12 ths, saying 6 are the half, and 1 is the sixth of 6.

In the second, we multiply, 1 st by 9, and next by 7, taking 1-12 th of each of the latter products by reckoning each product as 1-12 th of its usual value: thus, 7 times 10 are 70, 70-12 ths of an inch are 5 inches and 10-12 ths, carry 5, 7 times 17 are 119 and 5 are 124; 124 inches are 10 feet 4 in.

In the third, we multiply, 1 st, 17 feet by 9, of which the product is 153 feet; next 10 inches by 9, of which the product is 7 feet 6 in.; then 17 by 7, reckoning the product, 119, as inches, or 9 feet 11 in.; lastly 10 by 7, reckoning the product as 70-12 ths of an inch, or 5 in. 10-12 ths.

EXERCISES.

Multiply

Ex.	ft.	in.		ft.	in.		ft.	in.	
1.	36	10	by	4	6*	Prod. by Rule 1,	165	9	
2.	28	5	-	11	9	— —	333	10	9
3.	11	7	-	7	5	Prod. by Rule 2,	85	10	11
4.	13	6	-	9	7	— —	129	4	6
5.	63	4	-	13	8	Prod. by Rule 3,	865	6	8
6.	117	9	-	19	10	— —	2335	4	6

---

* N. B. In working the above Exercises it must be recollected, that to say we multiply by 4 feet 6 inches, means only to take 4 times and 6-12 ths, or 4½ times; for speaking strictly, we cannot say we multiply by 4 feet, that is, to say we take 4 feet of times.

APPLICATION OF DUODECIMALS.

In the measurement of surfaces the contents are usually givei in square feet and 12 ths, which are called inches, although as 144 square inches make 1 square foot, each 12 th is properly 12 square inches. Measurements are also made in yards of 9 square feet; in Carpenters' squares of 100 sq. ft. and in Bricklayers' square rods of 272 sq. ft. Land measure is taken in acres, Roods, &c. Solids are measured in cubic inches, cubic feet, cubic yards, and tons of 40 cubic feet.

The principles upon which measures in square feet and 12 ths of rectangular surfaces are obtained, are these; taking the length as so many square feet and 12 ths as there are feet and inches in the length, we have the superficial contents for 1 foot in breadth; and then multiplying these by the number of feet and inches in the given breadth, we obtain the whole superficial contents.

The solidity of rectangular solids is obtained from considering the number of square feet, &c. obtained from the length and breadth, as so many cubic feet, &c., and multiplying them by the number of feet and inches in the depth. Hence the common rule is: Multiply the length by the breadth for the *surface* and that product by the depth for the *solidity*.

Exercises.

Ex. 1. What are the superficial contents of the floor of a room, 18 ft. 7 in. long, and 11 ft. 10 in. wide?

Answer Sq. ft. 219 10 10.

Ex. 2. What are the superficial contents in square yards of the floor of a room 23 ft. 4 in. long, and 13 ft. 6 in. wide?

Answer 315 sq. ft. 0 in. or 35 yds. 0 ft. 0 in.

Ex. 3. A wall is in length 95 ft. 10 in., and in height 11 ft. 6 in., how many square rods of 272 sq. ft. are there, and what is the amount at £ 24 10 per rod of the standard thickness of 1½ bricks, the thickness of the wall being 2 bricks?

Answer Surf. 4 rods 14 ft. Cont. 5 rods 109 ft.
Amount £ 132 6 4.

Ex. 4. How many cubic yards are there in a solid of these dimensions, viz. length 18 ft. 4 in., breadth 7 ft. 10 in., depth 9 ft. 6 in.? Answer 50 yds. 14 ft. 3 in.

Ex. 5. The dimensions of a case are, length 8 ft. 6 in., breadth 5 feet. 7 in., depth 5 ft. 4 in., what are the cubic contents, and how much is the amount of the freight at 35 s. per ton of 40 cubic feet? Answer Contents 253 cubic ft. 1 in. Amount £ 11 1 5

INVOLUTION.

Involution is the formation of a series of numbers, by the multiplication of the given number and of each successive product by that number.

The first product is called the 2 nd power or square; the second product the 3rd power or cube; the third product the 4 th power or biquadrate; and the succeeding products are called the 5 th, 6 th, &c. powers.

Thus with the number 4.

4^1 or the 1 st power is = 4
4^2 or the 2 nd power or square = 16
4^3 or the 3 rd power or cube = 64
4^4 or the 4 th power or biquadrate = 256
4^5 or the 5 th power = 1024

The first power, or the given number, is called the root, and the number of the power is called the index of that power.

The corresponding number to any power may be found from the multiplication of the numbers corresponding to those powers, which by addition of their indices make up the index of the given power—thus, the 5 th power of 4 may be formed from the multiplication of the numbers corresponding to the 2 nd and 3 rd powers of 4,

as 16 × 64 = 1024.

Fractional and decimal numbers are involved in the same manner, and their powers possess the same properties as whole numbers.

Thus $\frac{3}{4}$ as a root or 1 st power, has
$\frac{9}{16}$ for the square or 2 nd power,
$\frac{27}{64}$ for the cube or 3 rd power.
So also 0.4 as a root or 1 st power, has
0.16 for the square or 2 nd power,
0.064 for the cube or 3 rd power.

Table of the first 6 powers of single figures.

1 st	1	2	3	4	5	6	7	8	9
2 nd	1	4	9	16	25	36	49	64	81
3 rd	1	8	27	64	125	216	343	512	729
4 th	1	16	81	256	625	1296	2401	4096	6561
5 th	1	32	243	1024	3125	7776	16807	32768	59049
6 th	1	64	729	4096	15625	46656	117649	262144	531441

It is seldom that any higher powers of numbers are used than squares and cubes; of these two powers it may be useful to recollect, that the square of any number is equal to 4 times the square of the half of that number; and that the cube of any number is 8 times the cube of the half of that number: thus,

the square of 6 is 4 times the square of 3,
or 36 is 4 times 9; and
the cube of 6 is 8 times the cube of 3,
or 216 is 8 times 27.

The square of any number formed by 2 figures is equal to the sum of the squares of the tens and units, and twice the product of the two figures:

thus with 26, or 20 and 6,

$$\begin{array}{lrcrcr} \text{the square of } 26 = & 20 & \times & 20 & = & 400 \\ \text{with} & 6 & \times & 6 & = & 36 \\ \text{and twice} & 20 & \times & 6 & = & 240 \\ \cline{6-6} \text{for} & 26 & \times & 26 & = & 676 \\ \cline{6-6} \end{array}$$

The cube of any number of two figures is equal to the sum of the cubes of the tens and units, and of 3 times the product of the two multiplied by the sum; thus with 26 or 20 and 6,

$$\begin{array}{lrcrcrcr} & 20 & \times & 20 & \times & 20 & = & 8000 \\ & 6 & \times & 6 & \times & 6 & = & 216 \\ \text{three times} & 20 & \times & 6 & \times & 26 & = & 9360 \\ \cline{8-8} \text{for} & 26 & \times & 26 & \times & 26 & = & 17576 \\ \cline{8-8} \end{array}$$

If a series of integers be formed of the odd numbers commencing with unity, as

1, 3, 5, 7, 9, 11, 13, 15, 17, 19,

the sum of any number of terms, commencing with the first, will be the square of the number of those terms: thus if we take 4 terms, 1, 3, 5, and 7, the sum is 16, or the square of 4.

If the series be separated thus;

1, 3 5, 7 9 11, 13 15 17 19, &c.

the sum of each division will be the cube of the terms in the division: as 7, 9, and 11 equal 27, the cube of 3; and 13, 15, 17, 19 equal 64, the cube of 4. The square of the number of the terms is the mean of the terms employed.

EVOLUTION.

Evolution is the extraction of roots.

The extraction of the roots of the second powers, is called the extraction of the square root, of the third powers, the cube root, &c.

When a number is given which is exactly a power, or which has a terminable root, either as a whole, a fractional, or a decimal number, it is called a rational number; when it has not a terminable root, it is called a surd number.

Thus 36 is exactly the square of 6, therefore as regards the square or second power it is a rational number; but as no number can be found that is exactly the cube root of 36, in the cube or third power it is therefore a surd number. The numbers 15625 64, and $\frac{1}{64}$ with its equivalent decimal 0.015625, are both squares and cubes.

TO EXTRACT THE SQUARE ROOT.

If the number is a whole number, separate it into periods of two figures each, beginning with the two lower figures of the whole numbers.

If the number is a decimal number, separate it into periods of two figures each, beginning with the two highest places of decimals.

If the given number is both a whole and a decimal number, separate the periods of the integers to the left, and the periods of decimals to the right, as with each of the separate numbers.

Then, 1 st, find the highest root contained in the highest period, make it both the divisor and quotient, and take and subtract the product as in division.

Next, bring down the following period, double the divisor figure, find how often it is contained in the increased remainder or resolvend, excepting the unit figure of the latter, make the number of times both the quotient figure and the unit figure of the divisor, and take and subtract the product as before.

In this manner proceed until either all the periods are used or the extraction is carried as far as required, increasing each succeeding divisor by the last figure obtained, finding how often it is contained in the tens of the resolvend, and having inserted the quotient figure as the unit figure of the new divisor, taking and subtracting the product.

N. B. The root of a fraction may be found by extracting the root either of each term, when the fraction has been reduced, if requisite, to its lowest terms, or of the decimal value of the fraction.

When the number of the figures of a whole number is not even, the highest period will contain only 1 figure, and when the number of the decimal figures is not even, the lowest figure must have a cipher annexed to it.

At the commencement of the operation, the divisor will sometimes appear to be contained in the resolvend a greater number than 9 times, but the proper quotient figure will never be greater than 9, and to try which will be the proper figure, the product should mentally be increased by 8 or 6, for the tens in the supposed unit figure 9 or 8.

When the lowest period has been employed, if there is a remainder the calculation may be continued, decimally, by annexing periods of 2 ciphers.

When the first higher figures have been obtained, some of the lower may be formed by common division.

The proof of the accuracy of the calculation is made by squaring the root, and adding in the remainder if any.

Example 1.

To extract the square root of 873.075.

$$\begin{array}{rl}
2\ ) & 8\dot{7}3.075\dot{0}\ (29.547 \\
2 & 4 \\
\hline
49\ ) & 473 \\
9 & 441 \\
\hline
585\ ) & 32.07 \\
5 & 29.25 \\
\hline
5904\ ) & 2.8250 \\
4 & 2.3616 \\
\hline
59087\ ) & 463400 \\
 & 413609 \\
\hline
 & 49791
\end{array}$$

PROOF.

$$29.547 \times 29.547 + 0.049791 = 873.075$$

N. B. When at least half the number of the intended figures in the root is obtained, we may continue the extraction by contracted division, thus,

59087) 49791 (8426
2521
158
40

making the whole 29.5478426.

EXERCISES.

Find the square root,

Ex. 1.	Of 119025	Root	345
2.	Of 106929	—	327
3.	Of 36372961	—	6031
4.	Of 22071204	—	4698
5.	Of 2268741	—	1506.23, &c.
6.	Of 3271.4007	—	57.19, &c.
7.	Of 4795.25731	—	69.247
8.	Of 4.37259	—	2.091
9.	Of 0.00032754	—	0.01809
10.	Of 1.270054	—	1.1269

EXAMPLE 2.

To find the square root of $\frac{63}{448}$.

$$\frac{63}{448} = \frac{9}{64}$$

The square root of $\frac{9}{64} = \frac{3}{8}$.

The same may be worked from the decimal value of the given fraction: thus,

$$\frac{63}{448} = 0.140625.$$

The square root of $0.140625 = 0.375 = \frac{3}{8}$.

EXERCISES.

Ex. 11. What are the square roots of $\frac{2304}{5184}$, $\frac{9216}{12544}$, $51\frac{21}{25}$, and $9\frac{43}{49}$. Answer $\frac{2}{3}$, $\frac{6}{7}$, $7\frac{1}{5}$, and $3\frac{1}{7}$.

Ex. 12. What are the square roots of $\frac{275}{341}$, $\frac{478}{549}$, $8\frac{5}{7}$, and $6\frac{2}{3}$. Answer 0.89802, &c., 0.933099, &c., 2.9519, and 2.5819.

TO EXTRACT THE CUBE ROOT.

Rule. Separate the given number into periods of three places each, beginning with the place of units, and making, if necessary, the lowest decimal period complete by annexing one or two ciphers.

Find the greatest root of the highest period, make this figure the first quotient figure, subtract its cube, and to the remainder bring down the next period.

Find the divisor by multiplying the square of the quotient by 300, and find how often it is contained in the resolvend.

Find the subtrahend by taking,

1 st. The cube of the last quotient figure.

2 ndly. The square of the last quotient figure, multiplied by 30 times the preceding quotient.

3 rdly. The last quotient figure, multiplied by 300 times the square of the preceding quotient.

Subtract their amount from the resolvend, bring down the next period, find the next divisor and subtrahend, and so continue the operation, placing a cipher in the quotient when the divisor is not contained.

Ex. To find the cube root of 22069.810125.

$$\begin{array}{rrl}
 & 22\dot{0}69.81\dot{0}12\dot{5}\ (\ 28.05 & \\
2^3 = & 8 & \\
\cline{2-2}
2^2 \times 300 = 1200\ ) & 14069 & \text{resolvend} \\
\cline{2-2}
 & 512 & - \ - \ 8^3 \\
 & 3840 & - \ - \ 8^2 \times 30 \times 2 \\
 & 9600 & - \ - \ 8 \times 300 \times 2^2 \\
\cline{2-2}
 & 13952 & \text{subtrahend} \\
\cline{2-2}
28^2 \times 300 = 235200\ ) & 117\,810 & \\
\cline{2-2}
280^2 \times 300 = 23520000\ ) & 117\,810\,125 & \text{resolvend} \\
\cline{2-2}
 & 125 & - \ - \ 5^3 \\
 & 210\,000 & - \ - \ 5^2 \times 30 \times 280 \\
 & 117\,600\,000 & - \ - \ 5 \times 300 \times 280^2 \\
\cline{2-2}
 & 117\,810\,125 & \text{subtrahend.} \\
\cline{2-2}
\end{array}$$

EXERCISES.

Find the cube root,

Ex. 1.	Of 389017	Root	73
2.	Of 10648	—	22
3.	Of 5735339	—	179
4.	Of 12.977875	—	235
5.	Of 33.230979637	—	3.215, &c.
6.	Of 0.001906624	—	0.124
7.	Of $\frac{243}{576}$	—	$\frac{3}{4}$
8.	Of $\frac{172}{475}$	—	0.71, &c.
9.	Of $4\frac{17}{27}$	—	$1\frac{1}{3}$
10.	Of $10\frac{1}{3}$	—	2.17, &c.

The cube root of a fraction, when reduced into its lowest terms, is extracted by either extracting the cube root of each term, or by extracting the cube root of the decimal value, when either term is a surd number.

From the facility with which the extraction of roots is performed with Logarithms, by only dividing the logarithm of the given number by the index of its power, and finding the corresponding number of the logarithm of the quotient, it is very seldom that the extraction of any other root than the square root is made by common numbers; and it is therefore here considered unnecessary to give any examples of the extraction of the higher roots, as of the biquadrate root, or the square root of a square root, &c.

Respecting the extraction of the cube root of a fraction, it is proper to observe, that as tables of the roots of numbers under 1000 generally accompany those scientific works in which such calculations occur, the root of each term may be taken from the tables, and the quotient be found for the decimal value.

Or, it may be proper to raise the denominator into a perfect cube, and then extract its root, and apply it as a multiplier to the tabular value of the surd numerator.

Thus, to extract the cube root of $\frac{7}{9}$,

$$\sqrt[3]{\tfrac{7}{9}} = \sqrt[3]{\tfrac{21}{27}} = \sqrt[3]{\tfrac{1}{27}} \times \sqrt[3]{21} = \tfrac{1}{3}\sqrt[3]{21},$$

and as the cube root of 21 is by the tables 2.758923, one third is 0.919641, which is the value of the cube root of 7-9 ths, or of the decimal 0.777, &c.

In this case the denominator is a square number, but if it is neither a square nor a cube number, the fraction must be multiplied in both terms by the square of the denominator:

thus, to extract the cube root of $\frac{7}{11}$,

$$\sqrt[3]{\tfrac{7}{11}} = \sqrt[3]{\tfrac{847}{1331}} = \sqrt[3]{\tfrac{1}{1331}} \times \sqrt[3]{847} = \tfrac{1}{11}\sqrt[3]{847},$$

but the tabular cube root of 847 is 9.461524, from which the value of 1-11 th is 0.860138, which is the value of the cube root of 7-11 ths.

If we form these principles into a rule for extraction of the cube root of a surd fraction, we have the following;

Rule. Multiply the numerator by the square of the denominator, extract the cube root of the product, and divide the result by the denominator.*

N. B. The cube root of any number which is the *exact* cube of a whole number of any *two* figures may be mentally found thus: having separated the lower three figures, according as the remaining figures are next above 1, 8, 27, 64, 125, 216, 343, 512, or 729, the tens of the root will be 1, 2, 3, 4, 5, 6, 7, 8, or 9.

Then if the *unit* figure of the given cube is 1, 4, 5, 6, or 9, the unit figure of the root will be the same figure; but if the unit figure of the cube is 8, 7, 3, or 2, the unit figure of the root will be its complement, or what it wants of 10, that is, 2, 3, 7, or 8.

Thus with 46656, the tens will be 3, because 46 is above 27 and under 64, and 6 being the unit figure, the unit figure of the root will also be 6, making together 36.

Or with 373248, the tens will be 7, because 373 is next above 343, and 8 being the unit of the cube, 2 its complement will be the unit of the root, making together 72.

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* The extraction of the *square root* of a surd fraction may be made in a similar manner: thus, the square root of 5-7 ths equals the square root of 35-49 ths, which is equal to 1-7 th of the square root of 35; that is, the square root of a fraction equals the square root of the product of the multiplication of the numerator by the denominator, divided by the denominator.

PROPORTIONS.

Proportion is the relation of quantities, which taken two and two have the same or equal ratios.

Thus, as the relation of 5 pence to 7 pence is equal to the relation of 20 oz. to 28 oz. these quantities form a proportion, or are in proportion; and they are expressed as such by saying that 5 pence are to 7 pence as 20 oz. to 28 oz.; or, as 5 pence are to 7 pence, so are 20 oz. to 28 oz.; or by characters, thus,

As 5 d. : 7 d. :: 20 oz. : 28 oz.*

In this order they are said to be in direct proportion, but if one of the pair of the terms is inverted, as in this arrangement,

5 d. : 7 d. :: 28 oz. : 20 oz.

they are said to form an inverse, or more correctly a reciprocal, proportion; and they are read, as 5 d. are to 7 d. so are 28 oz. inversely to 20 oz.

When four quantities are proportional, the two terms in each set may be multiplied or divided by the same number, without altering the equality of their relation, or making any alteration in its value.

As multiplying the first set of the above terms by 3,

15 d. : 21 d. :: 20 oz. : 28 oz.

Or dividing the same set of terms by 2,

5 d. : 7 d. :: 10 oz. : 14 oz.

And if either the *powers* or the *roots* be taken of the numbers of *all the terms* of a proportion, these powers or roots will be proportional; as of the above taking the squares,

25 : 49 :: 100 : 196.

If the powers be taken of the numbers of only one set of terms, they have then the duplicate, triplicate, &c. ratios of the other terms; as with

5 — 7 — 100 and 196,

the third term is then said to have to the fourth, the duplicate ratio of the first to the second; 5 being to 7 as 100 to 140, and 100 to 140 as 140 to 196.

---

* When the first three terms of a proportion are given to find a fourth, and they are made a form of question to be worked by the usual Rule of Three, if they are not all three similar quantities, the above characters should not be used; for the meaning of these characters, as arithmetically used, is, that the quantity placed on one side of this character : is produced from or compared with that placed on the other side; and the character :: is used to link together the different sets of terms.

When four quantities are proportional, the product of the numbers of the 1 st and 4 th terms, is equal to the product of the numbers of the 2 nd and 3 rd terms : thus, in the proportion

$$5 \text{ d.} : 7 \text{ d.} :: 20 \text{ oz.} : 28 \text{ oz.}$$
$$5 \times 28 = 7 \times 20.$$

This evidently arises from the nature of proportion ; for as the ratio, 5-7 ths of the first set of terms, is equal to the ratio, 20-28 ths of the second set of terms, and as the latter terms are each 4 times the former, the products may be resolved into

$$5 \times (7 \times 4) = 7 \times (5 \times 4);$$

and so for whatever may be the relation between the numbers of the first and third or the second and fourth terms.

If the first term of a proportion is greater than the second, the third is greater than the fourth ; if equal, equal ; and if less, less.

If the four terms of a proportion consist of similar quantities, the greatest and the least are together greater than the other two terms, as with the four terms, 5 s., 7 s., 20 s., and 28 s., which are in proportion, and of which the 5 s. and 28 s., are together greater than 7 s. and 20 s.

When the relation of one quantity to another is the same as the relation compounded of several relations, they are then said to form a compound proportion.

Thus, as the relation of 12 yards to 44 yards, is equal to the compounded relations of 7 pence to 9 pence, of 6 inches to 11 inches, and of 9 oz. to 14 oz., for the ratios,

$$\tfrac{7}{9} \times \tfrac{6}{11} \times \tfrac{9}{14} = \tfrac{12}{44},$$

these quantities are said to form a compound proportion, and are expressed by characters, thus,

$$\left.\begin{array}{lcl} 7 \text{ pence} & : & 9 \text{ pence} \\ 6 \text{ inches} & : & 11 \text{ inches} \\ 9 \text{ oz.} & : & 14 \text{ oz.} \end{array}\right\} :: 12 \text{ yards} : 44 \text{ yards},$$

which are read, as 7 pence are to 9 pence, as 6 inches are to 11 inches, and as 9 oz. are to 14 oz., so are 12 yards to 44 yards.

By separate proportions, the result is the same ;

$$\begin{array}{llcllcll} \text{for} & 7 \text{ pence} & : & 9 \text{ pence} & :: & 12 \text{ yards} & : & 15\tfrac{3}{7} \text{ yards}^{*} \\ & 6 \text{ inches} & : & 11 \text{ inches} & :: & 15\tfrac{3}{7} \text{ yards}^{*} & : & 28\tfrac{2}{7} \text{ yards}^{\dagger} \\ & 9 \text{ oz.} & : & 14 \text{ oz.} & :: & 28\tfrac{2}{7} \text{ yards}^{\dagger} & : & 44 \text{ yards} \end{array}$$

And the relation of 12 yards to 44 yards is said to be compounded of the relations of 12 yards to 15$\frac{3}{7}$ yards, of 15$\frac{3}{7}$ yards to 28$\frac{2}{7}$ yards, and of 28$\frac{2}{7}$ yards to 44 yards.

PROGRESSION.

Numbers or quantities are said to be in progression when they increase or decrease either by equal differences, or by the multiplication by the same number.

When the successive terms of a series have equal differences, they are said to be in *arithmetical progression ;* and when they are produced in succession by the operation of the same multiplier, they are said to be in *geometrical progression.*

In all progressions five things are taken into consideration; viz. the first term, the last term, the number of the terms, the difference or ratio of the terms, and the sum of the terms; any three of which being given the other two may be found.

ARITHMETICAL PROGRESSION.

Arithmetical Progression is when three or more numbers or quantities increase or decrease by a common or equal difference; as 2, 5, 8, 11, or 12, 10, 8, 6, 4, 2.

When the number of the terms is equal, the sum of the extremes is equal to the sum of the means; but when the number of the terms is unequal, the sum of the extremes is equal to twice the mean term. The sum of the extremes is also equal to the sum of any two terms at equal distance from the extremes. The number of the differences is equal to the number of the terms less 1.

With these data we have the following Rules.

To find the sum of the terms—Multiply the sum of the two extremes by half the number of the terms.

To find the difference between the terms—Divide the difference between the first and last terms, by the number of the differences, that is, by the number of the terms less 1.

To find the number of the terms—Divide the difference between the first and last terms by the common difference, and add 1 to the quotient.

To find the last term—Multiply the number of the differences less one by the difference, and to the product add the first term when the series is increasing, or take the difference between the product and the first term when the series is decreasing.

To find the first term—Multiply the number of the differences by the difference, and subtract the product from the last term when the series is increasing, or add the product to the last term when the series is decreasing.

EXERCISES.

Ex. 1. The first term of a series is 1, the common difference is 2, and the number of the terms is 12, what is the last term?

Answer 23.

Ex. 2. The first term of a series is 1, the 12 th is 23, and the number of the terms is 12, what is the sum of the series?

Answer 144.

Ex. 3. If the extremes be 3 and 19, and the number of the terms 9, what is the common difference, and the sum of the whole?

Answer Diff. 2, Sum 99.

Ex. 4. If a person travelling for 10 days, makes his first day's journey 20 miles, and increases his rate 5 miles each day, what distance will he have travelled by the end of his journey?

Answer 425 Miles.

Ex. 5. The hour circle of astronomical clocks is divided into 24 hours, and the hours are usually reckoned to 24 from noon of one day to noon of the following day. If, therefore, a clock of this sort were furnished with a striking apparatus like a common clock, how many strokes would it strike in the course of one circuit? Answer 300.

Ex. 6. What will be the value of 100 books, reckoning the first at 3 farthings, and each succeeding book at an increased value of 3 farthings?

Answer £ 15 15 $7\frac{1}{2}$.

Ex. 7. If 100 stones be placed in a right line, exactly a yard asunder, and the first a yard from a basket, what length of ground must a person go who gathers them up singly, and returns with them one by one to the basket?

Answer Miles 5 1300 yds.

Ex. 8. What is the last term, and also the sum of a decreasing series, in which the first term is $5\frac{1}{2}$, the common difference $\frac{3}{16}$, and the number of the terms 8?

Answer Last term $4\frac{3}{16}$, Sum $38\frac{3}{4}$.

Ex. 9. What is the last term of a decreasing series, in which the first term is 8.368, the number of the terms 6, and the common difference 1.014?

Answer 3.298.

Ex. 10. What is the sum of an increasing series, in which the first term is 8.368, the number of the terms 6, and the common difference 1.014. Answer 65.418.

GEOMETRICAL PROGRESSION.

Three or more numbers are said to be in Geometrical Progression, when each term in the series is produced from the preceding by the operation of the same ratio or multiplier; this multiplier being a whole or mixed number, that is greater than unity, when the series is increasing, and a fractional or decimal number, that is less than unity, when the series is decreasing.

In the numbers of any continued Geometrical Progression the product of the extremes is equal to the product of the means, when the number of the terms is even; and to the square of the mean term, when the number of the terms is odd: hence the geometrical mean between two terms is equal to the square root of their product; as with the numbers 4 and 64, 16 is the mean proportional, being the square root of the product 256; and this mean proportional, may be considered as being both the second and the third terms of a simple proportion, for 4 is to 16 as 16 is to 64.

When a number is divided into two such parts, that the ratio of the whole to one of the parts, is the same as the relation of that part to the remaining part, the number is said to be divided into *extreme* and *mean* ratio; of which parts, the larger is found by multiplying half the given number by the difference between the square root of 5 and unity, or by the square root of 5 less 1: thus, as the square root of 5 is nearly 2.236068, the half of any number multiplied by the factor 1.236068, gives nearly the greater of the two parts of the number which is to be divided into extreme and mean ratio.

thus for 20 $10 \times 1.236068 = 12.36068$ is the greater part nearly
$20 - 12.36068 = 7.63932$ is the less part —
for $20 : 12.36068 :: 12.36068 : 7.63932$
or $20 \times 7.63932 = 12.36068 \times 12.36068$.

If the first term of a series be unity, the succeeding terms will be the involved powers of the ratio: thus, in 2 series ascending and descending, if the ratios are 2 and $\frac{1}{2}$, the terms are

	1 st	2 nd	3 d	4 th	5 th	6 th	7 th
1 st series	1	2	4	8	16	32	64, &c.
2 nd series	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$, &c.

If the series commence with any other number than unity, the value of any term will correspond with the product of the first term multiplied by such a power of the ratio, as corresponds with the number of the terms less one: thus, if the first term be 5, and the ratio 2, then the 7 th term will be equal to 5 multiplied by the 6 th power of 2, or by 64; that is, to 320; for the series will be

1 st	2 nd	3 d	4 th	5 th	6 th	7 th
5	10	20	40	80	160	320

This corresponding power of the ratio is called its index, and in their application to the terms of a series, the indices commence with 0 and increase by 1; hence with the preceding terms, we have, as

Indices	—	0	1	2	3	4	5	6
Powers	—	1	2	4	8	16	32	64
Series	—	5	10	20	40	80	160	320

The indices of powers formed in this manner, are called the Logarithms of the powers considered as natural numbers; they possess the property that by the addition or subtraction of the indices or logarithms of two natural numbers, the amount forms the logarithm or index of the product or quotient of the natural numbers.

Thus, with the natural numbers 4 and 16 in the preceding formula, the indices or logarithms are 2 and 4; the sum of these is 6, which is the logarithm of their product 64.

On the contrary, taking 6, which is here the logarithm of 64, and subtracting 4, the logarithm of 16, the remainder 2 is the logarithm of 4, the quotient of the division of 64 by 16.

The involution and evolution of numbers are performed with their logarithms, by multiplying or dividing by the number of the power or root.

Thus, to cube 4, if we take its logarithm 2 and multiply it by 3 we have 6, which is the logarithm of 64, or the cube of 4.

So in the reverse, to extract the cube root of 64, if we take its logarithm 6 and divide it by 3 we have 2, which is the logarithm of 4, the cube root of 64.

The series of the powers usually employed for the tables of logarithms, is that which corresponds with the natural combinations of numbers into tens, the logarithms and numbers being,

Index or Log.	0	1	2	3	4	5	6
Nat. Numb.	1	10	100	1000	10000	100000	1000000

The arithmetical operations respecting numbers in Geometrical Progression, are commonly limited to the finding of the last or any assigned term in the series, and of the sum of all the terms in the series.

To find the last term of a Geometrical Progression.

Rule. Multiply the first term by such a power of the ratio as is denoted by the number of times less one.

Example.

To find the 7 th term of a series, in which the first term is 5, and the ratio is 4.

$4 \times 4 \times 4 = 64$ 3 rd power of the ratio.
$64 \times 64 = 4096$ 6 th power —
first term $5 \times 4096 = 20480$ 7 th term.

Exercises.

Ex. 1. What is the last term of the series having 9 terms, of which the first is 3, and the ratio 3? Answer 19683.

Ex. 2. The first term of a geometrical series is 2, the number of terms 12, and the ratio 5, required the last term.
Last Term 97656250.

To find the sum of a series in Geometrical Progression.

Rule. Multiply the last term by the ratio, and divide the difference between this product and the first term, by the difference between 1 and the ratio, and the quotient will be the sum required.

To find the sum of a series, the first term being 5, the ratio 4, and the number of the terms 7.

(as above) last term $20480 \times 4 = 81920$
$\frac{81920-5}{4-1} = \frac{81915}{3} = 27305$ sum required.

Exercises.

Ex. 1. The first term of a geometrical series is 1, the last term is 65536, and the ratio is 4, what is the sum of the terms?
Answer 87381.

Ex. 2. What would be the value of 12 books, the 1 st being rated at 1 d. the second 2 d. the third 4 d. and so on doubling the value for each book? Answer £ 17 1 3.

Ex. 3. What sum of money will be paid in a year by paying 1 farthing the 1 st month, 3 farthings the second, and so on tripling each payment to the end of the 12 months?
Answer £ 276 15 10

ALLIGATION.

Alligation is the method of determining the amount of the different quantities having different rates that will be required to produce a given rate.

It is usual to separate calculations of this nature into three cases, according to whether neither of the quantities is limited, whether one is limited, or whether the total is limited. The first is called Alligation Alternate, of which the second and third cases are considered as branches, and which are called Alligation Partial and Alligation Total.

CASE 1. ALTERNATE.

Rule. Arrange the given rates, so that the mean rate may be between the higher and the lower rates.

Then taking the given rates in pairs, find the difference between the mean rate and each of the two rates, and apply the difference to the alternate rate.

N. B. If the number of the given rates is odd, one upper or one lower rate must be at least twice used with the opposite rates.

Example.

To find what number of Pounds at 7 s., 8 s., 10 s., and 12 s. will be required to form a mean value with 9 s. per lb.

								Proof.	
	7	1			7	3	1	at 7 s.	7
9-	8	3	or	9-	8	1	3	- 8 s.	24
	10	2			10	1	2	- 10 s.	20
	12	1			12	2	1	- 12 s.	12
							—		—
							7	at 9 s. =	63

So also 3 lb. at 7 s., 1 lb. at 8 s., 1 lb. at 10 s., and 2 lb. at 12 s., make a total of 7 lb. worth 63 s. or 9 s. per lb.

It is to be observed, that when any arrangement of this sort has been made, the different results may be combined and the amounts used; as in the above, 4 lb. of the 1 st at 7 s., 4 lb. at 8 s., 3 lb. at 10 s., and 3 lb. at 12 s., make a total of 14 lb., which amount to 126 s., and therefore average 9 s. per lb.

N. B. What is usually called Alligation Medial, is in business called the finding of an average rate or price; for which see page 94.

CASE 2. PARTIAL.

Rule. Find any set of quantities which will produce a mean rate equal to the given mean rate; then if this obtained quantity of the given rate is equal to the given quantity, the other quantities will also be equal to the quantities required.

But if the obtained quantity is not equal to the given quantity, divide the number of the given quantity by the number of the obtained quantity, and multiply each of the obtained quantities by the quotient.

Example. To find how many lb., at the rate of 5 s. and 12 s per lb., are sufficient with 10 lb. at 7 s. per lb., to make a mean value of 8 s. per lb.

5 . . . 4 . . . . . in all 4 at 5 s.
8- 7 . . . 4 . . . . . *4 at 7 s.
12 . . 3+1 . . . . . 4 at 12 s.

* 10 ÷ 4 = 2½

lb. 4 × 2½ = 10 lb. at 5 s. = 50 s.
4 × 2½ = 10 lb. at 7 s. = 70 s.
4 × 2½ = 10 lb. at 12 s. = 120 s.
Total 30 lb. at 8 s. = 240

CASE 3. TOTAL.

Rule. Proceed as with Alligation Alternate, and if the total is not equal to the given amount, divide the number of the given total by the number of the obtained total, and multiply each obtained quantity by the quotient.

Example. To find how many ounces of Gold 18 carats, 20 carats, and 23 carats fine, will be required to make up 12 oz. of Gold 22 carats fine.

c. c.
18 . . . 1 . . . . . 1 at 18 = 18
22- 20 . . . 1 } in all 1 at 20 = 20
23 . . 4 2 } . . 6 at 23 = 138
8 at 22 = 176

12 ÷ 8 = 1½ c.
oz. 1 × 1½ = oz. 1½ at 18 car = 27
1 × 1½ = 1½ at 20 car. = 30
6 × 1½ = 9 at 23 car. = 207
Answer oz. 12 at 22 car. = 264

EXERCISES.

Ex. 1. What proportions of Silver 216 dwts. fine, and 231 dwts. fine, may be used to make a purity equal to standard, or 222 dwts. fine?

Answer 9 oz. 216 fine, and 6 oz. 231 fine.

Ex. 2. What proportions of Silver 210 dwts. fine, 217 dwts. fine, and 228 dwts. may be used to make a purity equal to standard?

Answer 6 oz. 210 fine, 6 oz. 217 fine, and 17 oz. 228 fine.

Ex. 3. What proportions of Gold 23 carats 1 gr. or 93 grs. fine, 22 carats 3 grs. or 91 grs. fine, and 18 carats or 72 grs. fine, are required to make the standard fineness of 22 carats or 88 grs. fine?

Answer 16 oz. 93 fine, 16 oz. 91 fine, and 8 oz. 72 fine.

Ex. 4. What proportions of Spirit of the following strengths, viz.

22 per cent. over proof or 122 strong,
7 per cent. over proof or 107 strong,
12 per cent. under proof or 88 strong,

are required to be mixed with 66 gallons of Spirits 10 per cent. under proof, or 90 strong, to make up a strength equal to proof, or 100 strong.

Answer 30 gal. 22 per cent. o.; 36 gal. 7 per cent. o.; 21 gal. 12 per cent. u.; and 66 gal. 10 per cent. u.

Ex. 5. What proportions of Spirit of the following strengths, viz.

10 per cent. over proof,
4 per cent. over proof,
12 per cent. under proof,
16 per cent. under proof,

are required to make up 100 gallons of Spirit 4 per cent. under proof?

Answer $28\frac{4}{7}$ gal. 10 per cent. o.; $19\frac{1}{21}$ gal. 4 per cent. o.; $19\frac{1}{21}$ gal. 12 per cent. u.; and $33\frac{1}{3}$ gal. 16 per cent. u.

Ex. 6. I have an order for 1200 pieces of printed Calicos, to average 11 d. per yard; what quantities of the following different rates can I supply to execute this; viz. 13¼ d. per yard, 11½ d. per yard, 10¼ d. per yard, and 8½ d. per yard.

Answer 500 ps. at 13¼ d., 150 ps. at 11½ d., 100 ps. at 10¼ d., and 450 ps. at 8½ d.

POSITION.

Position, Supposition, or the Rule of False, is the method of determining the true quantity from one or more assumed quantities. It is therefore divided into two parts, called Single and Double Position.

SINGLE POSITION.

Rule. Assume any quantity similar to that which is given, and work with it according to the conditions of the calculation.

Then if the assumed result is not the same as the given result, say, as the assumed is to the true result, so is the assumed quantity to the true quantity.

Example.

To find that number of which the half, quarter, and eighth amount to 49.

Assume the number required to be 40, then

	40	35 : 49 :: 40	for 56
		40	
$\frac{1}{2}$ …	20		$\frac{1}{2}$ … 28
$\frac{1}{4}$ …	10	35) 1960	$\frac{1}{4}$ … 14
$\frac{1}{8}$ …	5		$\frac{1}{8}$ … 7
		56 true result	
result	35		49

Exercises.

Ex. 1. Three numbers amount together to 120; the 1 st is once and one fourth of the second; and the second is once and one third of the third; what are they? Answer 50, 40, and 30.

Ex. 2. The price of a horse, chaise, and harness, is £ 100; the horse is worth twice the chaise, and the chaise is worth three times the harness; what is the value of each?

Answer £ 60, £ 30, and £ 10.

Ex. 3. In travelling a journey of 250 miles, supposing I increase my pace each day, one third, one fourth, one fifth, and one sixth of the preceding day's journey; how far should I go each day? Answer 30, 40, 50, 60, and 70 miles.

DOUBLE POSITION.

Rule. 1 st. Assume two numbers or quantities, and work with them according to the conditions of the calculation. 2 ndly. If neither of the results answers to the given result, find the error of each, and multiply each assumed number by its opposite error. 3 rdly. If the errors are both greater or both less than the given number, take their difference for a divisor, and take the difference of their products for a dividend; but if one error is greater and the other is less than the given number, take their sum for a divisor, and the sum of their products for a dividend, and in either case the quotient will be the number sought.

EXAMPLE.

To divide £ 200 between three persons, A, B, and C, so that A may have £ 6 more than B, and B £ 8 more than C.

Suppose A's share	£ 60	Suppose A's share	£ 80
B's —	54	B's —	74
C's —	46	C's —	66
£ 40 too little.	£ 160	£ 20 too much.	£ 220

£ 60 × 20 = £ 1200
80 × 40 = 3200
60) 4400

£ 73 6 8	A's share.
67 6 8	B's —
59 6 8	C's —
Proof £ 200 0 0	Amount.

EXERCISES.

Ex. 1. The ages of 3 persons amount to 180 years; the first is 6 years older than the second, and the third is 24 years younger than the second; what is the age of each?

Answer 72, 66, and 42 years.

Ex. 2. What are those unequal weights, to the less of which, if you add 5 lb. the amount will be double the greater weight, but if the same be added to the greater, the amount will be triple the less weight. Answer 3 lb. and 4 lb.

Ex. 3. What is the amount of that sum of money from which if $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ of it be taken, the remainder will be £ 25?

Answer £ 100.

N. B. For many of the purposes to which Double Position is applied, the following Rule is more convenient than the above.

PERMUTATIONS.

Permutation is the varying of the position of the different given things, without varying the number of the whole.

Rule. Multiply the numbers of the position of the given terms into one another.

Example. To find how many permutations or changes can be made with 6 things?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 = 720$ Answer.

EXERCISES.

Ex. 1. How many days can 5 persons be placed in a different position round a table at dinner? Answer 120 days.

Ex. 2. How many changes can be rung upon 10 Bells, and how long would they be ringing one over, supposing 10 changes to be rung per minute?

Answer, Changes 3,628,800
Minutes 362,880 = 252 days.

COMBINATIONS.

Combination is the taking of a less number of things out of a greater, and combining them together without regard to their order.

Rule. Take for a divisor the combined product of the series 1, 2, 3, 4, &c., up to the less number to be taken each time; then take for a dividend the continued product of a series of the same number of times, decreasing by 1 from the greatest number of times out of which the combination is to be made, and find the quotient of the latter product divided by the former, for the number of the combinations required.

Find by trial two numbers as near the true number as convenient; use each in the place of the one required, and find each result.

Then say, by proportion, as the difference of these results is to the difference of the two assumed numbers, so is the difference between the true result given and either of the former, to a correction which is to be applied, in addition or subtraction, to the assumed number whose result is used.

Thus, with the example, the difference of the results being 60, the difference of the assumed numbers being 20, and the difference between the true result (200) and the first result (160) being 40, we say, as 60 is to 20 so is 40 to 13 and one third, which, added to 60, gives A's share.

EXAMPLE.

To find how many combinations of 4 at a time, can be made out of 24 Letters of the Alphabet?

$1 \times 2 \times 3 \times 4 = 24$

$24 \times 23 \times 22 \times 21 = 255024$

$$24\ \big)\ 255024$$

$$\underline{10626}\quad \text{Number of combinations.}$$

EXERCISES.

Ex. 1. How many combinations can be made of 6 letters out of 12?

Answer 924 combinations.

Ex. 2. In how many ways can 4 men be taken for sentinels out of a company of 60 men?

Answer 487635 ways.

COMPOSITIONS.

Composition is the taking of a given number of things out of as many equal rows or sets of different things, one out of each row, and combining them together.

Rule. Multiply the whole number in each set as many times in succession together as there are sets of things.

EXAMPLE.

To find how many compositions can be made in 4 sets of 9 things each, so as to have 4 each time, viz. 1 out of each set.

$9 \times 9 \times 9 \times 9 = 6561$ Answer.

EXERCISES.

Ex. 1. In 6 sets of things of 10 in each, how many compositions can be made taking 6 each time, viz. 1 out of each set?

Answer 1000000.

Ex. 2. How many compositions can be made with the numbers on 6 dice of 6 faces each.

Answer 46656.

THE END.

A continuation of this work has been published under the following title:

AN APPENDIX

TO THE

COMMERCIAL ARITHMETIC,

EXHIBITING THE METHODS EMPLOYED BY MERCHANTS, BANKERS, AND BROKERS,

FOR THE VALUATIONS OF MERCHANDISE,

MENTAL PER CENTAGES,

INTEREST ACCOUNTS IN ACCOUNTS-CURRENT,

THE PUBLIC FUNDS, MARINE INSURANCES,

THE STANDARDING OF GOLD AND SILVER, THE

ARBITRATIONS OF EXCHANGE IN BILLS, BULLION, AND MERCHANDISE,

AND ACTUAL AND PRO-FORMA STATEMENTS OF

BRITISH AND FOREIGN INVOICES AND ACCOUNT-SALES.

A MANUAL OF FOREIGN EXCHANGES, in the direct, indirect, and cross operations of Bills of Exchange; including an extensive investigation of the Arbitrations of Exchange in Bills, Coin, and Bullion. Second Edition. By W. Tate.

KEYS to the above.

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INTRODUCTION.

The calculations here arranged as an Appendix to the "Commercial Arithmetic," are intended for the use of those young persons, who have gone through a general course of Arithmetic, particularly from that work; and consequently considering them to be well grounded in the elementary principles, I have not deemed it necessary to give much explanation of the methods in which the calculations are performed: but as a clear conception of the manner in which the results of the different parts of compound operations are to be applied, evidently depends upon an intimate acquaintance with the commercial regulations appertaining to the subject, I have entered at some length into the requisite details, in the hope of giving them not only that assistance which will facilitate their calculations, but also such full, and I believe I may confidently say such accurate, ideas of the practices of business, as will be of great service to them when they enter into active life.

Most of the Rules, it will be found, are either new, or are newly arranged in order to adapt them to the existing arrangements of mercantile affairs. Some of those used for the Valuations of Merchandise, are very useful in themselves, as being greatly abbreviated modes of working such calculations: they are also intended to have the effect of hereafter stimulating young persons to a similar consideration of the other classes of operations, that are peculiar to the line of business in which they may be placed. The Mental Per Centages have the same advantages, and are designed for a like purpose. In showing the different modes of making up Interest Accounts in Accounts-Current, I have explained a very valuable method of calculating the Interest forwards from the day of settling the last account, instead of the usual one of reckoning the same backwards from the day on which the present account is made up. This method has for some time past been used by foreign accountants, though it has scarcely been employed in this country, probably from its not being very easy to understand the principle upon which the adjustment of the Interest on the balance is to be applied; but I trust that the explanation I have given will remove this difficulty, and bring it, as it deserves, into more general use. In the branches of the Public Funds, Marine Insurances, and the Standarding of Gold and Silver, some particulars will be found that will be both amusing and instructive to the young student. The information they convey has been drawn from the best practical sources, and may therefore be depended upon for its strict coincidence with the usages of those departments. The remaining part of this Appendix is occupied with Exchange operations, and the forms and purposes of real and simulated mercantile accounts. The former of these subjects affords very good practice to the learner in the use of the Chain Rule, and will also be an easy introduction to a branch of calculations, that are daily becoming more and more necessary to be well understood by those who wish to make an efficient preparation for the employments of the Counting-house. The latter will be useful for initiating the pupil into the combined practice of those calculations of which, in the other parts of this work, he had only detached portions; while the explanations prefixed to the Foreign Invoices and Account-Sales will give him a great insight into the mercantile usages of other countries, and of the manner in which, from pro-formâ statements of the cost and charges, the arbitrated prices of merchandise are practically obtained.

The products to the exercises have been given at as much length as could consistently be done; and for the aid of those who may study this Arithmetic without the assistance of a Master, I have prepared a Key to the whole of the calculations in this Appendix and the Commercial Arithmetic, and I beg to refer to it for several modes of solution that are not generally known, but which, as they arise out of particular circumstances, I could not conveniently introduce amongst the General Rules in the body of the work.

W. TATE, Junr.

Commercial Academy,
Queen Street, London.

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PRACTICE.

RULE

For valuations by Dozens.

Take as many shillings per dozen as there are pence for each, and reckon 6 d. for ½ d. each, 3 d. for ¼ d. each, 1½ d. for ⅛ d. each, and 3 farthings for 1 sixteenth each.

EXAMPLE 1.

To find the amount of 89½ yds. at 7⅜ d. per yard.

£	s.	d.	
	7	4½	for 1 dozen
		7	
2	11	7½	for 84
	3	0⅞	- 5
		3⅝	- ½
£ 2	15	0	

89½ yards are 7 dozen 5½ yards.

For 7⅜ d. each, we take 7 s. and 4½ d., or 3 times 1½ d., per dozen.

EXERCISES.

Find the amount of

				£	s.	d.
Ex. 1.	62½ yards at	8½ d.	*Product*	£ 2	4	3
2.	37¼ yards at	9⅛ d.	—	1	8	4
3.	56½ yards at	10⅜ d.	—	2	8	10
4.	75¾ yards at	11¾ d.	—	3	14	2
5.	99⅛ yards at	14½ d.	—	5	19	9
6.	108⅜ yards at	15¼ d.	—	6	17	9
7.	113⅝ yards at	17¾ d.	—	8	8	1
8.	139⅞ yards at	19¼ d.	—	11	4	5
9.	145¼ yards at	$7\frac{3}{16}$ d.	—	4	7	0
10.	158½ yards at	$9\frac{7}{16}$ d.	—	6	4	8

RULE

For valuations by Scores.

Take as many pounds as there are shillings for each, and reckon 1 s. 8 d. per dozen for 1 d. each.

	d.		s.	d.		d.		s.	d.	
thus	1	each is	1	8	per score	1½	each is	2	6	per score
	2	—	3	4	—	3	—	5	0	—
	4	—	6	8	—	4½	—	7	6	—
	5	—	8	4	—	6	—	10	0	—

EXAMPLE 2.

To find the amount of 217½ yards at 5 s. 4½ d. per yard.

	£	s.	d.	
	5	7	6	per score
			10	
	53	15	0	for 200 yards
	2	13	9	- 10 —
	1	6	10½	- 5 —
		13	5¼	- 2½ —
£	58	9	0¾	

EXERCISES.

Find the amount of

							£		
Ex. 1.	31½ yards at	2 s.	3 d.	*Product*	£	3	10	10
2.	47¼ yards at	3 s.	6 d.	—		8	5	4
3.	65¾ yards at	4 s.	9 d.	—		15	12	4
4.	86½ yards at	6 s.	4½ d.	—		27	11	5
5.	91⅛ yards at	7 s.	6 d.	—		34	3	5
6.	101⅜ yards at	8 s.	10½ d.	—		44	19	8
7.	143⅞ yards at	10 s.	10 d.	—		77	18	8
8.	162⅛ yards at	13 s.	5 d.	—		108	15	2

EXAMPLE 3.

The combination of the principles of the two preceding Rules.

To find the value of 277½ yards at 10⅜ d. per yard.

277½ = 240 and 37½.

		£	s.	d.		
10⅜ £	=	10	7	6	value of	240 yards.
10⅜ s. × 3	=	1	11	1½	—	36
				10⅜	—	1
				5⅛	—	½
		£ 11	19	11		

The value of 240 yards, &c., at any number of pence per yard, &c., is evidently an equal number of pounds; and consequently the value of the same number at any fraction of a penny, gives the same fraction of a pound.

Therefore, according to this mode of calculation, the number of pence in the price being reckoned as so many pounds, (that is, the value of 240,) is to be multiplied by whatever number of times 240 may be contained in the given number of the quantity to be valued, and then the valuation of the remainder is to be made in dozens, or scores, and odd yards, &c.

EXERCISES.

Find the amount of

Ex. 1.	246½ yards at 4¼ d.	*Product*	£ 4	7	4
2.	251½ yards at 5½ d.	—	5	15	3
3.	268¾ yards at 6¾ d.	—	7	11	2
4.	482¼ yards at 8⅜ d.	—	16	16	7
5.	503½ yards at 9⅝ d.	—	20	3	10
6.	722¾ yards at 11⅞ d.	—	35	15	3
7.	737½ yards at 17¼ d.	—	53	0	2
8.	969¾ yards at 19½ d.	—	78	15	10

RULE

For the valuation of cwt. qrs. and lbs., at 20 s. per cwt.

Reckon every cwt. as 1 £, every quarter as 5 s., and double the number of the lb. as pence; with the addition of 1 d. for every 7 lb., and 14-100 ths of a penny for the remaining lb.*

If great precision is required, there may be 1 part added in for 2, 3, 4, or 5 lb., and 2 parts for 6 lb.

EXAMPLE 4.

To find the amount of 73 cwt. 1 qr. 17 lb. at 47 s. 6 d.

	£	s.	d.	
¼ ...	73	8	0.43	for 20 s.
			2	
	146	16	0.86	- 40 s.
½ ...	18	7	0.11	- 5 s.
	9	3	6.05	- 2 s. 6 d.
	£ 174	6	7.02	

This valuation as 20 s. per cwt. is thus made: 73 cwt. give 73 £, twice 17 d. with 2 d. (because there are 2 seven pounds in 17 lb.) give 36 d., or 3 s., which with 5 s. for 1 qr. give 8 s.; and as there are 3 lb. above the 2 sevens, or 14 lb., we add 3 times 14 parts, with 1 more, and obtain, for the amount, the above valuation.

Ex.						
1.	73 cwt. 1 qr. 16 lb. at	45 s.	*Prod.* £	165	2	8.15
2.	111 cwt. 2 qrs. 21 lb. at	62 s. 6 d.	—	349	0	5.62
3.	304 cwt. 3 qrs. 23 lb. at	57 s. 6 d.	—	876	14	11.20
4.	87 cwt. 0 qr. 13 lb. at	78 s.	—	339	15	0.66
5.	319 cwt. 1 qr. 19 lb. at	105 s.	—	1676	19	0.72
6.	246 cwt. 3 qrs. 25 lb. at	59 s. 9 d.	—	737	16	7.79
7.	193 cwt. 3 qrs. 22 lb. at	117 s. 6 d.	—	1139	8	8.44

N. B. The products of these Exercises are here put down in the 100 ths of the amount, in order to guide the learner; but for purposes of business the parts, if above 50, are called another penny.

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* At 20 s. per cwt., or 15 d. for 7 d., 1 lb. is worth $2\frac{1}{7}$ d., or 2 pence and 14-100 ths of a penny. The addition of 1 or 2 extra 100 ths is made in order to allow for the remaining fraction of 2-7 ths of 1-100 th in the value of each pound at 20 s. per cwt.

RULE

For the valuation of tons cwt. qrs. and lb. at 20 s. per ton.

Reckon every ton as 1 £, every cwt. as 1 s., every qr. at 3 d., and every 7 lb. as ¾ d.

EXAMPLE 5.

To find the amount of 37 tons 17 cwt. 3 qrs. 14 lb. at £ 24 10 per ton.

£	s.	d.
37	17	10½
		4
151	11	6
		6
909	9	0
18	18	11¼
£ 928	7	11¼

N. B. When the lb. are given otherwise than in 7 lb. 14 lb. or 21 lb., the preceding Rule for cwt. may be used, if the shillings in the price per ton make a fraction of £, for which a corresponding fraction can be taken of a shilling:* thus,

3 tons 13 cwt. 1 qr. 17 lb. at £ 47 10 s. per ton, are
73 cwt. 1 qr. 17 lb. at s. 47 6 d. per cwt.

EXERCISES.

Find the amount of

Ex. 1. 111 tons 17 cwt. 3 qrs. at £ 22 per ton.
2. 67 tons 15 cwt. 1 qr. 14 lb. at £ 13 15 0 per ton.
3. 134 tons 11 cwt. 3 qrs. 7 lb. at £ 17 10 0 per ton.
4. 82 tons 14 cwt. 2 qrs. 21 lb. at £ 35 5 0 per ton.
5. 104 tons 17 cwt. 1 qr. 15 lb. at £ 64 10 0 per ton.
6. 44 tons 10 cwt. 3 qrs. 23 lb. at £ 45 15 0 per ton.

PRODUCTS.

Ex. 1.	£ 2461	10	6	Ex. 2.	£ 931	16	5	Ex. 3.	£ 2355	6	9
4.	2916	7	9	5.	6764	1	3	6.	2038	1	2

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* Or, in the valuation, 1-8 th of a penny may be reckoned for every lb. above 7, 14, or 21 lb. with the addition in the product of 1-8 th of a penny for every £ 8 in the price.

RULE

For valuation of oz., dwts., and grains Troy, at 20 s. per oz.

Reckon every oz. as 1 £, every dwt. as 1 s., and half the number of grains as so many pence.

EXAMPLE 6.

To find the value of 341 oz. 11 dwts. 17 grains of Gold, at £ 3 17 10½ per oz.

	£	s.	d.	
$\frac{1}{10}$ . . .	341	11	8½	amount at 20 s.
			4	
	1366	6	10	for 4 £
$\frac{1}{16}$ . . .	34	3	2	- 2 s.
	2	2	8¼	- 1½ d.
	£ 1330	0	11¾	

EXERCISES.

Find the value of

Ex. 1. 67 lb. 11 oz. 17 dwts. 11 grs. of Gold at 77 s. 6 d. per oz.
2. 24 lb. 0 oz. 15 dwts. 17 grs. of Gold at 77 s. 9 d. per oz.
3. 35 lb. 10 oz. 12 dwts. 22 grs. of Gold at 80 s. 0 d. per oz.

4. 167 lb. 5 oz. 18 dwts. of Silver at 60½ d. per oz.
5. 231 lb. 6 oz. 16 dwts. of Silver at 60⅛ d. per oz.
6. 142 lb. 10 oz. 11 dwts. of Silver at 59⅝ d. per oz.

PRODUCTS.

Ex. 1.	£ 3161	10	2	Ex. 2.	£ 1122	13	1
3.	1722	11	8	4.	506	13	3
5.	696	2	11	6.	425	19	2

Rule

for finding the amount of a Rate per £.

Take the given sum as the amount at 20 s., and take parts for the given rate.

Example 7.

To find the amount of £ 679 9 4 at 4 s. 10½ d. per £.

	£	s.	d.	
$\frac{1}{4}$ ...	679	9	4	amount at 20 s.
$\frac{1}{40}$ ...	169	17	4	for 5 s.
	4	4	11	- 1½ d.
	£ 165	12	5	

Exercises.

Find the amount of

Ex.									Prod.		
1.	£ 187	10	6 at	1 s.	6	d. per £.	—	£ 14	1	3
2.	£ 143	6	3 at	2 s.	6	d. per £.	—	17	18	3
3.	£ 317	19	10 at	3 s.	4	d. per £.	—	53	0	0
4.	£ 222	12	6 at	7 s.	10	d. per £.	—	87	3	11
5.	£ 148	15	4 at	11 s.	9½	d. per £.	—	87	14	2
6.	£ 923	16	9 at	18 s.	9	d. per £.	—	866	1	11

Rules

for finding the amounts of Rates per cent.

Rule 1. Take parts for the rate out of the given amount.

2. Multiply the variable term of the rate in pounds, by the number of the hundreds of pounds in the given sum, and take parts for the remainder out of £ 100.

3. Multiply the given sum by the variable term of the rate, and divide the product by 100

EXAMPLE 8.

To find the amount of £ 649 at 8¾ per cent.*

	£				
$\frac{1}{10}$ ...	649	amount at 100 per cent.			
½ ...	32	9	0	—	5
½ ...	16	4	6	—	2½
	8	2	3	—	1¼
	£ 56	15	9		

Or take 10 per cent., and subtract 1-8 th for 1¼ per cent.

EXERCISES.

Find the amount of

Ex. 1.	£ 452	18	6 at 10	per cent.	*Prod.*	£	45	5	10
2.	£ 237	12	2 at 7½	per cent.	—		17	16	5
3.	£ 448	19	10 at 62½	per cent.	—		280	12	5

EXAMPLE 9.

To find the amount of £ 652 10 at 65⅜ per cent.

	£	s.	d.		
½ ...	65	7	6	amount of £ 100	
			6		
	392	5	0	—	600
$\frac{1}{20}$ ...	32	13	9	—	5
	1	12	8¼	—	2 10
	£ 426	11	5¼		

EXERCISES.

Find the amount of

Ex. 1.	£ 620	at £	11	11	0 per cent.	*Prod.*	£	71	12	2
2.	£ 475	at £	8	13	3 per cent.	—		41	2	11
3.	£ 990	at £	6	17	11 per cent.	—		68	5	4

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* The learner is to observe, that in Rates per cent. without any valuations being given, the two terms are of the same value: thus, 8¾ per cent. may be 8¾ of a £ out of £ 100; or 8¾ s. out of 100 s.: and so with any other quantity, as 8¾ yards out of 100 yards: but when a value is given to the variable part of the Rate, if it is in money, it always is intended to be out of £ 100; as 12 s. 6 d. per cent. means 12 s. 6 d. out of £ 100. In any other case both terms of the Rate are valued.

EXAMPLE 10.

To find the amount of £ 1187 10 6 at 12⅜ per cent.

	£	s.	d.
¼...	1187	10	6
			12
	14250	6	0
½...	296	17	7
	148	8	9
	14695	12	4
	£ 146	19	1

Calculations of this sort, which, properly, are calculations by the Rule of Three, are frequently better done by decimals, particularly when the multiplier is a large number.

In dividing by 100, we use the Rule, page 50, in the Introduction.

N. B. This Example and the following Exercises may be worked also by the method of Example 8.

EXERCISES.

Find the amount of

Ex. 1. £ 877 at 11 guineas per cent.
2. £ 472 at 5½ guineas per cent.
3. £ 846 14 4 at 2⅝ per cent.
4. £ 467 at 92½ per cent.

PRODUCTS.

Ex. 1. £ 101 5 10 Ex. 2. £ 27 5 2 Ex. 3. £ 22 4 6
Ex. 4. £ 431 19 6

PARTICULAR RULE

For the per centage in shillings and pence.

Multiply the principal by the number of the shillings, and take parts for the pence out of a shilling; then consider the pounds in the product as so many shillings, and divide them by 100, reducing the remainder (or the figures cut off) into pence.

EXAMPLE 11.

To find the amount of £ 1733 13 8 at 13 s. 9 d. per cent.

	£	s.	d.	
	1733	13	8	
			14	
	24271	11	4	for 14 s.
subtract	433	8	5	- 3 d.
s.	238,38	2	11	
d.	4,56			

Amount 238 s. 4½ d. called £ 11 18 5.

We here consider the £ 23838 as so many shillings, and dividing by 100, we reduce the remainder 38 s. into 456 pence, which we also divide by 100: there is no necessity for putting down the latter work, as only the pence in the product need to be expressed.

N. B. No notice is generally required to be taken of the remaining shillings and pence in the product of the principal, but if any great precision is required, the whole of the part cut off, (as the £ 38 2 11,) should be multiplied by 12, in the common way, and the pounds in the product should be reckoned as so many pence, and divided by 100.

In general, the shillings and pence in the given principal are rejected if less than 10 s., and are reckoned as another pound if above that sum.

EXERCISES.

Find the amount of

			£		
Ex. 1.	£ 837 at 11 s. per cent.	*Product*	4	12	1
2.	£ 2173 at 12 s. 6 d. per cent.	—	13	11	8
3.	£ 478 14 6 at 7 s. 7 d. per cent.	—	1	16	4
4.	£ 988 17 11 at 12 s. 7½ d. per cent.	—	6	4	10

MENTAL CALCULATIONS OF PER CENTAGES.

TO CALCULATE 10 PER CENT.

For the Pounds—Cut off the unit figure of the pounds in the principal, and the remaining figures will show the number of the pounds in the product.

For the Shillings—Take twice as many shillings as there are units of pounds, and add 1 s. to the product for 10 s. in the principal.

For the Pence—Reduce the given units of shillings into pence, take in the given pence, and divide the amount by 10 by separating the unit figure.

EXAMPLE.

To find the amount of £ 877 16 7 at 10 per cent.
Product £ 87 15 7.9-10 ths.

We here say £ 877 produce £ 87, and 7 over; then, twice 7 are 14 and 1 for the 10 s. are 15 s.; lastly, 12 times 6 are 72 and 7 are 79 pence, making 7 pence and 9-10 ths.

EXERCISES.

Ex 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
600	0	0	550	16	0	47	14	8
800	0	0	432	12	6	99	19	11
450	0	0	84	4	9	383	16	3
672	10	6	561	8	2	57	14	10
844	14	4	97	16	10	954	15	5
927	6	11	742	5	4	77	17	7

Products, being the amount of the

Ex.								
1.	Principals £	4294	11	9	Per Centages £	429	9	2.1
2.	—	2469	3	7	—	246	18	4.3
3.	—	1621	18	8	—	162	3	10.4

TO CALCULATE 5 PER CENT

For the Pounds and the Shillings—Consider the pounds in the principal as so many shillings, and value them in pounds.

For the Pence—Multiply the shillings by 6, take in half the number of the given pence, and divide the amount by 10; or, reduce the unit of shillings into pence, take in the given pence, divide by 20, and to the quotient add 6d. for 10 s. in the principal.

EXAMPLE.

To find the amount of £ 657 19 11 at 5 per cent.

Product £ 32 17 11.95-100 ths.

Taking the 657 as shillings, we obtain the £ 32 17; taking 19 s. 11 d. we put down 5 hundredths, because there is an odd penny, and carry 5 for the half of 11; then we say, 6 times 9 are 54 and 5 are 59, put down 9-10 ths and carry 5; 6 times 1 are 6 and 5 are 11.

Or the pence and parts may be obtained thus; 9 s. 11 d. are 119 pence, 20 in 119 gives 5 and 19 over; set down the 19-20 ths, and to the 5 pence add 6 pence because there is an odd pound; the 19-20 ths multiplied by 5 or divided by 2 may be set down as 95-100 ths.

EXERCISES.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d	£	s.	d.
400	0	0	520	10	6	337	15	7
67	10	0	44	17	10	67	14	2
844	6	0	536	13	7	824	7	11
90	0	0	97	16	4	53	10	5
456	17	8	357	19	6	263	13	3
55	12	6	42	0	0	95	17	11

PRODUCTS.

Ex.								
1.	Principals £	1914	6	2	Per Centages £	95	14	3.7
2.	—	1599	17	9	—	79	19	10.65
3.	—	1642	19	3	—	82	2	11.55

TO CALCULATE 2½ PER CENT.

For the pounds—Cut off the unit figure of the given pounds, and divide the other figures by 4.

For the shillings—Take half the number of the pounds remaining from the division of the pounds.

For the pence—Multiply the shillings by 3, and take in one fourth of the number of the pence; then divide the product by 10, and add in 6 d. if there should be an odd pound in the principal; or, reduce the units of shillings and pence into pence, divide by 40, and add in 3 d. for 10 s., and 6 d. for an odd pound.

Example.

To find the amount of £ 879 17 11 at 2½ per cent.

Product £ 21 19 11.375.

We here say, beginning with 879, 4 into 87 produces £ 21 and 3 over; the half of 39 is 19 and 1 over; and thus we get £ 21 19; then for the pence and parts we say, 3 times 17 are 51 and 2.75 d. the fourth of 11 d. are 53.75, which divided by 10 produces 5.375 d. and with 6 d. for the odd pound, produces 11 d. and 375-1000 ths of a penny.

Or for the pence, we may say, 7 s. 11 d. are 95 pence, 40 in 95 gives 2 and 15 over, set down 15-40 ths and carry 2, then 2 and 3 (for 10 s.) are 5, and 6 (for the odd pound) are 11, making 11 pence and 15-40 ths, or dividing the 40 ths by 4 375-1000 ths.

Exercises.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
448	10	0	747	18	6	937	10	4
97	12	4	4157	10	2	5200	7	3
346	7	8	826	6	10	732	16	11
83	10	2	7157	14	5	73	13	5
459	6	10	926	17	7	624	9	7
92	16	0	5375	18	10	5162	17	9

Products.

Ex.		Principals £			Per Centages £		
1.	Principals	£ 1528	3	0	Per Centages £ 38	4	0.9
2.	—	19192	6	4	— 479	16	1.9
3.	—	21031	15	3	— 525	15	10.575

TO CALCULATE 1 PER CENT.

For the pounds—Separate the units and tens of pounds in the principal, and the other figures will show the required number of pounds.

For the shillings—Consider the tens and units of pounds as shillings, and divide them by 5.

For the pence—Consider the remaining pounds as so many tens, and half the given number of shillings as so many farthings, subtracting 1 from 12 to 37 farthings, and 2 when the farthings are above that number.

EXAMPLE.

To find the amount of £ 1677 15 2 at 1 per cent.

Product £ 16 15 6¾.

The £ 1600 produce £ 16; £ 77, considered as shillings and divided by 5, produce 15 s. and 2 over; which, with 8, for half the number of the shillings, being reckoned 28, produce 27 farthings, or 6¾ d.

N. B. Instead of dividing the tens and units of shillings we may take double the number of the tens and add in 1, if the units of pounds are 5, or above: thus, for 77, we may take twice 7 (in the tens) are 14, and 1 (for 5 out of 7 in the units) are 15 and 2 over.

EXERCISES.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
600	0	0	475	12	8	1347	16	11
750	0	0	1640	18	2	872	19	2
812	14	6	87	14	4	5679	14	5
734	18	8	583	15	6	937	17	11
512	15	7	1444	13	2	6732	9	10
887	13	4	92	16	5	881	5	7

PRODUCTS.

Ex. 1.	Principals	£ 4298	2	1	Per Centages	£ 42	19	7½
2.	—	4325	10	3	—	43	5	2½
3.	—	16452	3	10	—	164	10	5½

TO CALCULATE ½ PER CENT.

Cut off the unit figure of the given pounds, and the others will show the per centage on shillings; then consider the units of pounds with half the number of the given shillings, as so many tens and units of 8 ths of pence, deducting 1 out of every 25.

N. B. If the pounds are above 5, reckon 6 d. for the £ 5 and apply the latter part of the rule to the remainder; and when this rule is used for a number of pounds only, we may reckon the units as so many pence, with the addition of 1 d. for 3, 4, 5, 6, and 7 pounds, and 2 d. for 8 or 9 pounds.

Example.

To find the amount of £ 887 13 4 at ½ per cent.

Product £ 4 8 9¼.

The 88 tens of pounds produce 88 shillings, £ 5 produce 6 d., and the 2 over with 7 for half the number of the given shillings, reckoned as 26 and divided by 8, produce 3¼ d., making the whole 9¼ d.

Exercises.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
3563	10	0	4474	8	8	1347	16	11
472	15	0	567	15	3	872	19	2
5142	8	4	8283	10	10	5679	14	5
673	16	3	456	17	11	937	17	11
4474	9	11	5375	8	6	6732	9	10
525	18	2	926	19	1	881	5	7

Products.

Ex. 1.	Principals	£ 14852	17	8	Per Centages	£ 74	5	3¼
2.	—	20085	0	3	—	100	8	6
3.	—	16452	3	10	—	82	5	2¾

TO CALCULATE ¼ PER CENT.

Multiply 5 s. by the number of hundreds of pounds, or take 1-4 th of so many pounds, and add to the product as many pence as there are tens of pounds in 6 times the remainder of the principal.

Example.

To find the amount of £ 1837 14 6 at ¼ per cent.

Product £ 4 11 10 6-10 ths.

18 times 5 s., or the 1-4 th of £ 18, are £4 10; we then say, 6 times 14 s. are 84 s. or £ 4 4, carry 4 for £ 4; 6 times 7 are 42 and 4 are 46, set down 6 as 6-10 ths and carry 4; 6 times 3 are 18 and 4 are 22; 22 pence are 1 s. and 10 d., which with the £ 4 10 make the whole product as above.

Exercises.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
540	0	0	1436	10	0	1537	13	8
600	0	0	917	14	2	679	11	2
450	0	0	3276	15	5	86	14	4
875	0	0	828	17	7	9	12	6
225	0	0	9164	15	3	4	16	4
960	0	0	982	13	11	95	11	10

Products.

Ex. 1.	Principals £	3650	0	0	Per Centages £	9	2	6
2.	—	16607	6	4	—	41	10	4
3.	—	2413	19	10	—	6	0	8

TO CALCULATE ⅛ PER CENT.

Multiply 2 s. 6 d. by the number of hundreds of pounds, or take 1-8th of so many pounds, and add to the product as many pence as there are tens of pounds in 3 times the remainder of the Principal.

Example.

To find the amount of £ 6378 14 6 at ⅛ per cent.

Product £ 7 19 5 6-10 ths.

63 times 2 s. 6 d., or the 8 th of £ 63, are £ 7 17 6; then, reserving these shillings and pence, we set down £ 7, and say, 3 times 14 s. are 42 s., or £ 2 2, for which we carry 2 for the £ 2; 3 times 8 are 24 and 2 are 26, set down 6 as 6 tenths, and carry 2; 3 times 7 are 21 and 2 are 23; 23 d. are 1 s. 11 d., which added to the 17 s. 6 d. reserved are 19 s. 5 d.

Exercises.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
840	0	0	1456	10	8	526	17	2
955	0	0	937	5	4	87	13	3
4170	10	0	6162	11	8	927	8	11
637	15	6	713	16	3	55	16	0
3405	10	6	4395	8	4	737	9	5
828	12	3	926	14	10	916	11	2

Products.

Ex. 1.	Principals	£ 10837	8	3	Per Centages	£ 13	10	11
2.	—	14592	7	1	—	18	4	9
3.	—	3251	15	11	—	4	1	3

TO CALCULATE $\frac{1}{10}$ PER CENT.

Reckon the thousands of pounds as so many pounds; divide the number of the hundreds and tens of pounds by 5, and consider the quotient as so many shillings. Consider the remaining pounds as so many farthings, subtracting 1 when they exceed 12, and 2 when they exceed 37.

N. B. Instead of dividing the hundreds and tens of pounds by 5, we may double the number of the hundreds and add in 1 for 5 in the tens of pounds.

Example.

To find the amount of £ 3786 17 10 at $\frac{1}{10}$ per cent.

Product £ 3 15 9.

£ 3000 produce £ 3; the 78 considered as shillings and divided by 5 produce 15 s., and 3 over; and £ 36 17, reckoned 37 parts or farthings, produce, with the deduction of 1, 36 farthings or 9 pence.

Instead of dividing the 78 by 5, we may say, twice 7 are 14 and 1 (for 5 out of 8 with 3 over) are 15.

Exercises.

Ex. 1.			Ex. 2.			Ex. 3.		
£	s.	d.	£	s.	d.	£	s.	d.
3000	0	0	4472	8	8	1347	16	11
1000	0	0	567	15	4	872	19	2
2750	0	0	8283	10	10	5679	14	5
3500	0	0	456	17	11	937	17	11
10000	0	0	5375	8	6	6732	9	10
750	0	0	926	19	1	881	5	7

Products.

Ex. 1.	Principals	£ 21000	0	0	Per Centages	£ 21	0	0
2.	—	20083	0	4	—	20	1	8
3.	—	16452	3	10	—	16	9	1

INTEREST ACCOUNTS

IN

ACCOUNTS-CURRENT.

The Interest upon Accounts-Current or running Accounts is reckoned in various ways.

1 st, With respect to the Principals or the amounts on which the Interest is calculated: in some accounts the Interest is reckoned upon the balances resulting from each operation or variation in the account; but it is the more usual practice to reckon it upon each sum, separately, on each side of the account, and to take the balance of these separate results.

2 ndly, With respect to the time; in some accounts it is reckoned in months and days, but in most mercantile accounts it is reckoned in days only.

3 rdly, When the Interest is reckoned upon the balances, the time is carried forward from one day of taking the balance to the next; but when the Interest is calculated upon the separate debits and credits, the time is commonly reckoned backwards from the day of settling the balance to each of the days in the account; and, in this case, the Interest on the sums which are due beyond the day of settlement, are added to the opposite side of the account, which has the effect of discounting such sums, and rendering them due on the day of the general settlement.*

4 thly, In accounts in which the Interest is reckoned in days; the statements are made out either with the actual amount of the Interest on each sum, or with the Interest numbers formed from the product of the Principals multiplied by the number of the days, calculating the balance of the Interest from the balance of these numbers.

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* Another method of settling such accounts is, by reckoning the Interest forwards, and adjusting the balance, on the plan shown in the third of the following examples.

EXAMPLE 1.

On the 30 th of June, there was due to me, upon an account, a balance of £ 1426 16 4; since which time I have received and paid the following sums, viz. July 10, I received £ 1000—July 18, I paid £ 124 7 6—August 22, I received £ 600—on Sept. 26, I paid £ 200—on Oct. 18, I received £ 400—on Nov. 24, I paid £ 500—on Dec. 1, I received £ 450, and on Dec. 11, I paid £ 500—it is required to find the balance of this account, with Interest at 4 per cent. made up to the 31 st of December.

	£	s.	d.		days	for			against		
June 30	1426	16	4		10	1	11	3			
July 10	1000	-	-	rec'd.							
for	426	16	4		8		7	6			
July 18	124	7	6	paid.							
for	551	3	10		35	2	2	3			
Aug. 22	600	-	-	rec'd.							
against	48	16	2		35	. . . .				3	9
Sept. 26	200	-	-	paid.							
for	151	3	10		22		7	3			
Oct. 18	400	-	-	rec'd.							
against	248	16	2		37	. . . .			1	-	2
Nov. 24	500	-	-	paid.							
for	251	3	10		7		3	10			
Dec. 1	450	-	-	rec'd.							
against	198	16	2		10	. . . .				4	4
Dec. 11	500	-	-	paid.							
for	301	3	10		20		13	2			
Interest	3	17	-	to Dec. 31		Balance		. .	3	17	0
Balance	£ 305	-	10	in my favour		£ 5	5	3	£ 5	5	3

In this mode of making out the account a balance is struck at the time of each receipt and payment; then the Interest is calculated upon the preceding amount for the time between the two dates, and the Interest is entered in the proper column according as the balance is for or against the party. To make the sums in the Interest columns equal, the balance of the Interest is added to the smaller amount.

EXAMPLE 2.

The preceding Account made out in Dr. and Cr. form with Interest Numbers.

Dr.—Mr. W. Ward, Liverpool, his Account Current with James Peacock.—Cr.

£	s.	d.		days.	Int. No.	£	s.	d.		days.	Int. No.
1426	16	4	June 30	184	262.568	1000	-	-	July 10	174	174.000
124	7	6	July 18	166	20.584	600	-	-	Aug. 22	131	78.600
200	-	-	Sept. 26	96	19.200	400	-	-	Oct. 18	74	29.600
500	-	-	Nov. 24	37	18.500	450	-	-	Dec. 1	30	13.500
500	-	-	Dec. 11	20	10.000				Balance of Numbers*		35.152
3	17	-*	Interest at 4 per cent.			305	-	10	Balance in my favour		
2755	-	10			330.852	2755	-	10			330.852

N. B. If the Account is made out in months and days, the amount of each Interest must be expressed instead of the Interest Numbers, as for the first two amounts in each side.

£	s.	d.		m.	d.	£	s.	d.	£	s.	d.		m.	d.	£	s.	d.
1426	16	4	June 30	6	-	28	10	9	1000	-	-	July 10	5	21	18	19	4
124	7	6	July 18	5	13	2	4	11	600	-	-	Aug. 22	4	9	8	11	10

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* 35.152 ÷ 9125 = £ 3 17.

EXERCISE,

to be worked after the three methods of the preceding Examples.

Dec. 31, 1829, there was due to me £ 863 17 10—Feb. 1, 1830, I received on account £ 241 8 8—on March 17, I received £ 150 0 0—on April 11, I received £ 64 13 0, and on April 29, £ 100 0 0—on May 16, I paid £ 300 0 0—on June 14, I paid £ 500 0 0—on July 16, I received £ 600—on Aug. 1, I received £ 307 0 7—on August 23, I paid £ 500—on Sept. 17, I paid £ 357 10 0—on Oct. 11, I paid £ 160 0 0—on Nov. 4, I paid £ 300 0 0, and on Dec. 20, I received £ 233 18 4.—What is the balance of this account, including Interest at 5 per cent. per annum, made up to the 31 st of Dec. 1830.

PRODUCTS.*

By the first method with Interest Numbers.

27.648—27.368—11.800—7.344—5.236—17.632—35.456
8.128—4.422—17.525—25.392—29.232—69.828—14.124
Balance of the Account £ 1325 12 3.

By the second method with days.

Dr. Side.

315.360—68.700—100.000
65.000—37.590—12.960—17.100.

Cr. Side.

80.253—43.350—17.160
24.600—100.800—46.664—2.574
Net Interest £ 41 5 6 Balance £ 1325 12 9.

By the third method, with months and days.

Interests Dr. Side.

£ 43 3 11—£ 9 7 4—£ 13 11 11—£ 8 17 8
£ 5 2 3—£ 1 15 5—£ 2 6 4.

Interests Cr. Side.

£ 10 18 8—£ 5 18 3—£ 2 6 9—£ 3 6 11
£ 13 14 8—£ 6 7 7—s. 7 1.

Net Interest £ 41 4 11 Balance £ 1325 12 2.

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* For the rules for calculating these products, see the Introduction, pages 104 and 106; and also the explanation of the following example.

EXAMPLE 3.

Dr.—Messrs. Prada & Co. of Rio Janeiro, their account with Pepys & Peacock.—Cr.

From June 30 to Sept. 30, 1830.—Interest at 4 per cent. per annum.

£	s.	d.	1830.	days.		Int. No.	£	s.	d.	1830.	days.		Int. No.
118	15	3	June 30	92		10.948	950	-	-	Sept. 4	26		24.700
800	-	-	Sept. 14	16		12.800	600	-	-	— 13	17		10.200
200	-	-	— 23	7		1.400	542	16	10	— 23	7		3.801
500	-	-	— 5	25		12.500	315	12	1	— 22	8		2.528
500	-	-	Oct. 6	6*	3.000		829	-	8	— 4	26		21.554
198	-	-	Dec. 31	92*	18.216		25	6	9	Feb. 16	226		5.650
300	-	-	Oct. 31	31*	9.300		160	-	-	Oct. 26	26*	4.160	
200	-	-	Nov. 7	38*	7.600		100	-	-	Nov. 14	45*	4.500	
100	-	-	Aug. 22	39		3.900				Numbers in *Dr. side			81.316
50	-	-	Sept. 4	26		1.300	10	14	7	Int. to Sept. 30 at 4 per ct.			
800	-	-	Nov. 23	54*	43.200		301	10	3	Balance to their debit			
50	-	-	Sept. 23	7		350							
			Numbers in* Cr. side			8.660							
			Balance of Numbers			97.891							
18	5	11	Commission and Postages										
3835	1	2				149.749	3835	1	2				149.749

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* For the explanation of these numbers, and general remarks on this account, see the following page.

EXPLANATION OF THE PRECEDING ACCOUNT.

This Account-Current of Pepys & Peacock, the party making out the account, commences with the balance in their favour on June 30,[*] and it is made up to Sept. 30.

The days are here reckoned backwards from Sept. 30 th to the respective days on which the sums of money are due before that period, and forwards to the days on which the amounts are due after that day; and each Principal (reckoning 10 s. and above as another £) is multiplied by the number of the days set against it, to form the Interest Number for that Principal.

In this account, the days for the sums not yet due, but for which, notwithstanding, the correspondents Messrs. Prada & Co. are debited or credited, are marked with an asterisk, and the Interest Numbers for these sums are entered in a separate column; but it is to be observed, that it is the practice in business to write these days and numbers in *red ink*, and to enter all the Numbers in one column: then the amount of the numbers in red on each side are entered on the opposite side, (using red ink for the words in italics in this account, but not for the Numbers, and writing the word *red* instead of using the asterisk,) and a balance is struck and entered of these numbers. The balance of these numbers is then valued, by dividing it by 9125, the divisor for 4 per cent., and an entry is made of the amount in the cash column on the opposite side of the account. Whatever further charges or allowances are to be made, are then entered on the proper side of the account, and the final balance of the account is taken.

In closing this account it therefore appears, that there is due to Messrs. Prada & Co. £ 10 14 7 for the balance of the Interest account,, and that after adding in the amount to be charged to them for commission and charges, there is finally a balance of £ 301 10 3 to be carried to their debit in their new account.

Before showing another method of working the Interest Calculations, it seems necessary to explain why the Interest Numbers on sums due in advance of the day to which the account is made up, should be entered on the opposite side of the account.

It being understood that the Interest Numbers are the representatives of the Interest amounts, we have to consider that, 1 st, in the practice of making up accounts to a certain day, the balance of the account is the amount then due.

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[*] There is one item afterwards entered on the Cr. side, which is due previous to the date of the last Balance; viz. £ 26 5 9, which is entered as due to Prada & Co. on Feb. 16, 1830. Such entries occur in business either as corrections to former entries made in error on the opposite side of the account, or as arising out of transactions, which have not come to maturity at the time of taking the previous balance:

2 ndly, by taking off the discount on a sum not yet due for the time in advance of any day, it is made due on that day: thus, if on the 30 th of September a person owes me a sum of money, or in other words, if I have a right to charge his account with any sum which I have engaged to pay for him, say £ 500, on the 6 th of October, I make that sum due on the 30 th of September, by allowing him the Interest on £ 500 for 6 days, which at 4 per cent is £ 3000 divided by 9125, or 6 s. 7 d.; consequently by deducting 6 s. 7 d. from £ 500 due on Oct. 6, I have £ 499 13 5 due on Sept. 30 th.—3rdly, in the practice of mercantile accounts, instead of making a deduction from one side of an account, the same is effected by making the amount that should be deducted an addition to the opposite side of the account; for it is the same in effect, to debit a person for £ 500, and to credit him for 6 s. 7 d., as to debit him for only £ 499 13 5, and hence it is that amount of these numbers (or of the actual Interests, if each is separately valued) is entered on the reverse side of the account to that on which these numbers for days in advance are entered.

Instead of this method of reckoning the days backwards, from the day of the present settlement of the account, a method has recently been introduced, particularly in foreign banking accounts, of reckoning the days forwards from the day on which the last balance was taken, and adjusting this result to the proper result, in the manner of the following Example, which will be explained after showing how this method operates upon the sums in the preceding account.

It may here be remarked, that the days employed in this next account, are either the difference between the days in the last account and the 92 days, the time which this account runs; or, for the days marked with an asterisk, they are 92 days more; and therefore the sum or difference of the two products equals the product of the principal multiplied by 92. Thus, for the second item on the Dr. side of Ex. 3 and 4, which is £ 800 due Sept. 14, in

Ex. 3, we have £ 800 × 16 = 12.800
4, — 800 × 76 = 60.800
The amount is 800 × 92 = 73.600

If the account were for the whole year, the sum or difference would be 365 days.

EXAMPLE 4.

Dr.—Messrs. Prada & Co. of Rio Janeiro, their account with Pepys and Peacock.—Cr.

From June 30 to Sept. 30, 1830.—Interest at 4 per cent. per annum.

£	s.	d.	1830.	days.	numbers.	£	s.	d.	1830.	days.	numbers
118	15	3	June 30	—	——	950	-	-	Sept. 4	66	62.700
800	-	-	Sept. 14	76	60.800	600	-	-	— 13	75	45.000
200	-	-	— 23	85	17.000	542	16	10	— 23	85	46.155
500	-	-	— 5	67	33.500	315	12	1	— 22	84	26.544
500	-	-	Oct. 6	98	49.000	829	-	8	— 4	66	54.714
198	-	-	Dec. 31	184	36.432	25	6	9	Feb. 16	134	*3.350
300	-	-	Oct. 31	123	36.900	160	-	-	Oct. 26	118	18.880
200	-	-	Nov. 7	130	26.000	100	-	-	Nov. 14	137	13.700
100	-	-	Aug. 22	53	5.300				on £293 18 11	92	27.048
50	-	-	Sept. 4	66	3.300	10	14	7	Balance of Int. & Nos.		97.891
800	-	-	Nov. 23	146	116.800	301	10	3	Balance to their debit		
50	-	-	Sept. 23	85	4.250						
			No. in red Cr. side		3.350						
18	5	11	Commis. & Postages								
3835	1	2			392.632	3835	1	2			392.632

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* This number in the Interest column should either be entered in red ink, or be annulled by crossing it with a line when carried to the opposite side.

EXPLANATION OF THE PRECEDING ACCOUNT.

In this account the days are reckoned from June 30 th, the date of the last balance, forwards, to each of the days on which each of the amounts becomes due, (that balance having no Interest to be calculated upon it,) and each Principal is, as before, multiplied by its respective number of days.

In this account one sum on the Cr. side is due on Feb. 16, or before the day of the last balance; the days for this item are therefore calculated backwards from June 30 to Feb. 16, and this alone forms what is called a *red* number to be added to the opposite side, to make it due, like the preceding balance of £ 118 15 3, on the 30 th of June, having also like it no interest or rather discount to be calculated in advance.

When these entries of the Interest numbers have been finished, the *balance of the Cash columns, as they now stand, is taken and multiplied by the number of the days from the day when the account commences to the day on which it is made up, and the product or Interest number is added to the* opposite *side.** *With this correction the balance of the Interest numbers is taken and entered, and the amount it gives of Interest is extended in the Cash column upon the same side of the account, and not, as before, on the opposite side of the account.*

To explain the reason that this method should produce the same effect as the preceding plan, we must observe, that by reckoning the days forwards as in this account from June 30, and applying the number of these days to the respective sums, we, in fact, reckon the Discount on each, and render them due on the day in which the account commences; then, when we strike a balance of the sums, exclusive of the Discount, and calculate the Interest for the days between the two periods of settlement, we add the result to the opposite side, for being Interest, its nature is opposite to that of Discount; this correction therefore settles the difference between the two methods, and the balance of the Interest is found as before, but, for the reason above given, it is entered upon the opposite side of the Interest account.

To show this upon a simple operation, let there be only taken one sum upon each side of an account, viz. £ 1000 on the Dr. side due to me August 2, and £ 700 on the Cr. side due by me Sept. 1.

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* That is, if the amount of the Dr. side of the account is the greater of the two sides, the product is added to the Cr. side.

Dr.	Cr.
£ 1000 due Aug. 2.	£ 700 due Sept. 1.

Then supposing the Interest to be 5 per cent. and the account to be made up to Sept. 30, the Interest on £ 1000 for 59 days will be £ 8 1 8, and the Interest on £ 700 for 29 days will be £ 2 15 7, making the balance in favour of the former £ 5 6 1, and the account according to the 1 st method will stand thus:

£	s.	d.		£	s.	d.	£	s.	d.		£	s.	d.
1000	-	-	Int. 59 ds.	8	1	8	700	-	-	Int. 29 ds.	2	15	7
5	6	1		-	-	-	-	-	-	Bal. of Int.	5	6	1
-	-	-		-	-	-	305	6	1	Bal. of acc.	-	-	-
1005	6	1		8	1	8	1005	6	1		8	1	8

According to the second method, reckoning the account forwards from June 30 to Sept. 30, making 33 days to Aug. 2 for £ 1000, 63 days to Sept. 1 for £ 700, and 92 days to Sept. 30 for £ 300, the balance of these sums, the account will stand thus:

£	s.	d.		£	s.	d.	£	s.	d.		£	s.	d.
1000	-	-	Int. 33 ds.	4	10	5	700	-	-	Int. 63 ds.	6	0	10
5	6	1	Bal. of Int.	5	6	1				Int. on £ 300 92 ds.	3	15	8
-	-	-		-	-	-	305	6	1	Bal. of acc.	-	-	-
1005	6	1		9	16	6	1005	6	1		9	16	6

In the first method there is due to me for Interest £ 8 1 8, and by me £ 2 15 7, leaving a balance in my favour of £ 5 6 1.

In the second method, I allow a discount of £ 4 10 5 on £ 1000, and charge a discount of £ 6 0 10 on £ 700, in order to make these sums due on June 30; I then charge the interest on £ 300, the balance of the Principals, which balance is reckoned due to me from June 30, and the difference £ 5 6 1 is the amount due to me as the balance of this charge and allowance.

The particular advantage of this method in large accounts, is, that the Interest columns, as they are called, can be filled in as soon as the sums and dates are entered, without waiting for any particular day being given, to which the account is to be made up: it can therefore be always kept in readiness, as it requires only one operation to adjust the Interest on the balance to any given day. The same might with propriety also be done by the other method, provided it was absolutely certain that the account would be made up to a certain fixed future day; but of this no one could make sure, and, consequently, in case of circumstances requiring an alteration in the day before fixed, every entry of the Interest would require to be corrected. In general, also, the numbers in red will be avoided.

It is proper further to remark, that from the practice of reckoning 10 s. and above as another pound, and rejecting less sums in the calculation of the Interest, and also from the balance being taken independent of these adjustments, a little variation will sometimes arise between the results of the two methods ; but if the balance be reckoned from the difference in the amounts of the pounds employed, both will exactly agree.

In banking accounts in which Interest is charged upon the debits, and allowed upon the credits, and in which variations almost daily occur in the balances, instead of either of the preceding methods, the following method has lately been adopted, in a large banking house of this metropolis, by which very much of the labour of calculating the Interest on country banking accounts is superseded.

The balances of the daily receipts and payments, or the balances of the debit and credit of the Ledger account, being entered in Dr. and Cr. columns for each month, the total amounts of the columns are taken : then, if the same rate of Interest is allowed as is charged, the balance of the amounts is found, and the Interest is calculated upon it for *one day* at the given rate per cent. ; but if, as is more usual, the rates charged and allowed are different, then the Interest is calculated for one day upon the separate amounts of both the debits and credits, and each amount of Interest, or the balance, is carried to the general account.

It is to be observed, that when any day occurs without having any sum charged, (as always for Sundays,) the preceding day's balance must be repeated for that day, and the same must continue to be done, if necessary, until some entry is made in the account.

It is also to be remarked, that banking accounts of this nature are usually made up every six months, to the last day of June and of December ;* and the Interest, by being charged to the general account, is then made part of the Principal, but that for the sake of convenience we have taken the account as only for three months, or as if the account were opened on Tuesday, the 1 st of April.† The Interest on the debit of the account is reckoned at 5 per cent. per annum, and on the credit at 3 per cent. per annum.

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[*] In the Court of Chancery, the accounts carrying Interest are commonly made up to the same days to which the Interest for the sums paid or received are charged or allowed for the elapsed days; but when the account remains without variation during either of these six months, the Interest is calculated for half a year, and it is legally allowed in these calculations to charge 10 s. and above as another £, any sum under 10 s. being rejected.

[†] The Sundays following form this date and day of the week, are marked by an asterisk in the table of the balances; and the balance of the preceding Saturday is entered to each Sunday as the balance for that day.

	APRIL.		MAY.		JUNE.*	
days	Dr.	Cr.	Dr.	Cr.	Dr.	Cr.
1	3972	—	—	1790	* —	1789
2	2051	—	—	1955	4000	—
3	—	475	460	—	—	628
4	—	1510	* 460	—	—	329
5	727	—	6590	—	500	—
6	* 727	—	2136	—	500	—
7	2075	—	858	—	500	—
8	945	—	—	1120	* 500	—
9	4751	—	—	1120	1734	—
10	1036	—	—	1120	1622	—
11	1036	—	* —	1120	3180	—
12	—	2175	748	—	2246	—
13	* —	2175	3864	—	1828	—
14	—	1150	3142	—	4154	—
15	4310	—	1780	—	* 4154	—
16	515	—	1160	—	1428	—
17	150	—	1673	—	3686	—
18	2127	—	* 1673	—	5932	—
19	330	—	486	—	8644	—
20	* 330	—	—	838	4382	—
21	8192	—	—	1372	5165	—
22	5210	—	—	545	* 5165	—
23	3125	—	3868	—	282	—
24	2160	—	4637	—	—	4781
25	500	—	* 4637	—	—	6383
26	—	954	4637	—	—	6383
27	* —	954	2268	—	—	2100
28	1030	—	2052	—	—	1742
29	—	1075	889	—	* —	1742
30	—	2500	—	692	1429	—
31	—	—	—	1789	—	—
	45299	12968	48018	13461	61031	25877

Dr.	Messrs. Smith & Co.	Cr.
45299	. . . . April	12968
48018	. . . . May	13461
61031	. . . . June	25877
£ 154348		£ 52306
at 5 per cent.		at 3 per cent.
for 1 day £ 21 2 10		for 1 day £ 4 6 0

EXERCISES.

Ex. 2. according to Example 3.

Dr.			1830.	Cr.			
£ 1130	14	6	June 30	£ 515	8	6	Sept. 17
614	8	3	July 17	300	0	0	— 1
500	0	0	Oct. 14	1500	0	0	Aug. 28
200	0	0	Sept. 1	733	9	6	Dec. 11
831	16	7	Nov. 6	822	19	9	Jan. 30, 1831
525	0	0	Dec. 31	17	10	0	Mar. 4, 1831
64	11	4	Jan. 27, 1831	536	18	6	Nov. 22, 1830
100	0	0	Mar. 11, 1831	200	0	0	Feb. 18, 1831

To be made up to December 31, 1830, at 5 per cent.
Adding Commission £ 10 3 7, and Charges £ 2 16 6.

Ex. 3. according to Example 4.

450	15	0	June 30	£ 200	0	0	July 22
600	0	0	Aug. 21	68	16	0	— 11
500	0	0	July 17	372	18	4	Nov. 16
500	0	0	Nov. 30	2165	11	6	— 22
811	14	0	Jan. 22, 1831	316	16	0	Dec. 10
37	9	6	Mar. 16, 1831	182	8	2	— 22
800	0	0	Dec. 31	200	0	0	Jan. 17, 1831
92	19	4	Nov. 14	200	0	0	Feb. 17, 1831
111	18	0	April 16, 1831	631	8	3	Mar. 1, 1831
68	17	8	Mar. 10, 1831	72	4	4	Dec. 28

To be made up to December 31, 1830, Interest 4½ per cent.
Adding Commission £ 14 16 6, and Charges £ 6 5 0.

PRODUCTS.

Ex. 2. Nos. Dr. 208.104 102.538 39.000 24.200 45.760
in red 1.755 7.000
Nos. Cr. 54.075 36.300 187.500 14.660 20.943
in red 24.690 1.134 9.800
Balance of Numbers 132.993 Interest Dr. £ 18 4 5
Balance of Account £ 628 11 1.

Ex. 3. Nos. Dr. 31.200 8.500 76.500 167.272 9.583
147.200 12.741 32.480 17.457
Nos. Cr. 4.400 759 51.847 314.070 51.671
31.850 40.200 46.400 153.964 13.032
Balance 436 × 184 = 80.224 Dr.—Balance of Nos. 125.036
Interest Dr. £ 15 8 4—Balance of Account £ 399 19 3

THE STOCKS.

The Stocks are the capitals upon which the Funded Redeemable Annuities[*] are calculated.

The capitals of the Bank and East-India Company are also called the Stocks of those Companies, and the capitals of the South-Sea Stock and Annuities are the remains of the former intended trading capital of the South-Sea Company.

The half-yearly Annuities are called dividends; they are 1½, 1¾, 2, or 2½ per cent., on 3, 3½, 4, and 5 per cent. Government Stocks, and at present 5¼ per cent. on India Stock, and 4 per cent. on Bank Stock.

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[*] These Annuities are called *funded*, because, formerly, they were payable out of funds created by particular taxes, but they are now paid out of the general national income, or what is termed the Consolidated Fund. The Stocks, or capitals of these Annuities, are also called "the Funds," and putting money into the Bank or into the Funds, is a common expression to denote the purchase of a Government Annuity.

They are also called *redeemable*, because the government retains the right of cancelling them, by paying the owner of the Annuity a sum of money equal to the amount of his Stock.

They also bear different denominations; 1 st, according to the rate at which the Annuity is calculated upon the Stock; as 3, 3½, 4, and 5 per cent. Stocks; 2 ndly, according to the time of their creation, as 3½ per cent., 1818; and, 3 rdly, according to other peculiarities attending them: thus, the Annuities which either upon their creation were charged upon different branches of the revenue, or which were separately classed, but which were afterwards combined, are called Consolidated; and those Stocks which formerly bore a rate higher than the present rate of annuity, are called Reduced.

The Stocks, or rather their Annuities, are called *unlimited*, because the Annuities are engaged to be paid without limitation of their duration, in distinction from the Annuities which are terminable at *limited* periods; as the Old Long Annuities which expire in 1860, and Government Life Annuities, which are granted in exchange for Stock; the latter Annuities are also called *irredeemable*.

The prices of the Stocks are the numbers of pounds in money that are given for £ 100 of each Stock, the variation being made in fractional parts of a half, quarters, or eighths.*

The brokerage on the purchase and sale of Stock,† is ⅛ of a £. or 2 s. 6 d. per cent., on the quantity of the Stock.

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* Thus the quotation of the price of 3 per cent. Consolidated Stock is made this day, May 2 nd 1832, at 84⅞-5, that is, 85; two prices being given, the lower at which it is bought by the Jobbers, and the higher at which it is sold, the difference ⅛ being what is called the turn of the market. When two Stocks bear the same rate of Annuity, and are payable at different periods, as the 3 per cent. Consols and Reduced, the difference in the price is usually equal to one quarter's annuity, that being the lower, of which the dividend was last paid.

† When Stock is sold, it is transferred in the books of the Stock at the Bank, from the credit of the seller's account to the credit of the buyer's, the account of the seller being debited, and the account of the buyer credited, for the amount of Stock sold, which may be for as small a sum as 1 d.; but the lowest amounts of Stock that can produce dividends, are,

5 s. 7 d. in 3 per cent. Stock, 4 s. 10 d. in 3½ per cent. Stock,
4 s. 2 d. in 4 per cent. Stock, 3 s. 4 d. in 5 per cent. Stock,

each sum producing a half-yearly dividend of 1 d.

The transfer books are shut about three weeks before, and remain so until about a fortnight after, dividends become due, during which time no public transfer can be made; but, after the dividends are due, it may be made during the time the books are shut, by an extra transfer, for which a fee is paid. No transfer can be made of Stock in the same day as that on which it is bought, or placed to the credit of an account, excepting by special permission of the directors.

The dividends on 3 per cent. Consols, New 3½ per cent., 3 per cent. 1726, South-Sea Stock, 3 per cent. South-Sea New Annuities, and 3 per cent. 1751, are due on Jan. 5, and July 5.

The dividends are, generally, begun to be paid to the public on the third day after they become due; and they are paid to the party in whose name the Stocks were placed at the time when the books were closed.

The dividends on 4 per cents. 1826, 3 per cent. Reduced, 3½ per cent.., and 3 per cent. Old South-Sea Annuities, are due April 5, and October 10.

Before the year 1800, these days, January 5, April 5, July 5, and October 10, were the Old Christmas, Lady, Midsummer, and Michaelmas days, on which days the dividends were made payable before the alteration of the style.

CASE 1.

To find the cost or the net proceeds of Government Stocks.

Rule. Multiply the quantity of Stock by the rate of the price, and divide the product by 100.

Or, multiply the price of £ 100 Stock by the number of hundreds, and take parts for the rest, if any, out of £ 100.

Add 1-8 th per cent. for Brokerage upon buying, or subtract it upon selling.

Ex. To find the cost of £ 655 Stock at 83½ per cent.

£ 655 × 83½ ÷ 100 =	£ 546	18	6	Value.
£ 655 at 1-8 th per cent. =		16	5	Brokerage.*
	£ 547	14	11	Whole Cost.

EXERCISES.

Ex. 1. Find the cost of £ 1200 Stock bought at 83½.
Ex. 2. Find the cost of £ 750 Stock bought at 79¼.
Ex. 3. Find the cost of £ 875 Stock bought at 85¾.
Ex. 4. What net amount shall I receive from my broker for selling £ 744 of Stock on my account at 91¼ per cent. ?
Ex. 5. What are the net proceeds of £ 564 15 10 Stock sold at 105½.
Ex. 6. What are the net proceeds of £ 823 17 7 Stock sold at 99½.

PRODUCTS.

Ex. 1.	Net Cost	£ 1002	0	0	Whole Cost	£ 1003	10	0
2.	—	594	7	6	—	595	6	3
3.	—	750	6	3	—	751	8	2
4.	Value	678	18	0	Net Proceeds	677	19	5
5.	—	595	17	1	—	595	3	0
6.	—	819	15	2	—	818	14	7

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* N. B. The Brokerage is never less than 1 s. however small may be the sum transferred. Some Brokers charge 1 s. for sums not exceeding £ 25, 1 s. 6 d. for sums from £ 25 to £ 50, 2 s. from £ 50 to £ 75, and 2 s. 6 d. from £ 75 to £ 100.

CASE 2.

To find the quantity of Stock that can be bought with a given sum of money.

Rule. Add $\frac{1}{8}$ for brokerage to the price of the Stock, and say, If this sum produce £ 100 Stock, what will the given sum produce?

EXAMPLE.

To find how much $3\frac{1}{2}$ per cent. Stock can be bought with £ 400, when the price is $98\frac{1}{2}$.

$$98\frac{1}{2} \text{ and } \frac{1}{8} = 98\frac{5}{8}$$

$$\text{If } \overset{\pounds}{98\frac{5}{8}} \text{ ——— } \overset{\pounds}{100} \text{ ——— } \overset{\pounds}{400}\,?$$

Answer £ 405 11 6 Quantity of Stock.

EXERCISES.

Ex. 1. What quantity of 3 per cent. Reduced Stock can be bought with £ 100, when the price is $91\frac{7}{8}$?

Answer £ 108 13 11.

Ex. 2. What quantity of $3\frac{1}{2}$ per cent. Stock can be bought with £ 1000, when the price is $98\frac{5}{8}$?

Answer £ 1012 13 2.

CASE 3.

To find the quantity of Stock that must be sold to produce a given sum of money.

Rule. Subtract $\frac{1}{8}$ for the brokerage from the price of the Stock, and say, If this remainder require £ 100 Stock, what will the given sum require?

EXERCISES.

Ex. 1. What quantity of 3 per cent. Consols must I sell to procure £ 100, when the price is $88\frac{1}{2}$?

Answer £ 113 3 1.

Ex. 2. What quantity of 4 per cent. Consols must I sell to procure £ 854 10, when the price is $105\frac{3}{8}$.

Answer £ 811 17 6.

CASE 4.

To Find the net proceeds of India, Bank, and South-Sea Stocks.

Rule. Make the calculation in the same manner as for Government Stocks, but subtract the expense of making the transfer, together with the brokerage on the sale of the Stock.

viz. Bank Stock under £ 25, 9 s.; above, 12 s.
India Stock under £ 10, 30 s.; above, 34 s.
South-Sea Stock under £ 100, 9 s. 6 d.; above, 12 s.

N. B. There is no expense of transfer to the buyer of these Stocks, excepting, that when the quantity of the Stock is small, the seller, or Jobber, will seldom be at the expense of the transfer except at an equivalent increased selling price.

EXERCISES.

Find the net proceeds of

Ex. 1.	£ 1000 India Stock at 243¼.	Net Proceeds	£ 2429	11	0
2.	£ 500 Bank Stock at 202.	—	1008	15	6
3.	£ 300 South-Sea Stock at 91½.	—	273	10	6

CASE 5.

To find the amount of the half-yearly Dividends on Stock.

Rule. Multiply the quantity of Stock by the rate of the Dividend, and divide the product by 100.

Or, Multiply the rate of the Dividend by the number of hundreds of pounds of Stock, and take parts for the remaining quantity, if there is any.

EXERCISES.

What is each half-yearly Dividend

Ex. 1.	On £ 700 in 3½ per cent. Stock?	*Prod.*	£ 12	5	0
2.	On £ 924 in 4 per cent. Stock?	—	18	9	7
3.	On £ 437 14 6 in 3 per cent. Stock?	—	6	11	3
4.	On £ 1000 India Stock at 5¼ per cent.?	—	52	10	0
5.	On £ 600 Bank Stock at 4 per cent.?	—	24	0	0

CASE 5.

To find the rate per cent. per annum that is made of money by investment in the Funds.

Rule. Find the amount of the proportionate dividend from the day when the last dividend was paid, and subtract it from the given price of the Stock, to find the net price. Then say, by proportion, As the net price is to the rate per cent. of the Annuity, so is £ 100 to the rate per cent. per annum made by the investment.

N. B. In using this rule, it is sufficiently accurate for any purpose for which it is employed to calculate the Interest in months and days of 30 to a month: great precision however would require the days in each half-year of the particular dividends to be taken, and also to reckon the growing dividend from the day when the last was paid.

Ex. To find the rate per cent. per annum made of money invested in the 3 per cent. Consols, purchased on the 15 th of May at 84¾ per cent.

from Jan. 5 to May 15 is 4 months 10 days.
4 months' dividend at 3 per cent = £ 1 0 0
10 days — — 1 8

Growing dividend £ 1 1 8

£ 84 15 less £ 1 1 8 = £ 83 13 4 net price of the Stock.
If £ 83 13 4 ——— £ 3 ——— £ 100?

Answer £ 3 11 8 the rate per cent required.

Otherwise, as the dividends on the 3 per cent Consols are due on the 5 th of January and July, and are paid on the 8 th of those months, and as there are 181 days in the first half-year, (on which the dividend is £ 1 10,) and 127 days from January 8 to May 15, (for which the proportionate dividend is £ 1 1,) the net price is £ 83 14; but if this is used instead of the above, the result will still be £ 3 11 8.

Ex. What rate per cent. per annum is made of money invested in 4 per cent. Consols, purchased on the 27 th July at 98⅞ per cent.? Time 3 months 22 days.

Answer. Grow. d. £ 1 4 10 Rate req. £ 4 1 11

N. B. The dividends on this Stock are due on April 5 and Oct. 10.

LONG ANNUITIES.

Long Annuities are Annuities which have been sold by government, originally, for the period of 99 years; they are due on April 5 and October 10, and they expire Jan. 5, 1860.

The price of these Annuities is the number of years' purchase at which they are sold, as $18\frac{5}{16}$, &c.

The brokerage is 2 s. 6 d. or $\frac{1}{8}$ per cent. upon the amount of the Annuity.

N. B. Besides the *Old* Long Annuities, other Terminable Annuities have recently been created in exchange for Stock, at 30 and 50 years from Oct. 10, 1829, and Jan. 5, 1830.

EXAMPLE.

To find the cost of a Long Annuity of £ 50 at $18\frac{3}{4}$ years' purchase.

£ 50 × $18\frac{3}{4}$ =	£ 937	10	0	Amount
	1	3	6	Brokerage
	£ 938	13	6	Whole Cost

EXERCISES.

Ex. 1. What sum of money shall I have to pay my Broker for purchasing on my account £ 100 per annum, in the Long Annuities, at $19\frac{1}{2}$ years' purchase.

Ex. 2. What is the cost of a Long Annuity for £ 64 12 6 at $20\frac{1}{4}$?

PRODUCTS.

Ex. 1.	Amount	£ 1950	0	0	Whole Cost	£ 1952	8	9
2.	—	1308	13	2	—	1310	5	10

EXCHEQUER BILLS.

Exchequer Bills are a sort of Promissory Notes, which entitle the holder to receive the sums they specify, with Interest from the day of their date to the day on which they are advertised to be paid; which is generally about a year from the day on which they were issued. On the selling of Exchequer Bills, the Interest is calculated to the day on which they are sold.

The present rate of Interest is 1½ d. per cent. per diem, and Commercial Exchequer Bills, or those issued in aid of particular objects, and not for general purposes, bear 2 d. per cent. per diem.

Exchequer Bills are generally at a premium, and the brokerage upon either a purchase or a sale is 1 s. for every £ 100.

N. B. Exchequer Bills of different amounts are printed with ink of different colours; viz.

£ 100 Bills with Red ink — £ 500 Bills with Blue ink
£ 200 — Yellow — £ 1000 — Black.

EXAMPLE.

To find the cost of an Exchequer Bill for £ 1000, dated March 30, and sold May 23, at 70 s. premium.

From March 30 to May 23 is 54 days.

£ 1000 at 1½ d. per cent. is 1 s. 3 d.

	£ 1000	0	0	Bill.
54 days at 1 s. 3 d.	3	7	6	Interest.
70 s. × 10 =	35	0	0	Premium.
		10	0	Brokerage.
	£ 1038	17	6	Whole Cost.

EXERCISES.

Ex. 1. What is the cost of 4 Exchequer Bills of £ 500 each, dated June 27, and bought July 20, at 46 s. premium?

Answer, Interest £ 2 17 6, Whole Cost £ 2049 17 6.

Ex. 2. What are the net proceeds of 2 Exchequer Bills of £ 500 each, and one of a £ 1000, dated March 30, and sold August 22, at 68 s. premium?

Answer, Interest £ 18 2 6, Net proceeds £ 2085 2 6.

INDIA BONDS.

India Bonds are issued by the East India Company, for £ 100, £ 300, £ 500, and £ 1000 each, and bear Interest, at present, at 3 per cent. per annum.

The Interest on India Bonds is reckoned from the 31 st of March, and from the 30 th of September, in months, and in the days of the month in which they are sold.

The brokerage is the same as upon Exchequer Bills; viz. 1 s. upon every £ 100 either purchased or sold.

Example.

To find the net proceeds of 2 India Bonds of £ 500 each, sold on 23 rd May, at 47 s. premium.

From March 31 to May 23 is 1 month 23 days.

£ 1000 for 1 month is £ 2 10 0
1000 for 23 days is 1 17 10
£ 4 7 10

	£	s.	d.	
	1000	0	0	Bonds.
	4	7	10	Interest.
£ 1000 at 47 s. =	23	10	0	Premium.
	1027	17	10	Amount.
		10	0	Brokerage.
	£ 1027	7	10	Net Proceeds.

Exercises.

Ex. 1. What is the cost of an India Bond for £ 1000, bought the 30 th of June, at 40 s. premium?

Ans. Interest for 2 m. 30 d. £ 7 9 4, Amount £ 1027 19 4.

Ex. 2. What are the net proceeds of 4 India Bonds for £ 1700, sold the 13 th of January, at 62 s. premium?

Ans. Int. for 3 m. 13 d. £ 14 11 4, Net proceeds £ 1766 8 4.

MARINE INSURANCES.

Marine Insurances are contracts to indemnify the owners of Goods and Vessels, and the proprietors of Freight, from any stipulated loss they may receive from the loss or destruction of the Ships and Cargoes.

The persons undertaking the risks of Insurances, are called Underwriters.*

The agreements between the assurers and the assured, are called Policies.

The consideration or remuneration charged by the assurer, is called a Premium.

The stamp duty on Policies from one part to another of the United Kingdom is 1 s. 3 d. per cent., if the Premium does not exceed 20 s. per cent.; but above that rate, the stamp duty is 2 s. 6 d. per cent.

On foreign Policies the duty is 1 s. 3 d. per cent. if the Premium does not exceed 15 s. per cent., 2 s. 6 d. per cent. if the Premium does not exceed 30 s. per cent., and 5 s. per cent. if above that rate.

Upon any part of £ 100, the duty is the same as for £ 100.

When a merchant effects an Insurance for a foreign correspondent, his commission is usually ½ per cent. If he procures the settlement of a loss, his charge is 2 per cent. upon the amount recovered.

An Insurance Broker makes no charge to the assured for effecting an Insurance, as he receives a remuneration for his trouble from the companies, or from the underwriters, but for procuring the settlement of a loss he makes a charge of ½ per cent.

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* These Insurances can be undertaken, on joint account or in partnership, only by the chartered companies, as the Royal Exchange, the London, the Mutual, the Alliance, &c. Assurance Companies, and the subscription of any other partnership name to a policy, is an illegality that would subject the person making it to a penalty of £ 500.

PART I.

Premiums and Charges of Insurance.

Rule. Find the amount of the given rate per cent. calculated upon the given capital, and add to it the amount of the charge for the Policy stamp,* with also the commission and del-credere if charged.

Risks not Foreign.

Ex. 1. What is the amount of the Insurance upon £ 1200, from Dublin to London, the premium being 15 s. per cent.?

Ex. 2. What is the amount of the Insurance on £ 1720, from Bristol to Guernsey, at 10 s. per cent.?

Ex. 3. What is the amount of the Insurance on £ 1285, from London to Aberdeen, at 12 s. 6 d. per cent.?

Foreign Risks.

Ex. 4. What is the amount of the Insurance on £ 1640, from London to Hamburgh, at 15 s. per cent.?

Ex. 5. What is the amount of the Insurance on £ 2000, from Gibraltar to London, at 2 guineas per cent.?

Ex. 6. What is the amount of the Insurance on 10000 Dollars, valued at £ 2250, from Vera Cruz to London, at 2¼ guineas per cent., with a commission of ½ per cent., and a charge for del-credere of ½ per cent.

Answers.

Ex.												
1.	Prem.	£ 9	0	0	Policy	£ 0	15	0	Amount	£ 9	15	0
2.	—	8	12	0	—	1	2	6	—	9	14	6
3.	—	8	0	7	—	0	16	3	—	8	16	10
4.	—	12	6	0	—	1	1	3	—	13	7	3
5.	—	42	0	0	—	5	0	0	—	47	0	0
6.	—	59	1	3	—	5	15	0	—	87	6	3

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* In all the following calculations the amount charged for the Policy is only that of the stamp duty; but it is to be observed, that most brokers charge an additional sum, as 5 or 10 per cent. for the advance of money for the Policy stamps.

PART II.

Returns of Premium.

CASE 1.

When the risk does not in any part take place, there is a return of Premium upon cancelment of the Policy.

Rule. From the rate per cent. of the Premium, subtract the brokerage, and the remainder will be the net rate per cent. of the return.

N. B. The brokerage is 1 s. in a guinea when the rate of the Premium is in guineas, and 1 s. in a pound when the rate is in pounds.

EXAMPLE.

To find the amount of the return of Premium upon £ 850, insured upon goods at 3½ guineas per cent., but which risk did not take place.

N. B. The Premium being 3½ guineas per cent., the net rate of the return of Premium is £ 3½ per cent.[*]

£ 850 at £ 3 10 per cent. = £ 29 15 Amount of the return.

EXERCISES.

Ex. 7. What is the amount of the return of Premium on goods insured at £ 1000, at 20 guineas per cent., which goods were not shipped?

Ex. 8. What is the amount of the return of Premium on goods insured at £ 1000, at £ 21 per cent., which risk did not take place?

PRODUCTS.

Ex. 7.	Rate of return	£ 20	per cent.	Amount	£ 200
8.	—	19 19	—	—	199 10

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[*] N. B. It is sometimes an agreement in the Policy, that if the risk does not take place, the underwriters shall retain a part of the premium, as ½ or ¼ per cent.: thus, if, in the above Example, ½ per cent. had been allowed to be retained, the return would have been at the rate of £ 3 per cent.

CASE 2.

When there is an over-insurance, the Premium is to be returned on the amount over-insured.

Rule. Find the amount of the Short Interest at the net rate per cent. of the return; and, as this amount is per-centaged among the underwriters, find the rate per cent. of this amount upon the whole amount insured, and then find the amount of the sum insured at this rate per cent.

EXAMPLE.

To find the amount of the return of Premium for over Insurance upon a Policy of Insurance effected on 40 hhds. of Sugar, valued at £ 25 per hhd., at 4 guineas per cent., only 32 hhds. having been shipped.

40 hhds. insured at £ 25 each . .	£	1000	0	0
32 — shipped		800	0	0
Short Interest	£	200	0	0

£ 200 at 4 per cent. £ 8 0 0.
If £ 1000 pay £ 8 what will £ 100?
Answer 16 s. rate per cent.
£ 1000 at 16 s. per cent. = £ 8 0 0.

EXERCISE 9.

What is the amount of the return of Premium for over Insurance, upon a Policy of Insurance for £ 3000, at 60 s. per cent.; the Interest shipped being 200 cases of Sugar valued at £ 5 per case, 930 Hides valued at £ 1 per hide, and 138 bales of Cotton valued at £ 6 per bale?

PRODUCT.

Interest £ 2758	0	0	Return	£ 6	17	11
Rate per cent. £	0 4	8	Amount	£ 7	0	0

N. B. In the calculation of the rate per cent. any fraction of a penny is usually reckoned as a penny, in order to give the assured the full amount, at least of the return.

N. B. See the note at the foot of the next page.

CASE 3.

When there is a stipulated return of Premium for convoy or the protection of Ships of War, it is calculated upon the amount of the Interest that arrives safe.

Rule. Find the amount of the return upon the amount of the Insured Interest that arrives safe. Then if the whole Interest[*] arrives safe, the return is to be reckoned upon the amount at the given rate, which is commonly about half of the Premium received by the underwriters. If only a part arrives safe, the amount of the return upon this part, with or without any other claim upon the underwriters, is to be per-centaged and calculated as in the preceding case.

EXAMPLE 1.

To find the amount of the return for convoy and arrival, upon £ 1675 at 70 s. per cent.

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       £
     1675
       70
     ----
s. 1172,50
     -------
     £ 58 12 6   Amount of the Return.
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EXERCISES.

Ex. 10. Find the amount of the return of Premium for convoy and arrival, on £ 1162 at 4 per cent.

Ex. 11. Find the amount of the return of Premium for convoy and arrival, on £ 3000 at 65 s. per cent.

PRODUCTS.

Ex. 10. £ 46 9 7. Ex. 11. £ 97 10 0

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[*] Interest, in Marine Insurances, signifies the actual or insured value of the property on which the Insurance is made.

Short Interest is the excess of the insured value above the value of the property exposed to the risk. In valued Policies, or when the value of the goods intended to be shipped is specified, the Short Interest is the insured value of the goods not shipped. In open Policies, or when the goods are not valued, the Short Interest is the excess of the Insured Interest above the value of what is termed Covered Interest, for which see Part IV.

EXAMPLE 2.

To find the amount of return of Premium for convoy and arrival, and for Short Interest upon an Insurance for £ 7000 at 8 guineas per cent., the amount of the valued Interest which was shipped and which has arrived safe with convoy being £ 5360, and the return for convoy and arrival being £ 4 per cent.

Interest insured . .	£ 7000
Valued Interest shipped . .	5360
Short Interest . .	£ 1640

	£	s.	d.
Return for convoy on £ 5360 at £ 4 per cent.	£ 214	8	0
— Short Interest on £ 1640 at £ 8 per cent.	131	4	0
	£ 345	12	0

£		£ s. d.		£
If 7000	———	345 12 0	———	100?

Answer £ 4 18 9 Rate per cent.

£ 7000 at £ 4 18 9 per cent. = £ 345 12 6 Amount of the return.

EXERCISES.

Ex. 12. What is the amount of the claim upon the underwriters for convoy and arrival at £ 6 per cent. upon an Insurance of £ 12000, the amount of the valued Interest arriving safe with convoy being £ 10800, the Premium being 12 guineas per cent., and the underwriters retaining by agreement ½ per cent. upon the return for Short Interest?

Ex. 13. An Insurance was effected on 50 hhds. of Sugar valued at £ 20 per hhd., and on 100 casks of Coffee valued at £ 15 per cask, at 10 guineas per cent., to return £ 5 per cent. for convoy and arrival, and the Interest shipped and which arrived safe with convoy was 30 hhds. of Sugar and 75 casks of Coffee. It is required the amount of the claim upon the underwriters for both of these returns.

PART III.

Amounts to be insured and Covered Interest.

Rule. Add together the per-centage charges of the Insurance; that is, the Premium and Policy, and also the Commission if charged, and subtract the amount from £ 100; then say, as the remainder is to £ 100, so is the given sum to the amount to be insured, or the amount of the Covered Interest.*

N. B. The amount to be insured, to include the Net Interest and the charges of Insurance, is to be reckoned at the next higher pound when there is any remainder in the division, because Insurances are not made upon fractional parts of a pound; but in calculations to find what is called Covered Interest, the result is usually taken at the amount produced in shillings and pence.

Example.

To find what amount must be insured to include £ 567 10 6, and the charges of Insurance; the Premium being 2½ guineas per cent., the Policy 5 s. per cent., and the Commission 10 s. per cent.

£ 2 12 6 with 5 s. and 10 s. = £ 3 7 6
£ 100 less £ 3 7 6 . . . = 96 12 6
If £ 96 12 6 require £ 100 what will £ 567 10 6 require?

Answer £ 588.

Exercises.

Ex. 14. What amount must be insured to include £ 629 14, and the charges of Insurance; the Premium being 40 s. per cent., the Policy 5 s. per cent., and the Commission 10 s. per cent.?

Ex. 15. What is the amount of the Covered Interest on an amount of goods of £ 472 11 10; the charges being, for the Premium 4 guineas per cent., Policy 5 s. per cent., Commission ½ per cent. upon Insurance, and 2 per cent. upon recovery of loss, and also Brokerage, on the same, ½ per cent.?

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* The amount of the Freight, or of the Goods and Charges, exclusive of the expense of the Insurance, is called the Net Interest, and the Covered Interest is the amount which will include the Interest and the cost of Insurance upon the amount of the Covered Interest.

PART IV,

Returns of Premium for Short Interest on Open Policies.

Rule. Find the amount of the Covered Interest and subtract it from the Insured Interest to find the amount of the Short Interest; find the amount of the Short Interest at the rate of the return; find the rate per cent. upon the Insured Interest; and, lastly, find the amount of the Insured Interest at this rate per cent.

Example 1.

To find the amount of the return for Short Interest upon a Policy of Insurance effected on £ 1500, by an agent, on the freight of a vessel at 5 guineas per cent., the amount of the actual freight being only £ 1132 10.

Premium 5 guineas, Policy 5 s., Commission 10 s. = £ 6 0 0.

£ 100 less £ 6 = £ 94.

	£		£		£ s. d.
If	94	———	100	———	1132 10 0?

	£ s. d.	
Answer	£ 1204 15 9	Covered Interest.
	1500 0 0	Insured Interest.
	£ 295 4 3	Short Interest.

The Premium at 5 guineas gives £ 5 per cent. for the Return.

£ 295 4 3 at £ 5 per cent. = £ 14 15 2.

	£		£ s. d.		£	
If	1500	pay	14 15 2	what will	100	pay?

Answer s. 19 9 d.

£ 1500 at 19 s. 9 d. per cent. = £ 14 16 3 Return.

Exercises.

Ex. 16. An Insurance was effected upon £ 5000 upon goods from Leghorn to London, as Interest should appear, at 3 guineas per cent., and the sterling amount of the Interest shipped was £ 3482 16 2; what was the amount of the return for Short Interest, guaranteeing the charges incident to the recovery of a loss at 2½ per cent.?

EXAMPLE 2.

To find the amount of the return of premium for Short Interest and for convoy and arrival, on an Insurance effected by an agent, on £ 2000 in goods as Interest should appear, at 9 guineas per cent., to return £ 4 10 per cent. for convoy and arrival; the net invoice amount of the goods shipped amounting to only £ 1364 10 6, and the goods arriving safe.

£ 100 less 9 guineas with 15 s. = £ 89 16 0.

£ s. d.		£		£ s. d.
If 89 16 0	———	100	———	1364 10 6?

	£	s.	d.	
Answer	£ 1519	10	4	Covered Interest
	2000	0	0	Insured Interest
	£ 480	9	8	Short Interest.

£ 480 9 8 at £ 9 per cent. = £ 43 4 10 for Short Inter.
1519 10 4 at £ 4 10 — = 68 7 7 - Convoy, &c.
£ 111 12 5

£		£ s. d.		£
If 2000	———	111 12 5	———	100?

£ 5 11 8 Rate of the Return.

£ 2000 at £ 5 11 8 per cent. = £ 111 13 4 Amount of Return.

EXERCISE 17.

Find the amount of the claim upon the underwriters for return for Short Interest and for convoy and arrival, on an Insurance effected by an agent, on £ 3000 per ship or ships, as Interest should appear and be declared, on Brazilian produce, at 12 guineas per cent., to return £ 6 per cent. for convoy and arrival; the goods shipped and arrived safe with convoy being valued in sterling at £ 2564 13 8.

PART V.

General and Particular Averages and Salvage Losses.

General Averages are contributions made by the owners of the ship, freight, and cargo, to defray the value of any property on board or belonging to the vessel, that is purposely thrown overboard, or destroyed for the preservation of the remainder; as also any expense which the general good of the whole may require to be incurred; as when, in cases of distress, there is a jettison of part of the cargo, or a cutting away of the mast, cable, &c. The owners of the property so destroyed also bear their part of the loss, and when this property has been insured, the underwriters are liable for the amount, however small it may be.

Particular Averages or Partial Losses, are the contributions to which the underwriters are liable, for the partial destruction of the property, by any of the accidents against which the Insurance has been made; as when part of the goods are either damaged or are wholly or partially destroyed, or when the whole of the goods are either damaged or partially destroyed by sea water.—When the property so damaged has been insured, the underwriters are liable for the actual or proportional loss, provided it is not exempted by the terms of the policy; as by the goods being warranted free of average, either entirely or under a certain per centage.

Salvage Losses are partial losses calculated as for total losses, with the deduction of the selling price of the goods preserved.

N. B. In maritime affairs, the term salvage applies to the remuneration paid to those who assist in saving the vessel or cargo when wrecked or in distress, or to the recaptors of a vessel which has been taken by an enemy.

In the following statements the valuations and proportional amounts are to be put in by the learner, who can check them by the results given with the products in the following page.

Statement of General Average on the Speedwell, Robert Simpson, Master, from Yarmouth to London, in consequence of getting aground near Harwich, and having the assistance of two Fishing Smacks, by which, and by throwing part of her cargo overboard, she was got off and brought into Harwich.

Paid, per agreement, to John Morris and James Daniel, of the smacks Thomas and Daniel		60	0	0
Notarial Charges, Agency, and Postages		4	14	0
Value of Barley thrown overboard				
Shipped . . . 690 quarters				
Landed . . . 509⅜ ths				
Thrown overboard 180⅝ at 31 s.	£			
Deduct Freight and Lighterage	27 7 6			
Add Freight of Barley thrown overboard		22	11	6
	£			
Statement		1	1	0
	£	340	18	4
Valuation of the Ship . . .	£ 800 0 0			
Cargo . .	964 11 1			
Freight . .	70 0 0			
Contributing Amount	£ 1834 11 1			
£ 340 18 4 on £ 1834 11 1 is £	per cent.			
£ 800 - - at £ per ct. is £				
964 11 1 - —				
70 - - - —				
	£ 340 18 4			

STATEMENT OF GENERAL AVERAGE per Dover, John Bloomfield, Master, accrued on her passage from Demerara to Liverpool, in consequence of cutting part of her rigging and receiving other damage, as appears by protest, &c.

Sundry cordages for fore-top-mast-stay, back-stays, running riggings, &c., as per B. & T. Morris's bill				
Cwt. 8 2 5 at 50 s.	£			
⅓ off new for old				
New top-sail fixing and fitting, as per J. Caley's Bill	£ 21 6 6			
⅓ off new for old				
Two surveys		2	2	0
Noting and extending protest		2	2	0
Postages			10	6
		33	3	8
Less proceeds of old cut rigging saved		1	8	0
£		31	15	8

Interest

£ 2200	Ship
400	Freight
3226	Cargo

If 5826 lose £ 31 15 8 what will £ 100 lose?
Answer £

London,
June 8th, 1833.

STATEMENT OF PARTICULAR AVERAGE Loss upon Sugars for the Barton, Capt. Edwards, from Jamaica to London.

Interest Insured 57 hhds. valued at £ 25 per hhd.	1425	0	0
5 hhds. if arrived safe would have sold as per certificates for	347	10	0
But being washed and damaged did sell for	134	1	4
Deterioration	213	8	8
Insured value of the 5 hhds. £	250	0	0
If £ 347 10 lose £ 213 8 8 then £ 250 will lose £			
£ on £ 1425 is £ per cent.			
£ 1425 at £ per cent. is £			

PRODUCTS.

Ex.		£	s.	d.		£	s.	d.
12.	Claim per cent. £	6	11	0	Amt. of Claim £	786	0	0
13.	—	6	11	0	—	163	15	0
14.	Insured Amt.	648	0	0				
15.	Covered Inter.	510	12	8				
16.	—	3720	19	0	Short Interest	1279	1	0
	Rate of Return		15	5	Amt. of Return	38	10	10
17.	Covered Int.	2959	16	4	Ret. for Short Int.	4	16	5
	Ret. for Convoy	177	11	9	Claim per cent.	6	1	7
	Amt. of Claim	182	7	6				

1 st General Average.

Net amount of Jettison £ 252 11 10
Contribution per cent. 18 11 8

Do. of Ship £ 148 13 4—Cargo £ 179 4 11—Freight £ 13 0 2

2 nd General Average.

Net amount of Cordage £ 14 4 10—Do. of Top Sail £ 14 4 4
Rate of Contribution £ 0 10 11 per cent.

Particular Average.

Proportionate Amount of Loss £ 153 11 0
Rate per cent £ 10 15 6—Amount of Claim £ 153 10 10.

STANDARDING OF GOLD AND SILVER.

To standard Gold or Silver is to reduce the given full weight of either metal, of any other purity than standard, to its equivalent standard weight.

Standard Gold is a mixture of metal containing 11 parts of pure Gold and 1 part of any other metal.

Standard Silver is also a mixed metal containing 37 parts of pure Silver and 3 parts of alloy.

The purity of these metals is found by a process called an assay,* in which an arbitrary pound weight of either metal is employed. The assay pound weight† for Gold is divided into 24 carats each of 4 grains; and for Silver, into 12 ounces each of 20 pennyweights. Hence,

Standard Gold is 22 carats fine and 2 carats of alloy;‡

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* When an assay is made for the purpose of finding the *absolute* purity of either of these metals, an assay pound of the metals is refined, and the quantity of fine metal produced, after making an allowance for the waste of Silver if that metal is tried, determines the purity of the metal; but when the assay is required to show the *relative* fineness to standard metal, an assay pound weight of metal of true standard purity is refined with the same weight of the metal to be proved, and the results are weighed one against the other; and if the result of the assay piece is *heavier* than that of the standard or trial piece, it is said to be *better*, and if *lighter*, *worse* than standard. Thus, if the fine Gold from the assay piece is 1 carat heavier than that from the trial piece, the Gold tried is said to be better 1 carat; that is, it is equal to 23 carats fine. If the fine Silver from the assay piece is 2 dwts. lighter than that from the trial piece, it is said to be worse 2 dwts., or it is 11 oz. fine.

† The assay pound for Gold generally weighs only a few grains, and that for Silver a few pennyweights.

‡ Standard purities of Gold and Silver are the purities of the coins of the kingdom,

Standard Silver is 11 oz. 2 dwts. or 222 dwts. fine, and 18 dwts. of alloy.

Mixtures of Gold and Silver are called Partings: when the Gold is greater in quantity the mixture is called a Gold Parting, and when the Silver is greater, a Silver Parting.

The Betterness or Worseness of Gold or Silver is the weight of the quantity of alloy to be added to or taken from the full weight, to reduce it to standard weight. The full weight is sometimes called the scale weight, and the standard weight the pay weight, because these metals are bought and sold by that weight.

An assay report is the account rendered by the assayer of the absolute or comparative purity of the metal tried.

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as determined from trial plates of the metals kept in the Exchequer; and all manufactured articles of these metals, except Gold watch cases, which are allowed to be made only 18 carats fine, and small articles of jewellery, must be of standard purity. To prove that they are of this fineness, they are required to be tried at one of the appointed Assay Offices, and they have the following marks stamped upon them, except they are articles that would be damaged by being stamped, and also watch cases with respect to the duty stamp.

1 st. The Maker's stamp, being generally the initials of his name or firm.

2 nd. The Office stamp, to show where the assay has been made; the stamp of London or Goldsmith's Hall being a Leopard's head crowned: of Sheffield, a Crown; of Birmingham, an Anchor; of Newcastle, 3 Castles; of Edinburgh, a Thistle; of Dublin, a Harp.

3 rd. The Date stamp, to show the year in which it was stamped or made. This stamp is a letter of the alphabet, J being rejected. It extends as far as U, the alphabet being changed every 20 years;

Thus, for 1796 the letter was A, for 1806 L
1816 — — a, - 1826 l
1836 — is to be 𝔄, - 1846 𝔏

4 th. The Standard stamp, which is a Lion passant for the regular standards, and a Crown with the number 18 for gold watch cases only 18 carats fine.

5 th. The Duty stamp, which is the King's head, to show that the duty has been paid.

TO STANDARD GOLD.

Multiply the full weight by the number of carats, and take parts for the grains and parts of a grain in the assay report, and then divide the product by 22 to find the Betterness or Worseness; which result is to be added to, or subtracted from, the full or scale weight to find the standard or pay weight.

EXAMPLE.

To find the standard weight of a bar of Gold weighing 11 lb. 10 oz. 13 dwts. 12 grs., reported Worse 2 car. 3⅜ grs.

	lb.	oz.	dwt.	gr.	
½ ...	11	10	13	12	Full weight.
				2	
	23	9	7	0	for 2 car.
½ ...	5	11	6	18	- 2 grs.
¼ ...	2	11	13	9	- 1 gr.
½ ...		8	18	8	- 2-8 ths.
		4	9	4	- 1-8 th.
22)	33	9	14	15	
	1	6	8	20	Worseness.
lb.	10	4	4	16	Standard.
oz.		124	4	16	Standard.

N. B. Gold in bars is not weighed lower than to half a dwt. or 12 grs.; it is weighed in pounds and valued in ounces.

To reduce the standard to fine weight, or to find the quantity of pure gold, take 1-12 th of the standard weight, and subtract the product.

	oz.	dwt.	gr.	
Thus, 1/12 ...	124	4	16	Standard.
	10	7	1	Standard alloy.
oz.	113	17	15	Fine.

TO STANDARD SILVER.

Multiply the full weight by the number of the pennyweights in the assay report, and divide the product by 222 to find the Betterness or Worseness; which result is to be added to, or subtracted from, the full weight to find the standard weight.

EXAMPLE.

To find the standard weight in 72 lb. 2 oz. 10 dwts. of Silver, reported Worse 7½ dwts.

	lb.	oz.	dwt.	
½ ...	72	2	10	Full weight.
			7	
	505	5	10	
	36	1	5	
222)	541	6	15	
	2	5	7	Worseness.
lb.	69	9	3	Standard.
oz.		837	3	Standard.

N. B. Silver in bars is not weighed lower than to a quarter of an ounce or 5 dwts.

To reduce standard to fine weight, take 3-40 ths of the standard weight, and subtract the product.

	oz.	dwt.	
Thus,	837	3	Standard.
	41	17	for 2-40 ths.
	20	18	- 1-40 th.
oz.	774	8	Fine.

EXERCISES.

Ex. 1. What is the standard weight of 2 lb. 7 oz. 13 dwts. of Gold, reported Worse 2 carats 3½ grains; and what is the value at £ 3 17 6 per oz. standard?

Ex. 2. What is the standard weight of 17 lb. 4 oz. 15 dwts. of Gold, reported Worse 1 car. 1¼ grs.; and what is the value at £ 3 17 10½ per oz. standard?

Ex. 3. What are the standard and fine weights of a Prussian Frederiok d'or, weighing 4 dwts. 7 grs., reported Worse 2 grs.; and what is the value at £ 3 17 10½ per oz. standard?

Ex. 4. What are the standard and fine weights of a Spanish Doubloon, weighing 17 dwts. 8½ grs., reported Worse 1 car. 0½ gr.; and what is the value at £ 3 17 10½ per oz. standard?

Ex. 5. What is the standard weight of 25 lb. 11 oz. of Silver, reported Worse 1 oz. 2 dwts.; and what is the value at 60 d. per oz. standard?

Ex. 6. What is the standard weight of 13 lb. 11 oz. 10 dwts. of Silver, reported Better 12½dwts.; and what is the value at 59⅝ d. per oz. standard?

Ex. 7. What are the standard and fine weights of a French Five-Franc piece, weighing 16 dwts. 1¾ grs., reported Worse 6 dwts.; and what is the value at 60 d. per oz. standard.?

Ex. 8. What are the standard and fine weights of a Spanish Pillar Dollar, weighing 17 dwts. 8 grs., reported Worse 7 dwts.; and what is the value at 60 d. per oz. standard.

PRODUCTS.

Ex.					
1.	Stand. weight oz. 27 10 7	Value £	106	12	5
2.	— — 196 5 22	—	764	6	6
3.	St. w. 4 dwt. 4.66 gr., F. w. 92.27 gr.	—		16	3.9
4.	— 16 dwt. 11.20 gr., — 362.26 gr.	—	3	4	1.4
5.	Stand. weight oz. 280 4	—	70	1	0
6.	— — 158 1	—	39	5	4
7.	St. w. 15 dwt. 15.32 gr., F. w. 347.17 gr.	—		3	10.9
8.	— 16 dwt. 18.88 gr., — 372.67 gr.	—		4	2.3

ARBITRATIONS OF EXCHANGE.

Arbitrations of Exchange are calculations to determine the rate of exchange produced by indirect Bills of Exchange,* Bullion, Coins, or Merchandise, purchased in one country and sold in another.

The arbitrated rates thus found are called Pars of Exchange.

GENERAL RULE.

Make an equation or statement of the given rates in the form of a continued proportion in the following order: Place the fixed term of the rate of the place making the operation as the term of demand; then when the fixed term of the required par is in the money of the place where the bills, &c. are *bought*, the *buying price* at that place is to be the *first rate* of the equation, and the *selling price* at the other place is to be the *second rate* of the equation; but when the fixed term of the required par is in the money of the place where the bills, &c. are to be *sold*, make the *selling price* the *first rate* and the *buying price* the *second rate* of the equation. In both cases, such extra rates are to be introduced or added as are necessary either to connect the terms of the rates, or to reduce the results to such quantities as the answer may require; and the whole is then to be worked by the Chain Rule.

Thus, as £ 1 or 240 d. is the fixed term of the rate between London and Paris, Amsterdam, Hamburg, Frankfort, &c. in arbitrations between London and these places, this sum of money must be made the term of demand; then the Sterling money in the buying rate must be made the first term of the 1 st equation, and its value the second term: that term of the selling rate which is like the last term must then be made the first term of the 2 nd equation, and its value in the money of the other place concerned must be made the last term; but as the fixed term of the rate between London and Lisbon, Leghorn, Madrid, &c., is in the money of those places, as 1 Milreis, 1 Pezza, 1 Dollar, &c., the fixed price is made the term of demand; the first rate of the equation is then the selling price, and the second rate (unless any intermediate connecting term is required) is the buying price.

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* Remittances of Bills of Exchange are said to be direct when they are drawn upon the place to which they are remitted; as, a direct remittance to Paris is a remittance of Bills drawn upon Paris: they are said to be indirect when they are drawn upon another place; as, when Bills on Amsterdam are remitted to Paris in order to be sold there.

An example of the introduction of an intermediate rate is given in the statement of the equation in the 7 th Exercise, in order to reduce the Maravedis in a Dollar into parts of a Ducat.

ARBITRATIONS OF BILLS OF EXCHANGE.

EXAMPLE.

To find what par of Exchange is established between London and Paris, by Bills on Madrid bought in London at 37¼ d. per Dollar, and sold in Paris at 15 Frs. 50 Cents per Pistole of 4 Dollars.

Statement 240 d. = 1 £?
Buying price 37¼ = 1 Dollar.
Selling price 4 = 15.50 Frs.

15.50 × 240 = 3720.00
37¼ × 4 = 149
3720.00 ÷ 149 = Frs. 24 96½ Cts.

Hence, Bills on Madrid bought in London at 37¼ d., and sold in Paris at 15.50, will produce Fr. 24 96½ Cts. for every £ 1 sterling.

EXERCISES.

Ex. 1. What par of Exchange is established between London and Paris, by Bills on Amsterdam bought in London at Flor. 12 1¾ Stiv., and sold in Paris at 57¼ Florins per 120 Frs.?

Statement 1 £?
1 = 12.08¾ Florins.
57¼ = 120 Francs.

Ex. 2. What par of Exchange is established between London and Paris, by bills on Vienna bought in London at Flor. 10 1 Kr. and sold in Paris at Francs 254 per 100 Florins?*

Ex. 3. What par of Exchange is established between London and Paris, by bills on Naples bought in London at 40$\frac{1}{8}$ d. per Ducat, and sold in Paris at 418 Francs per 100 Ducats?

Ex. 4. What par of Exchange is established between London and Hamburg, by bills on Paris, bought in London at 25.60 Frs., and sold in Hamburg at 187 Francs per 100 Marks?

Ex. 5. What par of Exchange is established between London and Hamburg, by bills on Leghorn, bought in London at 48$\frac{1}{2}$ d., and sold in Hamburg at 44 Schillings Banco per Pezza?

Ex. 6. What par of Exchange is established between London and Amsterdam, by bills on Hamburg, bought in London at Mks. 3112, and sold in Amsterdam at 35 Florins per 40 Marks?

Ex. 7. What par of Exchange is established between London and Amsterdam, by bills on Madrid bought in London at 37$\frac{1}{4}$ d. per Dollar of Plate of 272 Maravedis, and sold in Amsterdam at 100$\frac{3}{4}$ Florins per 40 Ducats each of 375 Maravedis?

Statement 240 d?
37$\frac{1}{4}$ = 272 Maravedis.
375 = 1 Ducat.
40 = 100$\frac{3}{4}$ Florins.

Ex. 8. What par of Exchange is established between London and Lisbon, by bills on Paris, bought in London at Frs. 25.65, and sold in Lisbon at 192 Reis per Franc?

Statement 1000 Reis?
192 = 1 Franc.
25.65 = 240 d.

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* For the answers or products of these and the following Exercises, see page 290.

ARBITRATIONS OF COINS.

In the valuation of Gold Coins of other countries, it is usual to rate them at 77 s. 10½ d. per oz. standard, this being the British mintage rate; and in the valuation of Silver Coins, to rate them at 60 d. per oz. st., as being the full average value of standard Silver.

Spanish Doubloons and Dollars are commonly valued by weight as articles of Merchandise.

EXAMPLE.

To find the par of Exchange established by French Gold Coin in comparison with British st. Gold at 77 s. 10½ d. per oz., the full standard weight of the 20 Franc piece being 97¾ grs. Troy.

Statement

$$\begin{array}{rcrl} & & 20 & \text{s.?} \\ 77\frac{7}{8} & = & 480 & \text{Gr. Troy.} \\ 97\frac{3}{4} & = & 20 & \text{Francs.} \end{array}$$

$$\frac{20 \times 480 \times 20}{77\frac{7}{8} \times 97\frac{3}{4}} = \frac{6144000}{243593} = \text{Frs. } 25 \;\; 22 \text{ Cts.}$$

In consequence of these fractions it is requisite to multiply both sides of the equation by 8 and by 4, therefore instead of ($77\frac{7}{8} = 480$ gr. Troy) we may use as the second term of the equation (623 s. $=$ 3840 gr. Troy.)

EXERCISES.

Gold pars at 77 s. 10½ d. per oz. standard.

Ex. 9. What par of Exchange is established between London and Amsterdam, from the st. weight of the 10 Florin piece being 102 gr. Troy?

Ex. 10. What par of Exchange is established between London and Berlin, from the st. weight of 100 Fredericks d'or, being 10156 gr. Troy, and their value in Prussian Currency being 550 Prussian Dollars?

Ex. 11. What par of Exchange is established between London and Petersburg, from the st. weight of the Imperial of 10 Silver Roubles, being 202 grains Troy, and the par of 100 Silver Roubles being worth 360 Banco Roubles, or Roubles of Exchange?

Statement 1 Banco Rouble?
360 = 100 Silver Roubles.
10 = 202 Gr. Troy.
3840 = 623 s. Sterling.
1 = 12 d.

Silver pars at 60 d. per oz. standard.

Ex. 12. What par of Exchange is established between London and Paris, the st. weight of the 5 Franc piece being 375½ grs. Troy?

Ex. 13. What par of Exchange is established between London and Amsterdam, from the st. weight of 10 Silver Florins being 1604 grs. Troy?

Ex. 14. What par of Exchange is established between London and Vienna, from the st. weight of 10 Austrian Species Dollars of 2 Florins each being 3902 grs. Troy?

Ex. 15. What par of Exchange is established between London and Hamburg, from the fixed valuation of the Cologne Mark weight of fine Silver being 27¾ Mks. Banco,* 8 Cologne Marks of fine Silver being reckoned equal to 65 oz. British st. Silver?

Ex. 16. What par of Exchange is established between London and Madrid from the estimate of 1000 Spanish Hard Dollars containing 835 oz. British st. Silver, 64 Hard Dollars being equal to 85 Dollars of Plate or of Exchange?

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* It is to be observed, that Banco money is only a valuation of bars of Silver, which are received by the bank of Hamburg at the rate of 27 Mks. 10 Sch. Banco per Cologne Mk. weight of fine silver, and paid at the rate of 27¾ Marks Banco per Cologne Mark fine. The Cologne Mark equals 3608 grs. Troy, and 8 Cologne Marks of fine silver are exactly 65 oz. and 1-111 th of an oz. Troy of British st. silver, but this fractional part is seldom used.

ARBITRATIONS OF BULLION.

EXAMPLE.

To find the par of Exchange established by the purchase of Gold in London, at 77 s. 6 d. per oz. st. fineness, or 11-12 ths fine, and the sale of it in Paris at the rate of 4½ per Mille Premium upon the fixed price of 3434 Frs. 44 Cts. per Kilogramme fine, which is reckoned equal to 15434 gr. Troy.

Statement 20 s.?
77½ = 1 Oz. St.
12 = 11 Oz. Fine.
1 = 480 Gr. Troy.
15434 = 1 Kilogramme.
1 = 3434.44 Francs.
1000 = 1004½ With Premium.

$$\frac{1004\frac{1}{2} \times 3434.44 \times 480 \times 11 \times 20}{77\frac{1}{2} \times 12 \times 15434000} = \text{Frs. } 25 \text{ } 38 \text{ Cts.}$$

EXERCISES.

Ex. 17. What par of Exchange is established between London and Paris by the purchase of Gold in London at 77 s. 6 d. per oz. st. and sale of it in Paris at 14½ per Mille Prem. with the following estimations.

10000 Oz. Troy = 311002 Grammes.

1000 Grammes of Gold of Eng. st. are worth 3148 Frs. 24 Cts.

Statement 20 s.?
77½ = 1 Oz. St.
10000 = 311002 Grammes.
1000 = 3148.24 Frs.
1000 = 1014½ With Prem.

Ex. 18. What par of Exchange is established between London and Paris by the purchase of Silver in London at 59½ d. per oz. st., and sale in Paris at 4 per Mille prem. with the following estimations?

10000. oz Troy = 311002 Grammes.

1000 Grammes of Eng. st. Silver are worth 202 Frs. 47 Cts.

Ex. 19. What par of Exchange is established between London and Amsterdam, from Gold bought in London at 77 s. 6 d. per oz. st., and sold in Amsterdam at 14 per cent. premium upon the fixed price of Flor. 1442 60 Cts. per Pond fine; the Pond containing 1000 Wigties of the same weight as French Grammes?

Statement 20 s.?
77½ = 1 Oz. St.
12 = 11 Oz. Fine.
10000 = 311002 Wigties.
1000 = 1442.60 Florins.
100 = 114 With Prem.

Ex. 20. What par of Exchange is established between London and Amsterdam, from Silver bought in London at 59½ d. per oz. st., and the sale of it in Amsterdam at the price of 103¾ Florins per Pond of fine Silver?

Statement 240 d.?
59½ = 1 Oz. St.
40 = 37 Oz. Fine.
10000 = 311002 Wigties.
1000 = 103¾ Florins.

Ex. 21. What par of Exchange is established between London and Hamburg, from the purchase of Silver in London at 59½ d. per oz. st. and the sale of it in Hamburg at the rate of 27 11/16 Marks Banco per Cologne Mark fine, it being estimated that 65 oz. Eng. st. Silver produce 8 Cologne Marks of fine Silver?

Statement 240 d.?
59½ = 1 Oz. St.
65 = 8 Cologne Marks fine.
1 = 27 11/16 Marks Banco.

ARBITRATIONS OF MERCHANDISE.

CASE 1.

When the whole Cost and the Net Proceeds are given.

EXAMPLE.

To find the course of Exchange established between London and Paris, from the purchase of Bullion in London, and sale of it in Paris, the Cost and Charges amounting to £ 27416 10, and the Net Proceeds yielding Frs. 702467.60.

	£ s.		Frs. Cts.		£
If	27416 10	———	702467 60	———	1 ?

Frs.
27416) 702467
Frs. 25 62¼ Cts.

EXERCISES.

Ex. 22. Find the rate of Exchange established between London and Calais, from the purchase of Indigo in London, amounting to £ 867 18 7, and the sale of it at Calais, the Net Proceeds being Frs. 23072.40.

Ex. 23. Find the rate of Exchange established between London and Amsterdam, from the purchase of Coffee in London, amounting to £ 1644 10 2, the Net Proceeds of the sale in Amsterdam being Flor. 19982 60 Cts.

Ex. 24. Find the rate of Exchange established between London and Hamburg, from the purchase of Drugs in London, amounting to £ 538 17 6, the Net Proceeds in Hamburg being Mks. 7835 12 Banco.

Ex. 25. Find the rate of Exchange established between London and St. Petersburg, from the purchase of Sugar in London, amounting to £ 1434 8 6, the Net Proceeds in St. Petersburg being Bo. Roubles 37166 27 Cop.

CASE 2.

When the Prices, Weight, and per Centage Charges are given.

Obs. 1. The per Centage of the Charges on the cost price is to be added to 100, and the same on the selling price is to be subtracted from 100; and in applying these, the former amount is to be on the same side of the Equation as the buying price, and the latter remainder is to be on that side which contains the selling price.

Obs. 2. When there is a per Centage Discount on either the buying or the selling prices, it is to be taken from 100, and the remainder is to be applied to the same side of the equation as that which contains the price on which the discount is allowed.[*]

EXAMPLE.

To find the rate of Exchange established between London and Hamburg from Coffee bought in London at 61 s. per cwt., and sold in Hamburg at 8 Schillings Banco per lb.; supposing that 112 lb. Eng. render 102 lb. Hamburg,[†] that the export charges in this country amount to 3 per cent., and the charges of the sale in Hamburg amount to 10 per cent.

$$\begin{array}{lrcrl}
\textit{Statement} & & & 20 & \text{s. ?} \\
 & 61 & = & 112 & \text{lb.} \\
 & 103 & = & 100 & \text{with Shipping Charges.} \\
 & 112 & = & 102 & \text{lb. Hamburg.} \\
 & 1 & = & 8 & \text{Schillings Bo.} \\
 & 100 & = & 90 & \text{with Hamburg Charges.}
\end{array}$$

$$\frac{90 \times 8 \times 102 \times 20}{103 \times 61} = 233\tfrac{3}{4}\ \text{Sch.} = \text{Bmf. } 14\ \ 9\tfrac{3}{4}\ \text{Sch.}$$

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[*] In Hamburg there are fixed discounts called Rabat upon the prices of a few articles of Merchandise, being what are called, $8\frac{2}{3}$ and $4\frac{1}{3}$ per cent.; but as these per centages are reckoned upon the net amount, instead of, as usual, upon the full amount, instead of making the rate of the Equation, for the former 100 and $91\frac{1}{3}$, it is $108\frac{2}{3}$ and 100, or 326 and 300.

[†] In arbitrations of Merchandise the relations of the weights are generally not made from the exact absolute standard weights of the two places concerned, but from the average net weights of the particular goods.

EXERCISES.

Ex. 26. Find the rate of Exchange established between London and Hamburg, from East India Sugar bought in London at 28 s. per cwt., and sold in Hamburg at 8 Grotes or 4 Sch. Bo. per lb. with an allowance of Rabat of 8⅔ per cent., or 326 Mks. full price for 300 net; supposing that 1 cwt. Eng. yields 103 Hamburg, that the shipping charges in this country amount to 3 per cent., and that the sale charges in Hamburg with loss of Interest, &c. amount to 12 per cent.

Statement 20 s. ?
28 = 1 cwt.
103 = 100 with Shipping charges.
1 = 103 lb. Hamburg.
1 = 4 Sch. Bo.
326 = 300. with Rabat.
100 = 88 with Sale Charges.

Ex. 27. Find the rate of Exchange established between London and Antwerp, from Cotton Wool bought in London at 6½ d. per lb. and sold in Antwerp at 42 Cts. per lb. of Antwerp; supposing that the shipping charges in this country amount to 2 per cent., that 108 lb. Antwerp are equal to 112 lb. Eng., and that the sale charges amount to 10 per cent.

Ex. 28. Find the rate of Exchange established between London and Hamburg, from Cochineal bought in London at 10 s. 6 d. per lb. with 2½ per cent. discount., and sold at Hamburg at 9½ Mks. Bo.; supposing 100 Eng. render 93 lb. at Hamburg, the shipping charges to be 2½ per cent., the charges at Hamburg to be 6 per cent., and the allowance for loss of Interest, &c. on the proceeds of the sales to be 2 per cent.

Statement 20 s. ?
10½ = 1 lb.
97½ = 100 with Discount.
102½ = 100 with Charges.
100 = 93 lb. Hamburg.
1 = 9½ Mks. Bo.
100 = 94 with Hamburg Charges.
100 = 98 with Interest.

ARBITRATIONS OF WEIGHTS AND MEASURES.

Arbitrations of Weights and Measures are calculations for determining the relations between the weights and measures of two countries, from their mutual relations to those of another country.

Rule. Make the weight, &c. for which the equivalent is required the term of demand, make that rate the first rate of the equation which contains a similar weight, &c., and make that rate the second rate of the equation which contains the weight, &c. that is similar to the equivalent quantity required.

N. B. It is sometimes necessary to introduce other terms to reduce the rates, as in the following examples.

Example 1. To find the weight in Hamburg lb. which is equivalent to 1 cwt. Eng.; the Hamburg lb. being reckoned equal to 10080 Dutch Asen, and the Dutch Mark of 5120 Asen being reckoned equal to 3798 gr. Troy, of which 7000 grains are equal to 1 lb. Avoirdupois.

Statement 112 Avoird. ?
1 = 7000 Gr. Troy
3798 = 5120 Asen
10080 = 1 lb. Hamburg.

$$\frac{112 \times 7000 \times 5120}{10080 \times 3798} = \frac{401408000}{3828384} = 104\tfrac{85}{100} \text{ lb. Hamburg.}$$

Ex. 29. Find the weight in Danish lb. equivalent to 1 cwt. English; the Danish lb. being reckoned equal to 10392 Dutch Asen, and 5120 Asen being estimated at 3798 gr. Troy.

Ex. 30. Find the weight in Bremen lb. that are equal to 1 cwt. English; the Bremen lb. being reckoned equal to 498¼ French Grammes, and 1000 Grammes being equal to 15434 gr Troy.

EXAMPLE 2.

To find the weight of the Vienna Mark in oz. Troy, the estimated weight of 1000 Marks being 280644 French Grammes, and 1000 Grammes being equal to 15434 gr. Troy.

Statement 1 Vienna Mark?
1000 = 280644 Grammes
1000 = 15434 Gr. Troy
480 = 1 Oz.

$$\frac{280644 \times 15434}{480000000} = 9\tfrac{238}{10000} \text{ Oz. Troy.}$$

Ex. 31. Find the weight of the Hamburg Mark in gr. Troy; the estimated weight of the Mark being 4864 Dutch Asen, and 5120 Asen being reckoned at 3798 gr. Troy.

Ex. 32. Find the weight in gr. Troy of the Spanish or Castile Mark of 4608 Granos; 20030 Granos being reckoned equal to 1 French Kilogramme, and 1 Kilogramme being equal to 15434 gr. Troy.

33. Find the weight in oz. Troy of the Frankfort Mark; 100 Marks being reckoned equal to 23395 French Grammes, 1000 Grammes being equal to 15434 gr. Troy.

EXAMPLE 3.

To find the contents in Imperial gallons of the Hamburg Ahm, which is reckoned to contain 7300 French cubic inches (Old Measure) 100000 French cubic inches being reckoned 121057 Eng. cub. in. and 277.274 Eng. cub. in. being the measure of an Imperial gallon.

Statement 1 Ahm?
1 = 7300 F. C. Inches
100000 = 121057 E. C. Inches
277.274 = 1 Imp. Gall.

$$\frac{121057 \times 73{,}00}{277274{,}00} = 31\tfrac{87}{100} \text{ Imperial Gall.}$$

EXERCISES.

Ex. 34. What are the contents in Imperial gallons of the Neapolitan Barile, from the estimates of 1534 Barili being equal to 1000 Milleroles of Marseilles, which are reckoned at 64377 French Litres, and of 1000 Litres being equal to 61028 Eng. cub. inches?

Ex. 35. What are the contents in Imperial bushels of the Austrian corn Metzen, which is estimated at 3100 old French cubic inches, 100000 of which are reckoned equal to 121057 Eng. cub. inches?

EXAMPLE 4.

To find the length in Eng. inches of the Prussian foot, 100 feet being reckoned equal to 13913 old Paris Lines of 144 to a foot; 1000000 Paris feet being reckoned equal to 324839 Metres, and 1000 Metres being equal to 39371 Eng. inches.

Statement 1 Prussian foot?
100 = 13913 Paris Lines.
144 = 1 Paris Foot.
1000000 = 324839 Metres.
1000 = 39371 Eng. Inches.

$$\frac{39371 \times 324839 \times 13913}{14400000000000} = 12\tfrac{356}{1000} \text{ Eng. Inches.}$$

EXERCISES.

Ex. 36. Find the length in Eng. inches of the Spanish Vara, reckoning 1000 Varas to be equal to 375897 Paris Lines, and the other estimations the same as in the preceding Example.

Ex. 37. Find the length in Eng. inches of the Rhineland foot, reckoning 1000 Rh. feet to be equal to 139129 Paris Lines, and the other estimates as above.

PER CENTAGES OF RATES OF EXCHANGE.

Per Centages of Rates of Exchange, are the parts of the rate of Exchange that are equivalent to given rates per cent.

CASE 1.

When the rate and the per Centage are given to find the part of the rate.

Rule. Multiply the rate of Exchange by the given rate per ct., and divide by 100.

EXAMPLE 1.

To find the amount of 1¾ per cent. upon the rate of Exchange on Paris at Frs. 25 45 Cts. per £ st.

Cts. 25.45 for 1 per ct.
12.72 - ½
6.36 - ¼

Cts. 44.53

Product 44½ Cts.

N. B. In the London quotations of the Paris, Amsterdam, and Genoa rates, the number of Francs, Florins, and Lire in the rates, is the number of Cents that are equal to 1 per ct. In the Frankfort rate 1½ Batzen or 6 Kreutzers are reckoned 1 per ct.

EXAMPLE 2.

To find the amount of 1¾ per cent. in the Hamburg rate of Bmf. 13 14 Sch., and in the Vienna rate of Florins 10 6 Kr. per £ st.

Mks.	13 14 Sch.		Flo.	10 6 Kr.	
	16			60	
Sch. Bo.	2.22	1 per ct.	Kreutz.	6.06	1 per ct.
	1.11	½		3.03	½
	.55	¼		1.51	¼
Sch.	3.88 or 3 9/10 Sch. nearly.		Kr.	10.60 = 10 6/10 Kr.	

EXAMPLE 3.

To find the amount of 1½ per cent. on the rates of Exchange of 48¾ per Milreis on Lisbon, and 36¾ per Dollar on Madrid.

48¾	d.	36¾	d.
4		4	
1.95	1 per ct.	1.47	
97	½	73	
2.92	or nearly 3 farthings.	2.20	or 2¼ farthings nearly.

N. B. In the Lisbon, Gibraltar, Leghorn, and Venice rates, ½ d. sterling is usually reckoned as 1 per ct., and in the Madrid and Naples rates, 1½ farthings are reckoned as 1 per ct.

It may be further observed, that Interest calculations on rates of Exchange are made in the same manner when the term is a fraction of a year. As for example, at 4 per ct. per annum, 3 months' Interest is 1 per ct.; 2 months', is 2-3 rds; 6 weeks', is ½ per ct.; 1 month's, is 1-3 rd per ct., &c.

At 5 per ct. per annum, 3 months' interest is 1¼ per ct. or 1-80 th of the rate; 2 months' is therefore 1-120 th, 6 weeks' is 1-160 th, and 1 month's 1-240 th of the rate.

For days, it is usual to multiply the rate by the number of days, and divide the product by 7200 for 5 per ct., by 9000 for 4 per ct., and by 12000 for 3 per ct., or to take parts for the number of the days out of these divisors.

EXAMPLE 4.

To find the net rate of Exchange of London on Paris, or the short price* of a bill drawn on Paris at 25.65 that has three months to run, and of another at 25.40 that has 12 days to run, at 5 per ct.

$\frac{1}{80}$ ...	25.65	Rate	$\frac{1}{600}$ ...	25.40	Rate.
	32	Int.		4¼	Int.
Frs.	25.33	Net.	Frs.	25.35¾	Net.

12 out of 7200 is the 1-600 th part.

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[*] The short prices of bills of Exchange are the prices of bills at sight, or at 3 days' sight; and they are generally nearly equivalent to the prices of bills at other periods of payment, after allowing for the Interest; which, as is explained in the note to the third of the following pages, is a subtraction from the full or long price when the price or variable part of the rate of Exchange is in foreign money, and an addition when it is in the money of the place where the estimation is made.

EXERCISES.

Ex. 38. How much is 1 per cent. on Frs. 25 50 Cts.?

Ex. 39. How much in Stivers, also in Cents, is 1 per ct. on Flor. 12 2 Stiv.?

Ex. 40. How much is 3 per ct. on Mks. 13 12 Sch.?

Ex. 41. How much is 2 per ct. on Flor. 10 10 Kreut.?

Ex. 42. How much is 1¾ per ct. on Lire 25.35?

Ex. 43. How much is 1¼ per ct. on Batzen 153?

Ex. 44. How much is 1½ per ct. on 47¾ d. per Pezza?

Ex. 45. How much is ¾ per ct. on 37¼ d. per Dollar of Plate?

Ex. 46. How much is 3¾ per ct. on 46¾ d. per Milreis?

Ex. 47. Find the short price on Paris, for Frs. 25 80 Cts., at 3 months, allowing 5 per ct. per annum.

Ex. 48. Find the net rate on Paris, the full rate being Frs. 25 87½ Cts., allowing Interest for 37 days at 4 per ct., and charges ⅞ per ct.

Ex. 49. Find the short price on Amsterdam for Flor. 12 6 Stiv. at 3 months, allowing 4 per ct. per annum.

Ex. 50. Find the net rate on Amsterdam, the full rate being Flor. 12 7½ Stiv., allowing Interest for 54 days at 5 per ct., and charges ⅞ per ct.

Ex. 51. Find the short price on Hamburg for Mks. 13 14 Sch. at 2 months, allowing 4 per ct. per annum.

Ex. 52. Find the net rate on Hamburg, the full rate being Mks. 13 13½ Sch., allowing Interest for 7½ months at 3 per ct. and charges ¾ per ct.

Ex. 53. Find the short price on Leghorn for 48¼ d. per Pezza, allowing Interest for 2½ months at 5 per ct.

Ex. 54. Find the net rate on Lisbon for 48½ d. per Milreis, allowing Interest for 137 days at 4 per ct., and charges 1¼ per ct.

CASE 2.

When the whole rate and a part are given to find the per Centage.

Rule. By Proportion, say, as the whole rate is to its part, so is 100 to the rate per cent.

EXAMPLE 1.

Find the per Centage of 33 Centimes in the rate on Paris of Frs. 25 70 Cts.

As 25.70 ——— 0.33 ——— 100.

2570) 3300

Product $1\frac{28}{100}$ per cent.

In general the product may be obtained by inspection, to any degree of nicety required for purposes of business; thus, reckoning in the above rate, that 26 Cents is 1 per ct., the remaining 7 Cents give an additional ¼ per ct., making the product 1¼ per ct.

EXAMPLE 2.

To find the per Centage produced by ¾ d. on the Leghorn rate of 47 ¾ d.

As 47¾ d. or 191 f. ——— 3 f. ——— 100.

191) 300

Product $1\frac{57}{100}$ per cent.

If the difference of 3 farthings be valued from the estimation of ½ d. in the Leghorn rate being equal to 1 per ct., the result will be 1½ per ct., or very nearly the same as above.

EXERCISES.

Ex. 55. What per Centage is produced by 20 Centimes in the Paris rate of Frs. 25.90?

Ex. 56. What per Centage is produced by 3 Stivers in the Amsterdam rate of Flor. 12 4?

Ex. 57. What per Centage is produced by 9 Schillings Bo. in the Hamburg rate of Mks. 13 12?

Ex. 58. If the rates on Paris are 25.30 at sight, and 25.60 for bills at 3 months, what is the per Centage difference of these rates, after allowing interest on the latter at 4 per cent. per annum.

Ex. 59. If the rate of London on Paris at 3 months is 25.75, and Paris on London at 3 months is 25.20, what is the difference in the rates per ct. on the cash prices of these bills, or the prices after the interest at 4 per ct. per annum is deducted from the London and added to the Paris price?*

Ex. 60. If bills on Paris at 3 months are 25.60, and by sending Amsterdam bills to Paris they yield me 25.55, what is the difference per ct. in the net rate, allowing 4 per ct. per annum for interest on the former rate, and $\frac{5}{8}$ per ct. for charges on the latter rate.

Ex. 61. If by sending goods to Amsterdam I establish a net arbitrated rate of Flor. 13 45 Cts., what is the rate of my gain per ct., if I have bills in return at 2 months at Flor. 11 95 Cts. after allowing for 6 months Interest on the arbitrated rate at 4 per ct.?

Ex. 62. If by sending goods to Hamburg I establish a net arbitrated rate of Mks. 14 15 Sch., what is the rate of my gain per ct., if I have bills in return at Mks. 13.10 at 2 months, after allowing for 8 months interest at 5 per ct.?

Ex. 63. If by sending goods to Leghorn I establish a net arbitrated rate of 44¾ d., what is the rate of my gain per ct. if I have bills in return at 3 months at 48¼ d., after adding 9 months interest at 4 per ct. to the arbitrated rate?

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* Obs. 1. Bills at sight, or the short Cash prices of bills, are dearer than bills at 3 months, because upon the latter there is a loss of the Interest for the time they have to run.

Obs. 2. The interest on the London price is subtracted; for this rate on Paris being in foreign money, the lower the number of Frs. is that are given in London for £ 1 sterling, the dearer the bills are: (for if the rate is 25.20, I shall have to give more for a bill of Frs. 10000, than if the rate is 25.60.)

Obs. 3. The Interest on the Paris price is added; for in Paris the rate is in the money of the country, and therefore the greater the price is that is given in Paris for £ 1 sterling, the dearer the bills are: (for if the rate is 25.60 in Paris, a bill for £ 100 will cost more than if the rate is 25.20.)

Obs. 4. The same principles apply to London rates in sterling; but they appear to operate the reverse way: thus, in the rate of London on Leghorn, the Interest is added, and in the rate of Leghorn on London it is subtracted.

Ex. 64. If by sending Bullion to Paris I establish a net arbitrated rate of 25.75, and have bills in return at 1 month at 25.25, what rate per ct. is my profit, supposing 27 days to elapse between the purchase of the Bullion in London and the investment of the proceeds in bills in Paris, that the extra charges for bill brokerage, &c., are ¼ per ct., and that interest is reckoned at 4 per ct. per annum?

Ex. 65. If I buy Gold in London at 77 s. 9 d. per oz. st. and sell it in Paris at 14½ per Mille premium, what profit or loss per ct. do I make in this operation, if I have bills in return at 25.17½ at sight, supposing that the charges of purchase, shipping, and carriage to Paris amount to ¾ per ct., that the sale charges there amount to ¾ per ct., and that 28 days elapse between the days of purchase and reimbursement, reckoning interest at 4 per ct.?

Ex. 66. If standard Gold costs in London 77 s. 9 d. per oz. st., and I contract to supply the Prussian Mint at the price of 219½ Prussian Dollars per Mark of fine Gold, and to draw for the amount on Berlin at Prus. Dol. 6 20 gro.; what rate per ct. is my profit or loss, supposing the charges of purchase, shipping, and transport to Berlin to amount to 1⅛ per ct., and that the Prussian Mark weighs 3609 grains Troy?

Statement			20	s.?
	77¾	=	1	Oz. St.
	12	=	11	Fine.
	1	=	480	Gr. Troy.
	3609	=	1	P. Mark.
	1	=	219½	Dollars.

ARBITRATED PRICES.

Arbitrated prices are rates or prices produced by Bullion or Merchandise, from the cost or selling price in one of the two places concerned, together with the arbitrated weights or measures, and the charges, allowances, and rate of Exchange; an arbitrated *cost price* being the price at which the article should be bought, to admit of its being sold without loss at the given selling price; and an arbitrated *selling price*, that at which it should be sold to cover the cost and charges.

EXAMPLE.

To find the arbitrated selling price, in Mks. Bco. per lb. in Hamburg, produced by the purchase of Indigo in London at 5 s. per lb., calculating that the export charges are 3 per ct., that 100 lb. English yield $92\frac{1}{2}$ lb. Hamburgh, that the freight and sale charges are 5 per ct., and that the course of Exchange is Mks. 13 10 Sch.

Statement

$$\begin{array}{rcrl} & & 1 & \text{lb. Hamburg?} \\ 92\frac{1}{2} & = & 100 & \text{lb. English.} \\ 1 & = & 5 & \text{s. Sterl.} \\ 20 & = & 13\frac{5}{8} & \text{Mks. Bco.} \\ 100 & = & 103 & \text{With export charges.} \\ 95 & = & 100 & \text{Less Hamb. charges.} \end{array}$$

$$\frac{10300 \times 13\frac{5}{8} \times 500}{92\frac{1}{2} \times 2000 \times 95} = \text{Mks. } 3\ 15\tfrac{7}{8} \text{ Sch. or 4 Mks. nearly.}$$

Obs. The London charges here increase the cost price, and therefore the 100 with these charges are placed on the side of the equation containing this price. The equation then reaches to the net selling price at Hamburg, where, from the charges being estimated at 5 per cent. on the full price, 100 full price yields 95 net price. Therefore as we want to find the full price we reverse this analogy, and make 95 net equal to 100 less charges.

See the note in page 297; and for the methods of finding arbitrated prices with fixed as well as per centage charges, see the last three of the Pro-formâ Invoices and Account-Sales.

Exercises.

Ex. 67. If Bengal Cotton is bought in London at 5¼ d. per lb., and the estimated export charges with freight and insurance amount to 10 per ct., what is the arbitrated selling price per 50 Kilogrammes at Marseilles, to defray all expenses, supposing the sale charges to be 6 per ct.; 100 Kilog. being equal to 220½ lb. Avoirdupois, and the course of Exchange being reckoned at Frs. 25.00?*

Ex. 68. What is the arbitrated selling price in London per Imperial gallon of Cognac Brandy bought at Bordeaux at 155 Frs. per 27 Veltes, the charges being estimated at 20 per ct.; 10 Veltes being reckoned equal to 16½ Imp. gal. and the course of Exchange being 25.20?

Ex. 69. If Cotton Wool costs at Alexandria 28 Dollars per Rottolo of 44 Okes, what is the equivalent selling price in London per lb. Avoirdupois, independent of charges, estimating that 3 Okes Egyptian weight equal to 8 lb. Avoirdupois, that the Dollar of Silver is worth 4 s. 10 d. per oz. Troy, and that 1000 Dollars weigh 866 oz. Troy?

Ex. 70. If Coffee costs in London 60 s. per cwt., and the exportation charges amount to 3 per ct., what is the equivalent selling price per 100 lb. in Antwerp, including the sale charges which amount to 8 per ct. on the selling price; supposing 100 lb. Eng. render net 96 Antwerp, and the proceeds to be remitted at the Exchange of 12 Flor. 10 Cts.?

Ex. 71. If Logwood is bought in London at £ 6 per ton, with a discount of 2½ per ct., and the shipping charges, Insurance, &c., are reckoned at 6 per ct., what is the equivalent or arbitrated sale price in Lire Fuori Banco per 100 lb. in Genoa, at the Exchanges of 6 Lire Fuori Banco for 5 Lire Nuove, and 26 L. N. per £ sterling, supposing the freight and sale charges to amount to 12 per ct., and that 3 cwt. Eng. render net 400 lb. in Genoa?

Ex. 72. What is the arbitrated selling price per lb. in London of Maranham Cotton Wool, bought in Maranham at 4500 Reis per Arroba of 32 lb., and drawn for at the Exchange of 31½ d. per Milreis, it being estimated that 100 lb. Portuguese render net 95 lb. Eng., that the charges at Maranham amount to 10 per ct., and that the charges in London, with freight and insurance, amount to 12½ per ct.?

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* See the note in the next page.

Ex. 73. If refined Sugar is bought in London at 60 s. per cwt., the charges for freight and insurance amount to 2¼ per ct., and the charges on the sales amount to 7 per ct., what is the equivalent selling price in Onze and Tari per Sicilian Cantaro of 100 Rottoli, reckoning the cwt. Eng. to be equal to 64 Rottoli, and the Exchange to be at 60 Tari per £ sterling?

Ex. 74. What is the arbitrated selling price per cwt. in London of Argol bought in Leghorn at 50 Lire effective per 100 lb., Exchange at 49 d. per Pezza d'oro; it being estimated that 112 lb. equal 150 lb. of Leghorn, that the Leghorn export charges are 5 per ct., that the London import charges are 15 per ct., that the discount on the selling price in London is 2½ per ct., and that the allowances on the weight are 2 per ct. more in Leghorn than in London; 5¾ Lire in Silver making 1 Pezza in Silver, 107 Pezze in Silver equal to 100 Pezze d'oro, or in the Gold valuation of money of exchange?

Statement 112 lb. Eng.?
112 = 150 lb. Leghorn.
100 = 50 Lire in Silver.
5¾ = 1 Pezza in Silver.
107 = 100 Pezze d'oro.
1 = 49 d. Sterling.
12 = 1 Sh.
100 = 105 with Leghorn Charges.*
85 = 100 less London —
97½ = 100 less Discount.
100 = 98 with difference of allowance.

Ex. 75. What is the arbitrated selling price per oz. of standard Gold in London, supposing that it is bought in Paris at the price of 14 per Mille premium, that the charges amount to 1 per ct., and that the course of Exchange at which the cost price is drawn on London is 25.85?

N. B. For the rates of estimation required to this and the following Exercises, see the Arbitrations of Bullion, page 281.

Ex. 76. What is the arbitrated selling price per oz. st. of bar Silver in London, supposing the price in Paris to be 6 per Mille premium, and the rate of Exchange on London to be 25.75?

Ex. 77. What is the arbitrated selling price per oz. st. of bar Silver in London, supposing the price in Hamburg equals 27 Mks. 9 Sch. per Cologne Mark fine, and the rate of Exchange on London is 1314?

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* In the application of per centage rates as terms of the equation, we must ob-

Ex. 78. **What is the arbitrated selling price in Paris per Kilogramme of fine Gold, supposing Gold to be bought in London at 77 s. 6 d. per oz. st., the proceeds to be remitted at 25.30, the export charges to amount to ¾ per ct., and the sale charges to ½ per ct.?**

Statement 1 Kilog. fine?
1 = 15434 Grains.
480 = 1 Oz.
11 = 12 Standard.
1 = 77½ Sh.
20 = 25.30 Frs.
100 = 100¾ with export Charges.
99½ = 100 less sale do.

N. B. *The working of this equation gives the price per Kilogramme including the premium, therefore the fixed price of Frs.* 3434.44, *which is the price of* 1 *Kilog. of fine gold, must be subtracted from the price, and then the remainder multiplied by* 1000 *and divided by* 3434.44 *will give the price or premium per Mille.*

Ex. 79. What is the arbitrated cost price in Paris per Kilogramme of fine Silver, to allow st. Silver to be sold in London at 60 d. per oz., the cost being drawn for at 25.60, the charges of transport and purchase amounting to 1 per ct. and the sale charges to ½ per ct.?

Statement 1 Kilog.?
1 = 15434 Grains.
480 = 1 Oz.
37 = 40 Standard.
1 = 60 d.
240 = 25.60 Frs.
101 = 100 with transport Charges.
100 = 99½ less sale Charges.

N. B. *The result of this calculation is, as the above, to be compared with the fixed price per Kilogramme of fine Silver, viz.* 218.89 *Frs.*

Ex. 80. What arbitrated selling price in Amsterdam per Pond or Kilogramme fine of Silver bought in London at 60 d. per oz. st. will cover the charges, supposing them to amount to ½ per ct. on the purchase, and 1 per ct. on the sales, and that the Exchange of Amsterdam on London is 12 Flor.?

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serve, that when the arbitrated *cost price* is required to be found, the arrangement of the terms is

100 with export charges = 100,
100 = 100 less sale charges;

but when the arbitrated *sale price* is required, the terms are reversed: as,

100 = 100 with export charges;
100 less sale charges = 100.

PRODUCTS.

Ex. 1. Fr. $25.33\frac{1}{2}$
2. — 25.44
3. — 25.00
4. Bmf. 13 11
5. — 13 $9\frac{3}{4}$
6. Flor. 12.03
7. — 11.77
8. Pence $48\frac{3}{4}$ n.
9. Flor. $12.08\frac{1}{2}$
10. P. D. 6 $20\frac{1}{4}$
11. Pence $10\frac{2}{3}\frac{9}{2}$
12. Fr. $25.56\frac{1}{2}$
13. Flor. 11.97
14. — 9.50
15. Bmf. 13 $10\frac{1}{2}$
16. Pence $37\frac{2}{3}\frac{3}{4}$
17. Fr. 25.63
18. — 25.50
19. Flor. 12.10
20. — 12.04
21. Bmf. 13 12 n.
22. Fr. 26 58.3
23. Flor. 12.15
24. Bmf. 14 $8\frac{5}{8}$
25. Pence $9\frac{1}{4}$
26. Bmf. 14 $7\frac{1}{4}$
27. Flor. 13 19
28. Bmf. 15 8
29. D. lb. 101.7
30. B. lb. 101.95
31. Grs. 3608.1
32. — 3550.6
33. Oz. 7 10 $10\frac{3}{4}$
34. I. G. 9.237
35. I. B. 1.6918
36. In. 33.385
37. — 12.356
38. Cts. $25\frac{1}{2}$
39. — 12.1
40. Sch. 6.6

Ex. 41. Kr. 12.2
42. Cts. 44.3
43. Bat. 1.91
44. Farth. 2.86
45. — 1.12 n.
46. Pence $1\frac{3}{4}$
47. Fr. 25 $47\frac{3}{4}$
48. — 25 54.3
49. Flor. 12 17.7
50. — 12 17.4
51. Bmf. 13 $12\frac{1}{2}$
52. — 13 7.68
53. Pence $48\frac{3}{4}$
54. — 49.8
55. Per cent. $\frac{3}{4}$
56. — 1.22
57. — $4\frac{1}{11}$
58. — 0.17
59. — 0.16
60. — 0.18
61. — 9.57
62. — 5.10
63. — $3\frac{1}{2}$
64. — 1
65. — $\frac{1}{3}$ Loss
66. — 2.1 Gain
67. Frs. 70.555
68. Sh. 3 $0\frac{7}{16}$
69. Pence 11.98
70. Fl. 37.79
71. L. f. b. 8.244
72. Pence 5.86
73. Oz. 10 10 n.
74. Sh. 61 9.6
75. — 77 7 n.
76. Pence 59.04
77. — 58.67
78. Prem. $13\frac{7}{8}$
79. — $1\frac{1}{4}$
80. Fl. 105.86.

FIXED NUMBERS.

A Fixed Number is a number which is either employed as the invariable term of a simple proportion, or as the result of the invariable terms of a compound or continued proportion.

The chief use of fixed numbers is to abbreviate those calculations which are frequently employed for the same purpose, but they are also used in the directions of formulæ for calculations in which no abbreviation is intended to be employed.

EXAMPLE 1.

In the Exchange between London and Leghorn, to exchange Sterling to Leghorn Pezze d'oro, and the reverse, at 47½ d. per Pezza, the equational statements are,

x £ Sterling?
1 — 240 d.
47½ — 1 Pezza.

x Pezza d'oro?
1 — 47½ d.
240 — 1 £.

Hence, in giving directions for such calculations, it is common to say, multiply the £ Sterling by 240, and divide the product by the rate of exchange; or, multiply the Pezze by the rate of exchange, and divide the product by 240. In each of which, 240 is called a fixed number, because it is always employed in these exchanges between London and Leghorn.

EXAMPLE 2. (See Arbitrations of Bullion, page 281.)

20?
77½* = 1 Oz. St.
12 = 11 Fine.
1 = 480 Grains.
15434 = 1 Kilo.
1 = 3434.44 Frs.
1000 = *1004½ with Sm.

Here the variable terms are, the price in London 77½ s., and the price in Paris 1004½ Frs., and the other terms are constant or fixed; compounding therefore the fixed terms, we have,

$$\frac{20 \times 11 \times 480 \times 3434.44}{12 \times 15434 \times 1000} = 1958.214 \text{ fixed number.*}$$

The use of which, in other arbitrations of Gold between London and Paris, is, that in multiplying by the variable rate on the right side of the equation, (or by adding parts for the rate per Mille to the fixed number,) and dividing the product by the variable rate on the left hand side of the equation, the result is obtained without any of the work which involves the fixed number: as with the above rates 77½ and 4½.

$$\begin{array}{rrl} & 1958.214 & \times\ 4\text{-}1000\text{ ths} \\ & 7.832 & \text{for}\ \ 4 \\ & 979 & -\ \ \frac{1}{2} \\ \cline{2-2} 77\frac{1}{2}\ ) & 1967.025 & \times\ 4 \\ \cline{1-2} 310\ ) & 7868.100 & \\ \cline{2-2} & \text{Frs. } 25.38\frac{1}{10} & \end{array}$$

It is to be observed, that in the forming of fixed numbers, the extent to which they are to be carried necessarily depends on the degree of exactness to which the final result is required to be obtained; thus in the above, the fixed number is nearly eight times the amount of the final result, and therefore to whatever nicety we carry the fixed number, it will serve to obtain the result with nearly eight times this precision.—If the fixed number is carried to centimes, we may get the result pretty accurate as far as millièmes; it is, however, to be observed, that when such fixed numbers are found as far as is thus judged necessary, the decimal valuation needs not be regarded, as the value of the figures in the result is always known; consequently in working for the fixed number, and generally in working all arbitrated equations, the numbers 10, 100, 1000, &c. on either side of the equation, may with propriety be omitted,* as in the above.

COMPARATIVE
ARBITRATED PARS OF EXCHANGE.

Comparative arbitrated Pars of Exchange, are the rates of Exchange established between two places from their courses of Exchange of bills on other places, for the purpose of comparing these Pars with the short prices of bills on either or both of the places of operation, in order to find whether indirect bills are better than direct bills for either making or obtaining remittances.

Example 1.

The prices of bills in London being

On Paris, short	Frs.	25 65	per £ sterling.
Hamburg	Mks.	13 13	— —
Amsterdam	Flor.	12 6	— —
Vienna	Flor.	10 4	— —
Madrid	Pence	36¾	per Dollar of Plate ;

and the prices of bills in Paris being

On Hamburg	Frs.	185½	for	100 Marks,
Amsterdam	Flor.	57½	-	120 Francs,
Vienna	Frs.	256	-	100 Florins,
Madrid	Frs.	15.70	-	4 Dollars ;

it is required to find whether any indirect paper is better than direct paper, for making remittances to Paris.

The object of these calculations may be thus explained. If I have to make a remittance to Paris, I may do so either by purchasing bills on Paris, for my correspondent to receive the amount from the parties on whom they are there drawn, or by purchasing bills upon some other place, and sending them to Paris to be sold by him ; and it is evident, that if the indirect bills will produce a greater amount in French money than direct bills for the same amount in sterling, it will be better to remit them, provided the advantage is sufficient to cover the extra charges. These calculations are therefore made to find how much the fixed term of our rate of Exchange on Paris, which is £ 1 sterling; will produce from indirect paper upon each place on which bills can be bought in London and sold in Paris, in order to compare these results with the direct rates, and find whether and how much any one of them may be better than the direct rate.

This subject requires much practical knowledge of the operations of bills of Exchange, and the learner is referred for further information to the " New Cambist or Manual of Exchanges," by W. Tate, sold by the publishers of this work.

1 st statement. For Hamburg paper.

1 £?
1 — 13 13 Mks. and Sch.
100 — $185\frac{1}{2}$ Francs.

Frs. $185\frac{1}{2} \times 13\frac{13}{16} \div 100 =$ Frs. 25 62 Cts.

2 nd Statement. For Amsterdam paper.

1 £?
1 — 12.30 Flo. (6 St. = 30 Cts.)
$57\frac{1}{2}$ — 120 Francs.

Frs. $120 \times 12.30 \div 57\frac{1}{2} =$ Frs. 25 67 Cts.

3 rd statement. For Vienna paper.

1 £?
1 — 10 4 Fl. and Kr.
100 — 256 Francs.

Frs. $256 \times 10\frac{4}{60} \div 100 =$ Frs. 25.77.

4 th statement. For Madrid paper.

240 Pence?
$36\frac{3}{4}$ — 1 Dollar.
4 — 15.70 Francs.

(Frs. $15.70 \times 240) \div (36\frac{3}{4} \times 4) =$ Frs. 25.63.

Comparison of the arbitrated Pars with the short price for direct bills on Paris.

Hamburg	25.62	with 25.65 is	3	Cents	lower.
Amsterdam	25.67	—	2	—	higher.
Vienna	25.77	—	12	—	higher.
Madrid	25.63	—	2	—	lower.

Hence it appears that bills on Vienna are better paper for remittances to Paris than direct paper, by 12 Cents per £ sterling, or about ½ per ct.

It is, however, to be observed, that indirect paper occasions an extra Brokerage of 1-10 th per ct., and also a Commission, unless the remittance is upon joint account, or the sale and purchase are free of Commission.

Respecting the term *short* affixed to the preceding rate of direct bills on Paris, it is to be remarked, that it refers to the prices of bills at sight, or rather to the net price of bills having any time to run, after taking off the discount for that time, because indirect bills are sold for present payment, and therefore to make a just comparison, the direct price must be brought as nearly as possible to the same point of time.

EXERCISES.

Ex. 1. Find whether bills on any of the following places give a higher arbitrated rate for making remittances to Paris, than direct bills on that place at the Exchange of 25.70 short.

	London Prices.	Paris Prices.
Hamburg	13 14	185
Amsterdam	12 7	57¼
Vienna	10 6	254
Madrid	36⅛	15.75

Ex. 2. Find whether bills on any of the following places give a higher arbitrated rate for making remittances to Paris, than direct bills on that place at the Exchange of 25.95 at 3 months,* discount 4 per ct. per annum.

London Prices.

Hamburg	13 13½	
Amsterdam	12 6	
Vienna	10 5	
Leghorn	48¼	(pence per Pezza)
Naples	39¾	(pence per Ducat)
Madrid	36¾	

Paris Prices.

Hamburg	186	
Amsterdam	57½	
Vienna	254½	
Leghorn	518	(Francs per 100 Pezze)
Naples	423	(Francs per 100 Ducats)
Madrid	15.80	

---

* This being the price of direct bills at 3 months, it must be discounted, to reduce it to the short price, by taking off 1 per ct. for the interest for 3 months at 4 per ct., 1 per ct. on Frs. 25.95 is reckoned at 26 Cents.

Ex. 3. Find whether bills on any of the following places give a higher arbitrated rate for making remittances to Amsterdam, than direct bills on this place at the Exchange of Flor. 12 1¾ St. (or Flor. 12.08¾ Cents) at 3 months, discount 4 per ct. per annum.

London Prices.

Paris . . .	25.60	
Hamburg . .	13 12	
Vienna . .	10 1	
Leghorn . .	48½	
Genoa . . .	25.35	(Lire Nuove per £ sterling)
Naples . . .	40⅛	

Amsterdam Prices.

Paris	57¼	Florins per	120 Francs.
Hamburg . . .	35	—	40 Marks.
Vienna . . .	36⅛	—	30 Vienna Flor.
Leghorn . . .	97¼	—	40 Pezze.
Genoa	47½	—	100 Lire N.
Naples	79¾	—	40 Ducats.

PRODUCTS.

Ex. 1. Paris short. 25.70.

Hamburg .	25.67	3	Cents	Worse.
Amsterdam . .	25.66	4	—	—
Vienna . . .	25.65	5	—	—
Madrid . . .	25.89	19	—	Better = ¾ per ct.

Ex. 2. Paris 25.95, short 25.69.

Hamburg . .	25.75	8	Cents	Better.
Amsterdam . .	25.67	2	—	Worse.
Venice . . .	25.66	3	—	Worse.
Leghorn . . .	25.76	7	—	Better.
Naples . . .	25.54	15	—	Worse.
Madrid . . .	25.79	10	—	Better = ⅖ per ct.

Ex. 3. Amsterdam 12.08¾, short 11.96¾.

Paris . . .	12.21¼	24½	Cents	Better = 2 per ct.
Hamburg . .	12.03	6¼	—	—
Vienna . . .	12.06	9¼	—	—
Leghorn . .	12.03	6¼	—	—
Genoa . . .	12.04	7¼	—	—
Naples . . .	11.92½	4¼	—	Worse.

BRITISH AND FOREIGN

INVOICES AND ACCOUNT-SALES.

An Invoice is a mercantile document containing a description of goods sold or sent for sale, with an account of the charges, if any, that are made to the buyer or consignee.

Inland Invoices are usually made out in the form of bills of parcels, containing in the head, the place, date, and names of the parties. Shipping or exportation Invoices are generally headed with a short account of the goods shipped, the names of the vessel and captain, the port of destination, the name of the consignee, and a specification of the account on which the goods are sent.

Account-Sales are accounts of the sales of goods, with the attendant charges, for the purpose of showing their net proceeds. When the goods have been sold on commission or on joint account, an Account-Sales is a document rendered to the sole or joint owner.

Pro-formâ Invoices and Account-Sales are statements of suppositious transactions, made out for various purposes. When separately made out, they are used to show the ordinary allowances and charges on goods, and consequently, with the prices, to show the estimated amount or net proceeds. When the two statements are combined, they serve to show either the probable result of the purchase and sales in an estimated gain or loss, or an arbitrated rate of exchange; or, when the price at one place, with the charges at both places of operation, and in foreign transactions, the rate of exchange, are given, what arbitrated price at the other place will cover the given price and all the expenses.

Inland Invoices are sometimes distinguished as being buying or selling Invoices, according as they relate to goods bought or goods sold. Thus the first two of the four following Invoices, which are made out in the form of Bills of Parcels, are the buying Invoices of Pepys & Peacock, and the second two are the selling Invoices of the same parties.

London, 4 January, 1834.

Messrs. Pepys & Peacock,

Bought of G. Swainson & Co.

20 Puncheons Rum.

No. 1.	Gall.	97	No. 11	Gall.	101
2.	-	95	12	-	94
3.	-	97	13	-	94
4.	-	101	14	-	99
5.	-	94	15	-	96
6.	-	97	16	-	92
7.	-	96	17	-	93
8.	-	99	18	-	92
9.	-	103	19	-	96
10.	-	90	20	-	90

Gallons at 3 s. 6 d. on board £ 335 6 0

London, 11 January, 1834.

Messrs. Pepys & Peacock,

Bought of L. & F. Lamb,

10 Hhds. prime Kentucky Leaf Tobacco.

	cwt.	qr.	lb.
No. 22.	11	3	17
23.	12	2	3
24.	12	0	6
25.	13	2	18
26.	15	1	22
27.	14	2	4
28.	17	3	17
29.	15	0	27
30.	12	3	9
31.	11	2	19

Shrinkage 17 lb. each.

1 0 3 Damage on 5 numbers.

Net = lb. at $3\frac{3}{4}$ d. . . . £ 236 12 10

N. B. For the explanation and products of the calculations belonging to this section, see the end of these statements, page 327.

London, 4 January, 1834.

Mr. Caleb Stewart,

Bought of Pepys & Peacock.

26 Hhds. middling Jamaica Coffee.

10 hhds.	77	1	27	Tare	8	0	6	
10 -	75	3	2	-	7	3	22	
6 -	44	0	20	-	4	2	10	
								Draft 5 lb. per hhd.
Net				at 70 s. . . . . £				
				Discount 1 per cent. . .				
								£ 608 15 1

London, 8 January, 1834.

Messrs. Wood & Rolls,

Bought of Pepys & Peacock.

20 Bales Sea Island Cotton.

No. 43.	2	2	6	No. 51.	2	3	4
45.	2	1	12	48.	2	2	22
35.	2	1	27	71.	2	1	19
37.	2	2	11	75.	2	2	25
52.	2	2	3	76.	2	1	15
67.	2	1	18	60.	2	1	23
65.	2	0	24	69.	2	2	9
66.	2	2	11	46.	2	2	1
56.	2	2	13	68.	2	2	0
50.	2	1	26	82.	2	2	22
10 Bs.				10 Bs.			
				10 Bs.			

Draft 1 lb. per Bale

Tare 4 lb. per cwt.

Net 48 2 13 = lb. at 7⅛ d. £

Discount 1⅛ per cent.

£ 159 4 6

Invoice of 10 Chests of Indigo shipped on board the Harmony, Capt. Richards, for Hamburg, consigned to Messrs. T. & C. Meyer, by their orders and on their account and risk.

T. M.M. C.	10 Chests Indigo.												
	Gross.			T. & D.		Net.							
No.	cwt.	qr.	lb.	qr.	lb.	lb.							
1	3	0	17	3	2		. . . at 8 s. 9 d.						
2	3	0	27	3	24	1060	- 8 s. 6 d.						
3	3	0	20	3	1								
4	3	0	11	3	5								
5	3	1	4	3	8								
6	2	3	23	3	14		- 7 s. 3 d.						
7	2	3	26	3	5	983	- 6 s. 11 d.						
8	2	3	2	3	1								
9	3	0	6	3	13								
10	3	1	4	3	16								
							£				991	14	7
	Charges.												
	Entry, bond, &c.							£ -	12	6			
	Packing, mending, and lot money							2	10	0			
	Cartage, wharfage, and shipping							1	2	6			
	Lighterage, watching, and bills of lading							1	0	6			
	Brokerage on £ 991 14 7 at ½ per cent.												
	Insurance on £ 1100 at 12 s. 6 d.												
	Policy							-	16	6			
							£						
	Commission on £ 1009 13 3 at 2 per cent.												
							£				1029	17	1

London, 10 Sept. 1833,
Errors Excepted.
Pepys & Peacock.

Invoice of 10 Butts of Zante Currants, shipped on board the Bolsano, Capt. Wright, for Altona, consigned to Messrs. Danner & Co. for sale on joint account of themselves and the shippers in halves.

No.			£	s.	d.
	10 Butts Currants.				
No.	Butt. cwt. qr. lb.				
1	1 21 0 14				
2	1 20 3 11				
3	1 21 2 6				
4	1 21 1 24				
5	1 20 3 25				
6	1 22 0 2				
7	1 21 1 16				
8	1 21 0 22				
9	1 20 3 11				
10	1 20 3 17				
	Tare 18 lb. per cwt.				
	Net 178 0 22 at 66 s. per cwt.	£			
	Discount 1 per cent.				
		£			
	Less Duty on cwt. 178 1 16 at 44 s. 4 d.		186	14	7
	Charges.				
	Entry, Bond, and Certificate .	£ - 12 6			
	Wharfage and Shipping . . .	2 2 6			
	Cooperage and Marking . . .	1 10 0			
	Lighterage and Watching . .	1 7 0			
	Bills of Lading and petty Charges	- 9 0			
	Bill brokerage, Stamps, & Postages	- 15 0			
	Brokerage ½ per cent. on £ 582 3 4				
	Insurance on £ 200 at 15 s. .				
	Policy	- 3 0			
		£			
	Commission 2 per cent. on £ 198 1 10 .				
	London, 5 Oct. 1833.	£	202	1	1

ACCOUNT-SALES of 30 Puncheons of Rum received per Alathea, Phillips, from Jamaica, and sold on account of Abel Smith, Esq.

1833								
Aug.	20	Sold to Jones, Scott, & Co. the whole.						
				103-101				
				101- 97				
			95-91	104- 98				
		98- 94	95-89	95- 89				
		95- 89	92-88	97- 93				
		94- 88	97-93	102-100				
		90- 81	96-92	94- 89				
		99- 95	98-94	100- 94				
		98- 92	99-94	97- 95				
		103-100	97-90	96- 91				
		101- 97	96-91	———				
		103-100	98-92	947				
		96- 92	——— . .	914				
		———	. . . . .	928				
		Gallons at 3 s. .						
		Landing Charges 5 s. per Puncheon						
					£	425	17	0
		Charges.			£ s. d.			
Mar.	14	Insurance on £ 300 at 50 s. Policy 15 s. Commis. 30 s.						
Aug.	8	Freight on 2789 Gall. at 6 d.						
		Primage, Pierage, and Trade .			1 16 3			
	22	Dock Rates at 10 s. per 100 gal.						
		Interest, Stamps, and Entry .			- 14 0			
		Brokerage ½ per cent. . . .						
		Commission 2½ per cent. . .						
		Net Proceeds in Cash Sept. 26.			£	317	2	11

ACCOUNT-SALES of 180 Bags of Pimento, received per Diamond, Bruce, from Jamaica, and sold on account of Abel Smith Esq.

1833								
Aug.	14	Sold at Public Sales the whole.						
		20 Bags 19 2 7						
		20 — 20 1 12						
		20 — 19 2 8						
		20 — 21 0 6						
		20 — 21 0 10						
		20 — 20 1 9						
		20 — 20 1 3						
		20 — 19 3 23						
		20 — 20 0 6						
		180						
		Draft 1 lb. per Bag.						
		Tare 4 lb. per cwt.						
		Net = lb.						
		Tret						
		lb. Net at 5 d.						
		2 Bags, Sweepings Net 168 - 2 d.						
		£						
		Discount 1 per cent. . .						
		£				388	5	10
		Charges.	£	s.	d.			
Mar.	14	Insurance on £ 360 at 50 s. Policy 20 s.—Commis. 36 s.						
July	11	Freight on 181 1 27 at ¾ d. per lb.						
		Primage, Pierage, and Trade	3	11	3			
		Dock Rates on 181 1 27 at 1 s. 2 d.						
		Lotting and Fees of Entry .	1	8	6			
		Interest and Stamps	-	17	6			
		Broker. on 388 5 10 at 1 per ct.						
		Commis. on do. 2½ -						
		Net Proceeds in Cash, Sept. 19. £				282	18	6

ACCOUNT-SALES of 430 Doubloons and 37 Pieces of Gold, received by the Exmouth Packet from Rio de Janeiro, and sold for account of Messrs. Gore, Paira, & Co.

1833						
May	10	Sold to the Bank of England.				
		430 Doubloons melted into 2 Bars.				
		No. lb. oz. dw. gr. c. gr. Worsen.				
		1. 15 6 17 12 Worse 1 0½ oz.				
		2. 15 6 11 0 Worse 1 1¼ -				
		Worseness				
		lb.				
		oz. Standard at 77 s. 10½ d.				
	14	37 Pieces of Gold melted into 1 Bar.				
		No. lb. oz. dw. gr.				
		3. 17 9 17 12				
		Betterness at 2¾ grs.				
		lb.				
		oz. Standard at 77 s. 10½ d.				
		Charges.				
		Melting Charges.				
		Grains 18½ dwts. at 3 s. 10 d. £				
		Melting at 8 d. per lb. . . .	1 12 8			
		Porterage	- 4 0			
		£		2233	19	10
		Freight on £ 2232 5 7 at 2½ per cent. . . . . . . £				
		Carriage from Falmouth ¼—				
		Assays 15 s.—Booking 6 d. .				
		Broker. ⅛ per ct. on £ 2232 5 7				
		Com. ½ per ct. on £ 2233 19 10				
		Net Proceeds in Cash, May 12. £		2157	17	5

ACCOUNT-SALES of 6 Bars of Silver received by the Aurora, Fleming, from Hamburg, and sold on account of Messrs. Le Breton and Co.

1833 Aug.	7	Sold to the Bank of England.			

No.	lb.	oz.	dw.		dw.	Worsen.		
1	22	10	0	Better	2½	-	-	-
2	27	4	10	Worse	6	-		
3	25	6	0	—	7	-		
4	25	11	0	—	7½	-		
5	26	7	10	—	9	-		
6	23	8	0	—	1 oz. 6	-		
					Worseness	6	3	5
					No. 1. Betterness			
Wor.								
lb.								

oz. 1750 17 Standard at 59⅞ d. . .							
			£	s.	d.			
Freight as per Bill of Lading .			1	17	6			
Melting at 20 s. per 100 lb.	1 10 4							
less grains . . .	5 2							
			1	5	2			
Weighing, Porterage, &c. . .				7	0			
Mint Assays, 6 at 2 s. 6 d. .								
Brokerage ⅛ per cent. . . .								
Commission ½ per cent. . .								
Net Proceeds £						429	16	9

PRO FORMA INVOICE AND ACCOUNT-SALES of 5 Chests of Bengal Indigo sent from London to Hamburg.

5 Chests Indigo.						
Gr. 1775 Tare 474						
Dft. 5						
Net at 5 s. . . . . £						
	£	s.	d.			
Entry, Bond, and Clearing . . .	-	15	0			
Lot Money, Casing, Marking, Cartage, Wharfage, and Shipping .	2	14	0			
Brokerage and Bills of Lading .	1	17	0	4	6	0
Insurance on £ 400 at 7 s. 6 d. .						
Policy at 1 s. 6 d. .						
£				330	2	0
Rendered at Hamburg.						
Gr. 1648 lb. Tare 440						
Dft. 8 . . ½ per ct.						
Net lb. at 4 Mks. . . . . Bmf.						
Freight 30 s. at 13.8 . . Bmf.						
Primage 15 per cent. . . .						
Bmf.						
Landing Charges . . Cmf. 7.8						
Duty ¼ per cent.						
Warehousing and Porterage 6.0						
Stadte Duty. 8.12						
Cmf. 46.4						
at 123 per cent. Bmf.						
Fire Insurance ⅛ per cent. . . .						
Brokerage . . . . ⅙ do. . . .						
Commission and Del Credere 3 per cent.				250	15	0
Net Proceeds Bmf.				4549	1	0
£ 330 2 0 on B. Mks. 4549 1 -						
is B. Mks. per £ Sterl.						

PRO FORMA INVOICE AND ACCOUNT-SALES of 100 Bags East India Coffee sent from London to Antwerp.

cwt. qr. lb. Gross 125 2 8 Tare 2 } per bag. 3 2 8 Dft. 2 }				
at 60 s. . . .				
	£ s. d.			
Entry, Bond, and Custom-House Charges	1 10 6			
Lighterage, Wharfage, and Shipping	6 5 6			
Brokerage ½ per cent.				
	£	375	12	7
Rendered at Antwerp.				
Gross 13499 lb.				
2 per ct. allowance for weights				
2 per cent., Tare				
Net 12964 at 7¾ sous . . .	Flor.			
Discount 1½ per cent. . . .				
	Flor.	4948	20	
	Florins.			
Insurance on Flor. 5000 at ½ per ct.				
Freight and Primage	60.60			
Duty on 5777 Kilog. at 2 Flor. per 100 Kilog.				
Syndicat 13 per ct. on do.				
Landing and Housing	9.40			
Commis. 2 per cent. on Fl. 5023.55				
Net Proceeds	Flor.	4622	17	
£ 375 12 7 on Flor. 4622.17 is Flor. cts. per £ Sterl.				

PRO FORMA INVOICE AND ACCOUNT-SALES of Logwood sent from London to Genoa.

1598 Pieces Logwood			
Tons 10 12 2 26			
Draft 12 lb. per ton.			
Net at £ 6 5 s.			
Discount 2½ per cent. .			
	64	9	5
Entry, Bond, and Clearing . £ 1 5 6			
Wharfage, Lighterage, & Shipping 1 15 0			
Brokerage and Bills of Lading . - 10 3			
Insurance on £ 80 at 20 s. . . -			
Policy . . . . . - 1 6			
£	68	17	8
Rendered at Genoa.			
Cantari . . . 218.15			
½			
lb.			
Tare of usage 16 per ct.			
Net . . . 282.08 lb. at 8⅝ Lire per 100 lb.			
Freight and Primage . Lire 179 14 0			
Landing, Housing, Weighing . 40 0 0			
Warehouse Rent, Porterage, &c. 37 0 0			
Brokerage ½ per cent. . . .			
Commission and Del Credere 3 per cent.	342	4	0
L. F. B.			
Lire Nuove	1752	05	
£ 68 17 8 on L. N. 1752.05 c.			
is L. N. per £ Sterl.			

ACCOUNT-SALES of 17 Bars of Silver received per the Good Intent, and sold on account of Messrs. Pepys & Peacock, London.

Mks. l. l. grs.	Mks.	l.	gr.			
12399 10½ at 14 5½.. Fine	11086	7	17			
at Bmf. 27.10½ per Fine Mark						
Charges.						
	Bmf.	s.	pf.			
Receiving from on board, Carriage, &c.	138	4	0			
Stadte Dues	436	14	0			
Commission ½ per cent. . .						
Bmf.				304502	11	0
Hamburg, 14 Dec. 1832. P. S. Heisch.						

INVOICE of 8 Ingots of Silver sent from Hamburg viâ Osnabruck, Naarden, and Antwerp, to Messrs. Laurent & Co. of Paris, for account and risk of Messrs. Pepys & Peacock, London.

1 Ingot	Mks.	l.	gr.			
Mks. 115.14½ at l. 15.13½ Fine						
7 Ingots						
Mks. 866.4 at l. 15.14½ . . .						
Fine Mks.						
at Bmf. 27.8½ per fine Mk.						
Charges.						
Commission ½ per cent.. .	Bmf.					
Petty Charges	27	12	0			
Bmf.				26861	8	0
Hamburg, 10 th August 1832. P. S. Heisch.						

Compte de Vente de 8 Lingots d'Argent reçus de Hamburg, et vendus pour le Compte de Messrs. Pepys & Peacock, à Londres.

1 Lingot titre Prix. Kilo. 27.120...984...Frs. 215.39..				
7 Lingots Kilo. 202.590...989.......216.48..				
Frs.				
Prime 5½ per mille				
Frs.			49971	39
Frais à déduire.				
	Frs.	Cts.		
Transport de Hamburg et Factage	477	45		
Essayeur à 75 cts. per Lingot .				
Péseur à 25 - — . .				
Transports dans Paris et menus frais	10	60		
Provision à ½ per cent. . . .				
Net Produit . . . . Frs.			49225	48
Paris le 18 Août, 1832 Laurent & Comp^{ie}.				

Note of Purchase and Sales.

Purchase Aug. 10 th, drawn at 3 days sight.

Bmf. 26861.8 at 13.14 due Aug. 23 rd

Interest to Sept. 21, 29 days . . .

Sales Aug. 18 th, remitted at 1 month's date

Frs. 49225.48 at 25.20, due Sept. 21 st. . . .

Profit

£ on £ is nearly ½ per cent.

ACCOUNT-SALES of 200 Boxes Raisins, received from London, per the Frederick, Capt. Jones, and sold here on account of Messrs. Pepys & Peacock of that place.

200 boxes at 17 Roub. cash per box . . .			
Charges.			
Duty on Poods 126.31 at 540 cop. . . . . Rs.			
Quarantine Dues at 2 per cent. .			
Rs.			
Custom House charges 4 per cent.			
Receiving and delivering 45 cop. per box			
Freight, Sound Dues, and Watching	127.27		
Brokerage ½ per cent.			
Commis. and extra charges 3 per ct.		1062	48
Net Proceeds	Roub.	2337	52
St. Petersburg, 25th Aug. 1832			
Roub. 2337.52 at 10⅔¼ d. = £			

ACCOUNT-SALES of 10 Bales of Longcloths, and 5 Bales of Salampores, received per Triton, G. Rogers, Master, and sold on account of Messrs. Pepys & Peacock of London.

Ps. Doll.				
10 Bs. Longcloths 300 at 7				
5 - Salampores 300 - 3.4.8				
	Dollars	3112	6	0
Charges.				
Lighterage, Landing, & Cartage	Doll. 8 0 0			
Receiving, Stowing, &c.	3 6 0			
Store Rent ½ per cent.				
Brokerage ½ per cent.				
Commission 2½ per cent. . . .		120	5	4
	Dollars	2992	0	12
Gibraltar, 29 th Sept. 1832.				
Doll. 2992.0.12 at 51 d. = £				

ACCOUNT-SALES of 100 Bags Coffee, received per the Hardy, Capt. W. Stone, and sold on account of Messrs. Pepys & Peacock, London.

Net lb. 15192 at 16 grotes. . .			
Charges.			
Freight as per Bills of Lading £ 8 2 at 612 . . . L. d'or R. D.			
Duty at Elsfleth	37.24		
Lighterage 2½ R. D. per Last on 15284 lb.			
Cranage and Cartage 3 gr. per Bag			
Cooperage, &c. 14 gr. do. .			
Warehouse Rent and Petty Expenses	24.15		
Brokerage ¼ per cent.			
Commission & Del Credere 3 per ct.			
Net Proceeds L. d'or R. D.		3122	0
Bremen, May 27 th, 1832.			

ACCOUNT-SALES of 60 Bags Sugar, received per the Dolphin, Hy. Jukes, Master, and sold as follows for Messrs. Pepys & Peacock, London. By Petroni & Co. Constantinople.

10 Bags 567 Okes Tare 10 do. Net 557 at P. 4 15 c. Piast.		
50 Bags 2782 Tare 50 Allowance 4.25 Net 2727.75 at 4.10 .		
Piast.		
Charges.		
Freight & Primage £ 6 17 6 at 74 P.		
Duty on Cant. 82 5 Okes at 900 120		
Dritti 10 per cent. . .		
Brokerage at 2 per cent.		
Loss of Money 1 per cent.		
Commission 2 per cent.		
Net Proceeds Piast.	11634	27

ACCOUNT-SALES of Spelter imported per Hastings, Wyld, and sold on account of Messrs. Pepys & Peacock, London.

Ramnerain, Dey, & Co. for Cash less 4 mo. 3 da. Disc. at 10 per ct.				
3997 wg. Cwt. 982 1 24 or Fy. Maunds 1473 27 14 at Rs. 21 6 per Maund. and at 116 per 100 Sicca Rupees				
Less Disc. as above				
Charges.		26227	9	3
Customs' Duty . . . . Rs.	774 8 0			
Landing and Delivering Charges, Watching, and Weighing	172 0 0			
Commission at 5 per cent. . .				
Rupees		23969	11	0
Calcutta, May 17 th, 1832.				

INVOICE of 20 Bags of Cotton, shipped per Hero, W. Bruce, Master, for Liverpool, to the consignment of Messrs. Pepys & Peacock, London.

No. 1 to 20.—20 Bags. Wg. Arr. 101 28 lb. at 3500 Reis per Arr.			
Charges.			
Duty on Arr. 100 20 lb. at 600 Rs. Rs.			
Contribution 100 Rs. per Bag . . .			
Mending and Marking	6.400		
Porterage and Bills of Lading . .	1.241		
Commission 3 per cent.			
Reis		439	376
Maranham, July 1 st, 1831.			

SALES OF GUNPOWDER received from on board the Diligent, Capt. Dickson, and sold on account of Messrs. Pepys & Peacock of London,

by John Pritchard.

Mar.	28	Sold to Dodd & Brown,		
		21 Half Barrels Net 1050 lb. at 50 cts.		
Nov.	18	Sold by Auction, by J. Harris.		
		10 Half Barrels Net 500 lb. at 36 cts.		
		6 do. 300 - - 34		
		13 do. 650 - - 32		
		3 Whole do. 300 - - 32		
		U. S. D.		
		Charges.		
		doll. cts.		
		Advertising and Carriage to Magazine 14.75		
		Inspection 50 cents per ½ Barrel . .		
		Receiving 10 - — . . .		
		Warehouse Rent, Storing, &c. . . 20.10		
		United States Duty on 2800 lb. at 8 cents		
		Auction Commission on D. 586 at 2½ per cent		
		Commission on Doll. 1111 at 2½ per cent		
		Del-Credere on do. at 2½ per cent. .		
		Net Proceeds U. S. D.	748	34
		New York, Dec. 24, 1834.		
		Remitted for the above, Draft of J. Mills on Baring & Co., London, at 60 days' sight, at the exchange of 8¾ per cent. £ Sterl.		

PRO FORMA INVOICE AND ACCOUNT-SALES of German Wool, bought in London and sold in Boston.

16 Bales German Wool.				
Net 7879 lb. at 1 s. 4 d.				
Discount 5 per cent.				
	£			
Charges in London.				
Custom House and Shipping Charges	£3 3 8			
Brokerage ½ per cent.				
Insurance on £525 at 30s. per ct.				
Policy Stamp at 1 s. 6 d.				
	£			
Commission 2½ per cent.				
	£	525	16	7
Exchange at 108 per cent.	U. S. D.	2523	98	
Charges in Boston.				
Freight on 8176 lb. at ½ d. per lb. and Primage 5 per cent. .	£ 17 17 9			
Exchange at 4 s. 6 d. U. S. D.				
Duty on 8000 lb. at 4 cts. . . U. S. D.				
on £517 value at 40 per cent.				
	1312.64			
Custom House & local Charges	29.20			
Fire Ins. ⅛ per cent. on 4489.69				
Commission 5 — —				
Guarantee 2½ — —				
Interest 9 months at 6 per cent.				
	U. S. D.	4489	69	
Weight in Boston.				
Gross 8176 lb.				
Tare 224				
Net 7952 at Cents per lb.	U. S. D.	4489	69	

PRO FORMA INVOICE AND ACCOUNT-SALES of Georgia Cotton bought in New York and sold in London.

100 Bales weighing (no Tare allowed)				
Gross 38180 lb. at 10 Cts. per lb. U. S. D.				
Charges in New York.				
Cartage . . . . U. S. D.	6.25			
Brokerage 12 Cts. per Bale . .				
Insurance on U. S. D. 4100 at 1½ per cent.				
U. S. D.				
Commission 5 per cent. . . .				
U. S. D.				
Exchange at 4.80 . . . £		852	12	8
Charges in London.				
Freight on 38180 at ½ d.. . £				
Primage 5 per cent. . .				
£				
Landing, Warehousing, &c. . . .	17 4 1			
Fire Ins. ⅛ per ct. on £ 994 7 6				
Brokerage ½ — —				
Commission 2½ — —				
Del Credere 1 — —				
£				
Discount 1½ per cent. on £ 1009 10 4				
£		1009	10	4
Weight in London.				
Gr. 340 3 16				
Dr. 1 lb. per Bale.				
340 0 0				
Tare 4 lb. per cwt.				
Net 327 3 12 = lb. at d. per lb. £		1008	16	10

Pro Forma Invoice and Account-Sales of Indigo bought in London and sold in New York.

10 Chests, weighing Net. 2576 lb. . at 4 s. 6 d. . . £				
Charges in London.				
Custom House and Shipping Charges	£ 6 4 6			
Brokerage ½ per cent. . . .				
Insurance on £ 610 at 30 s. . .				
Policy Stamp at 1 s. 6 d. .				
	£			
Commission 2 per cent.				
	£	610	7	4
Exchange at 108 per cent. U. S. D.				
Charges in New York.				
Freight 140 ft. at 30s. per ton £				
Primage 5 per cent. . .				
	£ 5 10 3			
at 4 s. 6 d. per Dollar U. S. D.				
Duty on £ 600 at 4.80				
U. S. D. 2880				
at 15 per cent. U. S. D.				
Discount 1½ per cent. .				
Entry, Cartage, &c.	27.51			
Fire Ins. ⅛ per ct. on U. S. D. 3833.80				
Brokerage ½ — —				
Commission 5 — —				
Del Credere 2½ — —				
Interest 6 months at 6 per cent. .				
	U. S. D.	3833	80	
Weight in New York.				
Net 2590 lb. at per lb.	U. S. D.	3833	20	

EXPLANATIONS AND PRODUCTS OF THE PRECEDING INVOICES AND ACCOUNT-SALES.

INVOICE OF RUM.

This purchase having been made for exportation, the term *on board* affixed to the price, means that the seller has agreed to deliver the goods on board the vessel, free of all shipping charges.

PRODUCT.

Number of gallons 1916.

INVOICE OF TOBACCO.

The weight given to each hhd. is the remainder after having deducted the Tare. The Shrinkage is a regular customary allowance. The allowance for damage is settled between the buyer and seller.

PRODUCTS.

Amount of weight cwt. 137 3 2 Shrinkage cwt. 1 2 2
Net weight lb. 15145

INVOICE OF COFFEE.

Coffee is usually sold in London for cash in 1 month, with the allowance of 1 per cent. on British Plantation Coffee, and 2½ per cent. on foreign Coffee.

PRODUCTS.

Tare and Draft cwt. 21 3 0 Net cwt. 175 2 21
Amount £ 614 18 1 Discount £ 6 3 0

INVOICE OF COTTON.

The Tare is to be reckoned upon the remainder after the Draft has been subtracted. The Discount is the allowance, in London, for payment in cash at 1 month from the day of sale.

PRODUCTS.

Weights 10 bales cwt. 24 3 11 10 bales cwt. 25 3 0
Tare cwt. 1 3 6 Net lb. 5445 Value £ 161 13 0

SHIPPING INVOICE OF INDIGO.

The Gross weights, the Tares and Drafts, and the Net weights of the chests, are here arranged in columns, upon the latter of which the amounts are to be extended at the prices charged, taking the total of those Net weights which are charged at the same price.

The amount of the Charges is to be extended and added to the value of the goods, and the Commission is to be charged upon the sum of these two amounts.

The subscription to the Invoice contains the place and date of shipment; the words *errors excepted,* to give the Consigner the power, if necessary, of correcting any errors or supplying any omissions that may hereafter be discovered; and it concludes with the signature of the firm.

Similar subscriptions are annexed to all commercial accounts, but in copying them into the books they are generally omitted, except the dates which are retained in the copies of Invoices.

PRODUCTS.

	£	s	d		£	s	d
1 st Amount	116	16	3	2 nd Amount	450	10	0
3 rd —	84	9	3	4 th —	339	19	1
Brokerage	4	19	2	Commission	20	3	10

SHIPPING INVOICE OF CURRANTS.

In this Invoice the goods are charged at the long price, or the price duty paid, from which amount, after deducting the amount of the discount, the duty is deducted. Hence the Consignee is charged for only the Net amount. The drawback of the duty is calculated upon the Net Landing weight, and the selling price, upon the Net weight produced by re-weighing the goods to the buyer.

The Commission is charged, as in the preceding Invoice, upon the amount of the goods and charges.

PRODUCTS.

Gross weight	cwt. 212	1	8	Gross Amount	£ 558	1	0
Discount	£ 5	17	8	Duty	395	8	9
Brokerage	2	13	3	Commission	3	19	3

IMPORTATION ACCOUNT-SALES OF RUM.

The contents of these 30 Puncheons are arranged in three divisions, each containing the full and ullage contents of 10 Puncheons. The ullage contents are the quantities of Spirits contained in the casks, as found by gauge at landing; the value is to be found from these contents. In the sale of Rum, by the importer, it is customary to charge the purchaser 5 s. per Puncheon for Landing Charges, in addition to the amount of the Spirits.

The first item of the Charges is the expense of the Insurance; and here it is to be observed, that in the West India trade it is customary to charge a separate Commission on the amount insured. The Freight is charged on the landing gauge, in addition to which a charge is made for Primage (originally a donation to the captain of the Vessel) and also for Pierage, which is a contribution towards the expense of Ramsgate Pier. These three charges are paid to the ship's broker, but the additional charge for trade is a contribution paid by the importer to the West India Trade Association. The Dock Rates are paid to the West India Dock Company for warehouse rent, &c. The charge for Interest is for advance of money for Freight, Dock Rates, &c.; and that for Entry is for Custom House Fees. The Brokerage and Commission are charged by the broker and agent on the gross amount of the sales.

The amount of the charges is taken from that of the sales, and the remainder is the net proceeds, which is stated to be due to the owner of the goods on Sept. 26; because the goods are sold at 1 month's credit, and then the money is to be received on the following Saturday.

PRODUCTS.

Insurance	£ 9	15	0	Freight	£ 69	14	6
Dock Rates	13	18	10	Brokerage	2	2	7
Commission	10	12	11	Amt. of Charges	108	14	1

ACCOUNT-SALES OF PIMENTO.

This Account-Sales, like the last, is of West India produce, but it has been sold at public sales, and consequently the Brokerage is 1 per cent.

PRODUCTS.

Gross weight	cwt.	182	1	0	Tare	cwt.	6	1	23
Tret	lb.	750			Net	lb.	18759		
Gross amount	£	392	4	3	Insurance	£	11	16	0
Freight		63	10	6	Dock Rates		10	11	9
Brokerage		3	17	8	Commission		9	14	2

ACCOUNT-SALES OF GOLD.

As it is stated in the Account-Sales, this Gold is sold at two sales, the first of the Doubloons, and the second of the remainder. Both parcels are stated to be sold to the Bank of England, which is always done when the brokers of that company are employed to effect the sales, the name of the actual purchasers not being declared by them. The full weights are to be reduced to standard weights, (see "The Standarding of Gold and Silver,") and the value of each is to be calculated at the given price. To their amount is to be added the balance of the Melting Charges, that is, the remainder after deducting the charge for melting and porterage, from the value of the grains or the result of the scrapings of the melting pots, which grains are kept by the melters, and allowed for in their account.

Respecting the following charges it may be observed, that this Gold having been brought to this country by a government packet, and brought from the port it arrived at to the Bank of England, the charges for Freight and Carriage, as well as those for the Mint Assays and Porterage, are paid to the Bank, as the Company are answerable to the captain of the vessel for the amount.

PRODUCTS.

Worseness No. 1.	oz.	9	11	3	No. 2.	oz.	11	2	14
Stand. Weight	oz.	352	14	19	Value	£	1373	9	7
Betterness, No. 3.	oz.	6	13	16	Stand. Wt.	oz.	220	11	4
Value of No. 3.	£	858	16	0	Value of Grs.	£	3	10	11
Amt. of Freight		55	16	2	Carriage	£	5	11	7
Brokerage		2	15	10	Commission		11	3	4

ACCOUNT-SALES OF SILVER.

This Silver was imported by a private vessel; it was re-melted and sent to the Bank, and sold by their brokers. With respect to the allowance for Grains it will be seen, that their value, as is always the case with Silver, is of less amount than the expense of melting, and consequently the difference forms one of the charges to be subtracted from the value of the Silver.

PRODUCTS.

Worseness, No. 2.	oz.		8	18	No. 3.	oz.		9	13
4.			10	10	5.	lb.	1	0	19
6.	lb.	2	9	5	Betterness 1.			3	2
Standard Wt.	oz.		1750	17	Value	£	436	16	0
Brokerage	£	0	10	11	Commis.	£	2	3	8

Pro Forma Invoice and Account-Sales of Indigo.

In Hamburg accounts it is usual to employ three columns for Marks, Schillings, and Pfenings, but the latter is seldom used except for 6 Pfenings or ½ Sch. In this Account-Sales, the amount of the Freight, 30 s., is to be exchanged at 13 Mks. 8 S. per £ sterling, and the Primage is to be added. The amounts of various charges are given in Current Marks, and the sum is to be exchanged into Banco, at the rate of 123 Marks current for 100 Marks Banco. It may be useful here to observe, that Currency is the coin of Hamburg, and it is rated at 34 Marks current to 1 Cologne Mark weight of fine silver,* while Banco money is valued from bars of Silver, reckoning 27¾ Marks Banco to the same weight of fine Silver. Hence 100 Marks Banco are worth a little less than 123 Marks Current, and the difference between these values is called the Agio. The Stadte Duty is a toll levied upon goods passing up or down the Elbe, at Stadte, a town in the Hanoverian territories.

When the net proceeds of the sales have been found in Hamburg Banco, the Invoice amount in sterling money is to be proportioned upon this result, to find the *arbitrated* rate of exchange yielded by the transaction; and then the rate of the profit or loss is found by the merchant, by comparing it with the *actual* rate of exchange at which he can either draw or have bills remitted to him.

Products.

Net lb. in London	1296	Value	£ 324 0 0
— in Hamburg	1200	—	Bo. Mks. 4800 0
Freight, &c.	Bmf. 23 5	Duty	Ct. Mks. 24 0

Charges Ct. Mks. 46 4 at 123 per cent. = Bmf. 37 10

Rate of Exchange Bmf. 13 12½ Sch. per £ sterling.

Pro Forma Invoice and Account-Sales of Coffee.

In the valuation of the Sales it is to be observed, that although the price is given in Sous of 20 to a Florin, the amount is to be reckoned in Florins and Cents, the Sou being equal to 5 Cents.

The Syndicat is calculated upon the amount of the Duty, which is reckoned upon the entered weight of the goods in Kilogrammes, at 2 Florins per 100 Kilogrammes.

The arbitrated rate of Exchange is to be reckoned as in the last.

Products.

Allowance for weights	270 lb.	Tare	265 lb.
Amount of Sales	Flo. 5023.55	Duty	Flor. 115.54
Syndicat	— 15.02	Commission	— 100.47

Rate of Exchange Flor. 12 30½ Cts. per £ sterling.

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* The Cologne Mark weighs 3608 grains or 7 oz. 10 dwt. 8 grs., and this weight of fine Silver produces 8 oz. 2 dwt. 12 grs. of standard Silver, which, at 5 s. per oz., is worth 40 s. 7½ d. from this valuation the worth of the Mark in Banco and Currency, as well as the par of Exchange, can be easily estimated.

PRO FORMA INVOICE AND ACCOUNT-SALES OF LOGWOOD.

The Genoese weight is given in Cantari of either 100 Rottoli or 150 lb., therefore by reckoning the 218 Cantari 15 Rottoli as 21815 lb., and adding to them their half, they are reduced into lb. The Tare of usage being a per centage on the net weight, it is to be found at the rate of 16 Tare on 116 Gross, for which we are to multiply the Gross by 4, and divide the product by 29, reckoning the remainder as another pound. The valuation is made at 8⅔ Lire per 100 lb.; in which we must notice that 20 Soldi make 1 Genoese Lira, or 1 Lira Fuori Banco, and 12 Denari, 1 Soldo; but the Denari, in this Account-Sales, are throughout rejected. The net proceeds in Lire Fuori Banco, are to be rendered into Lire Nuove and Centisimi, the money of exchange, and also the present money of the Sardinian States, at the rate of 6 Lire F. B. for 5 Lire Nuove, 9 Soldi F. B. being reckoned as 45 Centisimi F. B.

PRODUCTS.

Net	Tons 10 11 2 10	Value	£ 66 2 5
Tare of Usage	lb. 4514	Amount of Sales	Lire 2444 13
Brokerage	Lire 12 4	Commission	- 73 6
Net Proceeds in Lire F. B.	2102 9		

Rate of Exchange L. N. 25.43 per £ sterling.

ACCOUNT-SALES OF SILVER AT HAMBURG.

The weight is given in Cologne Marks, each of 16 Loths, and the Report is given in Assay Loths and Grains of 18 to 1 Loth. The fine weight is to be found, by multiplying the full weight by the number of the Assay Loths, and taking parts for the 4½ Grains and the 1 Grain, and dividing the product by 16, the number of Loths in an Assay Mark. The valuation is made upon the fine weight, in working which, we may assume the Marks and Loths in the weight as so many Marks and Schillings Banco, and multiply them by 27, and take parts for 10½ Sch.; and the calculation is rendered easier by reckoning the 7 L. 17 Gr. as 8 Loths, and allowing 2 Sch. in the amount for the extra Grain.

PRODUCTS.

Amount	Bmf. 306610 14	Commission	Bmf. 1533 1

INVOICE OF SILVER SENT FROM HAMBURG TO PARIS.

The fine weights and the value are to be calculated according to the directions to the preceding Account-Sales.

PRODUCTS.

Fine wt. 1 ingot F. Mks. 114 1 9 7 ingots F. Mks. 855 11 10
Fine weight Mks 969 13 1 at Bmf. 27 8½ Bmf. 26700 4

ACCOUNT-SALES OF SILVER SOLD IN PARIS.

The valuation of Silver in France is usually made on the full weight at tariff prices corresponding with the assay report of the purity, reckoning Silver which is 900-1000 ths fine at 197 Francs per Kilogramme,* and it is sold at a variable premium per Mille, or per 1000 Francs, according to the market value of Silver.

Hence the value of each full weight is to be found by multiplying the price by the number of the Kilogrammes, as Frs. 215.39 by 27.120, and cutting off the lowest three figures in the product. In the same manner the premium is to be found.

The charge for Factage is for Commission at each place of transmissal from Hamburg, by the persons at Osnabruck, Naarden, and Antwerp, (see the last Invoice,) who received and forwarded the consignment.

N. B. The note appended to this Account-Sales, is a memorandum of the result of the transaction. It appears that the amount of the preceding Invoice have been drawn for at 3 days' sight from the date of the Invoice, upon the party in London for whom the purchase was made; and that the net proceeds of this Account-Sales have been remitted from Paris to London, in bills at 1 month's date. According, therefore, to the times at which these bills become due, there is a loss of 29 days' Interest, which, reckoned at 5 per ct., is added to the amount of the purchase, and then this latter amount being subtracted from the proceeds of the sales, shows the net profit to be £ 9 14 9, or nearly ½ per ct.

It forms a useful exercise to compare and note the difference between the French and Hamburg full weights, from the rates of 3608 grains Troy to the Cologne Mark, and 15434 grains to the Kilogramme; and also to compare the relative assays, by reducing the French reports of the fineness into their equivalent Hamburg reports; as for 0.984 fine, which, considered as the decimal of a Mark, and multiplied by 16 to reduce it into Loths, gives 15.744 Loths, or very nearly 15 Loths 13½ Grains.

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* Thus the two given tariff prices may be found by multiplying Frs. 197 by 984 and by 989, and dividing the product by 900.

PRODUCTS.

1 st amount	Frs. 5841.37	2 nd amount	Frs. 43856.68
Premium	273.34	Commission	249.86
Cost in sterling	£ 1935 19 3	Interest	£ 7 13 10
Amount of Sales	1953 7 10	Profit	9 14 9

SALES OF RAISINS SENT FROM LONDON TO ST. PETERSBURG.

In calculating the duty, it must be noticed, that the 31 are Russian pounds, of which 40 make 1 Pood. The Quarantine dues are reckoned upon the amount of the duty, and the Custom House charges upon the amount of both.

PRODUCTS.

Amt. of Sales	Roub. 3400.00	Duty, &c.	Roub. 698.28
Custom H. Charges	27.93	Net P. in ster.	£ 103 15 9

SALES OF CLOTHS AT GIBRALTAR.

It is to be noticed, that 12 Reals make 1 Dollar, and 16 Quartos, 1 Real.

PRODUCTS.

Amt. of Long Cloths	D. 2100 0 0	of Salampores	D. 1012 6 0
Rent and Brokerage each	15 6 12	Commission	77 9 12

Net Proceeds in sterling £ 635 16 3.

SALES OF COFFEE AT BREMEN.

The selling price is here given in Grotes, 72 of which make a Bremen Rix Dollar. The Bremen Rix Dollar is called a Rix Dollar Louis-d'or, because 5 of them are reckoned equal to a German Pistole, or German Louis d'or, or a Prussian Frederick d'or, which being worth nearly 16 s. 6 d. sterling, makes the par of Exchange of Bremen on London about 606 Rix Dollars Louis d'or per £ 100 sterling. The amount of the Freight in sterling is turned into Bremen money at 612 R. D. per £ 100 sterling.

The lighterage is reckoned upon the full weight, here given as 15284 lb. at 2½ R. D. per Last of 400 lb.

PRODUCTS.

Amt. of Sales	R. D. 3376 0	Freight	R. D. 49 41
Lighterage	9 40	Cranage, &c.	4 12
Cooperage	19 32	Brokerage	8 32
Commission, &c.	101 20		

SALES OF SUGAR AT CONSTANTINOPLE.

The weights are here given in Okes. The Oke contains 400 Drams, but it is also nominally divided into 100 parts, each of 4 Drams, 10 Okes are about 28 lb. avoirdupois. The valuation is made in Piastres and Cents, the latter of which are sometimes called Aspers, but correctly, an Asper is the 40 th part of a Piastre.

The amount of the freight in sterling is exchanged into Piastres, at the rate of 74 piastres for £ 1 sterling. The duty is reckoned on the net weight in Cantars, and Okes of 40 to a Cantar, at the rate of 900 Aspers per Cantar; and in this calculation of the duty 120 Aspers are reckoned equal to 1 Piastre.

PRODUCTS.

Amt. of 10 bags	Piast. 2311.55	50 bags	Piast.	11183.77
Freight	508.75	Duty		615.94
Dritti	61.59	Brokerage		269.91
Loss of money	134.95	Commission		269.91

SALES OF SPELTER AT CALCUTTA.

The weight of this Spelter or Zinc in cwt. qr. and lb. is rendered into factory Maunds, Seers, and Chattacks, at the rate of 3 Maunds for 2 cwt. The Maund contains 40 Seers each of 16 Chattacks. The quarter of a cwt. therefore equals 15 Seers, and 7 lb. equal 60 Chattacks.

The valuation is made at 21 current Rupees 6 Annas per Maund, the Rupee being divided into 16 Annas each of 12 Pice; and the amount is rendered into Sicca Rupees, at the Batta or allowance of 16 per cent.; 116 Current Rupees being reckoned equal to 100 Sicca Rupees.

In calculating the discount, 30 days are reckoned as 1 month, and in using Pice, they are not entered lower than 3.

PRODUCTS.

Amt. in Cur. Rupees	31500	4	0	in Sicca Rupees	27155	6	0
Discount	927	12	9	Commission	1311	6	3

INVOICE OF COTTON SHIPPED AT MARANHAM.

In calculating the amount of the Cotton as well as of the duty, it is to be noticed that 32 lb. make 1 Portuguese arroba.

PRODUCTS.

Net Cost	Reis 356.563	Duty	Reis 60.375
Commission	12.797		

SALES OF GUNPOWDER AT NEW YORK.

The auction commission is calculated on the amount of the Powder sold at public auction, and the agent's commission, as well as the charge for Del Credere, is reckoned upon the whole amount of the sales.

The amount of the net proceeds is exchanged into sterling, by reckoning the Dollar at 4 s. 6 d. currency, and then £ 108¾ currency as equal to £ 100 sterling.

PRODUCTS.

21 half Bs.	U. S. D. 525.00	10 half Bs.	U. S. D. 180.00
6 —	102.00	13 —	208.00
3 whole Bs.	96.00	Inspection	28.00
Duty	224.00	Auct. Commission	14.65
Commission	27.78	Del Credere	27.78

Remittance in sterling £ 154 16 7.

PRO FORMA INVOICE AND ACCOUNT-SALES OF GERMAN WOOL.

The object of this and the two following statements,* is to find the least arbitrated price at which the respective goods must sell for to cover the cost prices and the charges attendant on the shipment and sales. Thus in the present statement it is required to find the least price at which German Wool bought in London must sell for in Boston, to cover the cost price in London and the usual charges in the two countries.

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* These statements, with some slight variations, have been extracted by the kind permission of the Author, from the first edition of a highly valuable and elaborate work, called "Statements and Calculations on the Commerce of the United States," by W. F. Reuss, Esq.

The London Invoice amount is turned into United States currency at the exchange of 108 per cent. on the fixed par of 4 s. 6 d. per Dollar, or 40 Dollars for £ 9 sterling, or by first exchanging the sterling amount into U. S. Dollars and Cents, and then adding to the result the premium of 8 per cent. The Freight is valued at the fixed par without any premium. The Duty is calculated in two parts; the first being upon the entered weight, at 4 Cents per lb., and the second upon the London Invoice value, exclusive of Insurance. The latter is calculated by reducing the value here taken at £ 517 into U. S. currency, at the Custom House Par of U. S. D. 480 Cents per £ sterling, and taking 40 per cent. on this amount. The per centage charges (including 4½ per cent. for loss of Interest for the time that may be considered to elapse between the time of payment for the goods in London, and the time when the remittances for the proceeds will arrive at maturity) are calculated upon the amount of the cost and all charges in the following manner: The amount of the Invoice and the preceding charges is U. S. D. 3945.32, and these per centages amount to 12⅛, which taken from 100 leave net 87⅞; we therefore say, as 87⅞ net is to 100 gross, so are U. S. D. 3945.32 net to U. S. D. 4489.69 gross, and upon this amount the per centages are to be calculated. Lastly, to find the arbitrated price required, we take the estimated net Boston weight, and say, if 7952 lb. should sell for U. S. D. 4489.69, what should 1 lb. sell for? The answer to which proportionate statement gives the rate of 56$\frac{46}{100}$ Cents per lb., that rate being sufficient, as appears by the result, to cover the cost price and all the estimated charges.

PRODUCTS.

Gross amt. in London	£ 525 5 4	Net amount	£ 499 0 0
Insurance	8 6 6	Commission	12 16 6
Freight & Prim.	U. S. D. 79.50	Duty on wt.	U. S. D. 320.00
Duty on Value	992.64	Insurance	5.61
Commission	224.48	Guarantee	112.24
Interest	202.04	Amount of Charges	1965.71

Rate of remunerating price Cents 56$\frac{46}{100}$ per lb.